IMPROVED WEIGHTED POINT-MATCHING TECHNIQUE FOR TM-WAVE SCATTERING BY DIELECTRIC CYLINDERS

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1. INTRODUCTION
In this paper, the scattering of TM plane electromagnetic waves by dielectric cylinders is considered and a novel weighted point-matching technique is proposed in order to make the matrix involved in the integral equation formulation a symmetric matrix. Applying the Cholesky decomposition, this symmetrization allows the solution of the matrix equations with roughly half the storage and half the computation time as compared to those for asymmetrical ones. Numerical results are presented to illustrate the procedure.

2. INTEGRAL EQUATION FORMULATION
The 2-D integral equation satisfied by the total electric field intensity for the scattering of TM plane electromagnetic waves by a dielectric cylinder of an arbitrary cross-sectional shape can be written as [1]

\[ E(r) + \frac{(jk^2/4)}{\int_0^{2\pi a_j} \int_0^{R_{ij}} H_0^{(2)}(kr) \left[ \epsilon(r') - 1 \right] E(r')r' \, dr' \, d\phi} = E_i(r) \] (1)

where the integration is performed over the cylinder cross section and

\[ R = |r - r'| \] (2)

Using the point-matching technique and approximating the integral over each cell by the integral over an equivalent small circular region of the same area, with radius \( a_j \), as

\[ \frac{(jk^2/4)}{\int_0^{2\pi a_j} \int_0^{R_{ij}} H_0^{(2)}(kr) \, r' \, dr' \, d\phi} = \left\{ \begin{array}{ll}
(j/2)\pi k a_j \, J_1(ka_j) H_0^{(2)}(kR_{ij}) & j \neq i \\
(j/2)\pi k a_i H_1^{(2)}(ka_i) + 1 & j = i
\end{array} \right. \] (3)

we obtain finally a system of linear algebraic equations in the form

\[ \sum_{j=1}^{N} (j/2)\pi k a_j (\epsilon_\eta - 1) J_1(ka_j) H_0^{(2)}(kR_{ij}) E_j \\
+ \left\{ 1 + (\epsilon_\eta - 1)(j/2)\pi k a_i H_1^{(2)}(ka_i) + 1 \right\} E_i = E_i^i, \quad i = 1, 2, \ldots, N \] (4)

where \( R_{ij} = |r_i - r_j| \) and \( E_i^i \) and \( E_i \) are the values of the incident electric field intensity and the total electric field intensity, respectively, associated with the
The system of equations in (4) can be written in a matrix form as

\[ [Z_0] \mathbf{E} = \mathbf{E}^i \]  

(5)

where \( \mathbf{E}^i \) and \( \mathbf{E} \) are column matrices whose elements are the incident and the total fields \( E^i_j \) and \( E_i \), respectively, and \([Z_0]\) is a square matrix which is not symmetric.

### 3. WEIGHTING FACTOR FOR SYMMETRIZING \([Z_0]\)

The computer time and memory required to solve the system of linear equations involved can be reduced significantly if the matrix \([Z_0]\) in (5) were transformed into a symmetric one, so that the Cholesky decomposition could be employed. To get such a symmetric matrix, we first multiply both sides of equation (4) by \( k a_i J_1(k a_i) \). We have

\[
\sum_{j=1}^{N} (j/2) \pi k^2 a_i a_j J_1(k a_i) J_1(k a_j) H^{(2)}_0(k R_{ij})(\epsilon_{\gamma_i} - 1) E_j \\
+ k a_i J_1(k a_i) [1/(\epsilon_{\gamma_i} - 1) + (j/2)\pi k a_i H^{(2)}_1(k a_i) + 1](\epsilon_{\gamma_i} - 1) E_i \\
= k a_i J_1(k a_i) E^i_i, \quad i = 1, 2, \ldots, N
\]  

(6)

This is equivalent to using testing weighted Dirac delta functions of the type \( k a_i J_1(k a_i) \delta(r_i - r_j) \), \( i = 1, 2, \ldots, N \)

(7)

After transferring the factor \( \epsilon_{\gamma_i} - 1 \) into the matrix \( \mathbf{E} \), we derive the new system of linear equations

\[ [\mathbf{Z}] \tilde{\mathbf{E}} = \tilde{\mathbf{g}}^i \]  

(8)

with

\[
Z_{ij} = \begin{cases} 
(j/2)\pi(k a_i)(k a_j)J_1(k a_i)J_1(k a_j) H^{(2)}_0(k R_{ij}), & i \neq j \\
 k a_i J_1(k a_i) [1/(\epsilon_{\gamma_i} - 1) + (j/2)\pi k a_i H^{(2)}_1(k a_i) + 1], & i = j 
\end{cases} \quad (9)
\]

\[
\tilde{g}_i = (\epsilon_{\gamma_i} - 1) E_i 
\]

(10)

\[
\tilde{g}_i = k a_i J_1(k a_i) E^i_i 
\]

(11)

The far field scattering cross section width is obtained from

\[
\sigma(\phi) = \frac{\pi^2 k}{|E^i|^2} \left| \sum_{i=1}^{N} (\epsilon_{\gamma_i} - 1) E_i a_i J_1(k a_i) e^{i k(x_i \cos \phi + y_i \sin \phi)} \right|^2
\]

(12)

where \( x_i \) and \( y_i \) are the Cartesian coordinates of the center of the \( i \)-th cell.

Since \([Z]\) is a symmetric matrix, the Cholesky decomposition [2] is used for the solution of the matrix equation in (8), with the algorithm described in [4].
such that the CPU time and the memory space are reduced to roughly one half [5,6].

4. NUMERICAL RESULTS

The proposed procedure was applied to a 2-D lossless dielectric circular shell illuminated by a plane wave. A number of 120 uniformly distributed cells was used. The scattering cross section width is shown in Fig. 1(a) and the electric field distribution in the shell is shown in Fig. 1(b). They agree with the data in [1] quite well. The results for a dielectric cylinder of a square cross section illuminated by a plane wave are presented in Figs. 2(a) and 2(b). In this case a number of 196 cells was used.

It is worth mentioning that for the case of TE-wave scattering, the initial matrix is symmetric[7] and the Cholesky decomposition can be applied directly.

5. CONCLUSION

The integral equation for 2-D TM-wave scattering of dielectric cylinders has been solved by the method of moments with a new weighted point-matching technique. By introducing a supplementary weighting factor and by transferring the factor \( \epsilon_r - 1 \) into the corresponding column matrix, the square matrix involved becomes symmetric. The resultant system of linear equations is solved by the Cholesky decomposition. As a consequence, this procedure yields a substantial reduction in the computational effort.

REFERENCES


Outer Radius = 0.3λ
Inner Radius = 0.25λ
$\varepsilon_r = 4$

$E'$

$\sigma(\phi)/\lambda$

$\phi$ (degree)

(a)

(b)

Fig. 1. Normalized scattering echo width and total electric field in a dielectric circular shell: —— computed results; + + + results in [1].

$2a = 0.32\lambda$
$\varepsilon_r = 4$

$\sigma(\phi)/\lambda$

$\phi$ (degree)

(a)

(b)

Fig. 2. Normalized scattering echo width and total electric field distribution within a dielectric cylinder of square cross section.