ELECTROMAGNETIC SCATTERING BY HEMISPHERICAL BOSSES
ON AN INFINITE PLANE SURFACE

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INTRODUCTION

An analytic solution to the problem of multiple scattering of a plane electromagnetic wave by an array of hemispherical bosses on a perfectly conducting infinite plane surface is obtained by using the solution of the scattering by an array of full spheres, on the basis of an image technique. The solution of this problem is of relevance in analyzing the scattering by three-dimensional rough surfaces. The system considered is replaced by the array of complete spheres in the absence of the conducting plane, but with the given incident plane wave and also a supplementary, image plane wave, chosen such that the boundary conditions for the total field are satisfied at all the points where the conducting plane is located in the original problem. Numerical results are presented for the normalized backscattering cross section versus the incident angle for different system of spheres.

FORMULATION OF THE PROBLEM

Consider a linear array of N perfectly conducting hemispherical bosses with different radii and unequal spacing, along the z-axis, lying on the perfectly conducting y-z plane, as shown in Fig. 1(a). A plane wave of unit electric field intensity, whose propagation vector \( \mathbf{k} \) lies in the x-z plane and makes an angle \( \alpha \) with the z-axis, is incident on the hemispheres. Its incident electric field is assumed to be in the \( \mathbf{\hat{y}} \) direction. Thus [1,2]

\[
\mathbf{E}_i = e^{j \mathbf{k} \cdot \mathbf{r}} \mathbf{\hat{y}}
\]

(1)

where

\[
\mathbf{k} = k \sin \alpha \mathbf{\hat{x}} + k \cos \alpha \mathbf{\hat{z}}
\]

with \( k \) being the wave number. An image technique is applied, by which the conducting hemispherical bosses are replaced by complete spheres in free space, in the presence of the original incident plane wave and of a second plane wave with an electric field intensity \( \mathbf{E}_i' \) of unit amplitude, oriented in the \( -\mathbf{\hat{y}} \) direction, the propagation vector \( \mathbf{k}' \) being at an angle \( \alpha \) with the z-axis below the y-z plane, as shown in Fig. 1(b). Thus

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\[ \vec{E}_i = -e^{jk' \cdot \vec{r}} \]  

where

\[ k' = k \sin(-\alpha) \hat{\phi} + k \cos \alpha \hat{\phi} \]

The incident electric field is expanded in terms of spherical vector wave functions, defined with respect to the center of the pth sphere, i.e.

\[ \vec{E}_i(r_p, \theta_p, \phi_p) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ P_p(m,n) \overline{N}^{(1)}_{mn}(r_p, \theta_p, \phi_p) + Q_p(m,n) \overline{M}^{(1)}_{mn}(r_p, \theta_p, \phi_p) \right] \]

where \( p \) takes integer values from 1 to \( N \), while \( \overline{M}^{(1)}_{mn} \) and \( \overline{N}^{(1)}_{mn} \) are spherical vector wave functions of the first kind defined in terms of spherical Bessel functions. The expansion coefficients in equation (3) are obtained by using the orthogonality properties of the vector wave functions [3], i.e.

\[ P_p(m,n) = -j^n e^{jkd_p \cos \alpha} \frac{(2n+1)(n-m)!}{n!(n+m)!} P_n^m(\cos \alpha) \]

\[ Q_p(m,n) = -j^n e^{jkd_p \cos \alpha} \frac{(2n+1)(n-m)!}{n!(n+m)!} \frac{\partial}{\partial \alpha} P_n^m(\cos \alpha) \]

The expansion of the secondary incident electric field is obtained from equation (3) by replacing the incident angle \( \alpha \) by \( -\alpha \) in equations (4) and (5), in the form

\[ \vec{E}_i'(r_p, \theta_p, \phi_p) = -\sum_{n=1}^{\infty} \sum_{m=-n}^{n} (-1)^{(m+n)} \left[ P_p(m,n) \overline{N}^{(1)}_{mn}(r_p, \theta_p, \phi_p) + Q_p(m,n) \overline{M}^{(1)}_{mn}(r_p, \theta_p, \phi_p) \right] \]

The scattered field from the pth sphere can be expanded in terms of spherical wave functions as

\[ \vec{E}_s(r_p, \theta_p, \phi_p) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ A_{pE}(m,n) \overline{N}^{(3)}_{mn}(r_p, \theta_p, \phi_p) + A_{pM}(m,n) \overline{M}^{(3)}_{mn}(r_p, \theta_p, \phi_p) \right] \]

where \( A_{pE} \) and \( A_{pM} \) are unknown scattering coefficients due to TM waves and TE waves, respectively. \( \overline{M}^{(3)}_{mn} \) and \( \overline{N}^{(3)}_{mn} \) are spherical vector wave functions of the third kind which represent outgoing waves, associated with the spherical Hankel functions of the first kind.

The boundary condition for the total electric field at the surface of the pth sphere requires that

\[ \hat{n}_p \times \vec{E}_{total}(r_p, \theta_p, \phi_p) = 0 \quad ; \quad p = 1,2, \ldots, N \]

where \( \hat{n}_p \) is the outward unit normal to the surface of the pth sphere, and \( \vec{E}_{total}(r_p, \theta_p, \phi_p) \) is the total electric field which is given by

\[ \vec{E}_{total}(r_p, \theta_p, \phi_p) = \vec{E}_i(r_p, \theta_p, \phi_p) + \vec{E}_i'(r_p, \theta_p, \phi_p) + \sum_{q=1}^{N} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ A_{qE}(m,n) \overline{N}^{(3)}_{mn}(r_q, \theta_q, \phi_q) + A_{qM}(m,n) \overline{M}^{(3)}_{mn}(r_q, \theta_q, \phi_q) \right] \]

Substituting (9) into (8) and using the translation addition theorem [4], we obtain finally a system of equations in terms of the unknown scattered field coefficients,
which has the matrix form
\[
\vec{A} = \vec{L} + T \vec{A}
\]  
(10)

where \(\vec{A}\) and \(\vec{L}\) are the column matrices of the unknown scattered field coefficients and incident field coefficients, respectively, and \(T\) is a square matrix containing the translation addition coefficients. The solution of equation (10) yields the scattered coefficients in equations (7) and (9), i.e.
\[
\vec{A} = (I - T)^{-1} \vec{L}
\]  
(11)

It can be shown that the \(y\)- and the \(z\)- components of the total field in (9) are zero at \(x=0\), and therefore the boundary conditions are fully satisfied in the original system.

RESULTS AND CONCLUSIONS

Typical numerical results are presented in Figure 2 which shows the normalized backscattering cross section versus the angle of incidence \(\alpha\) for two systems of hemispherical bosses on an infinite perfectly conducting plane. Figure 2(a) presents the normalized backscattering cross section for a linear array of equispaced three identical hemispherical bosses. The electrical radius of each hemispherical boss is \(k a=0.5\), while the electrical separation between successive hemispherical bosses is \(k d=2.0\). The solid curve shows the scattering by the array of full spheres while the dotted curve shows the scattering by the hemispherical bosses. It can be seen that the magnitude of the backscattering cross section is approximately zero at small angle of incidence since the incident field and its image are equal in magnitude and opposite in phase. Also the magnitude of the backscattering cross section for complete spheres is higher than that due to the presence of hemispherical bosses. Figure 2(b) presents the scattering by a system of five hemispherical bosses of the same electrical radii and separation as in Fig. 2(a).

REFERENCES


Fig. 1. Geometry of the scattering system: (a) hemispherical bosses (b) full spheres.

Fig. 2. Normalized backscattering cross section patterns versus $\alpha$ for a system of (a) three and (b) five identical hemispherical bosses with $ka=0.5$, $kd=2.0$. 