INTRODUCTION

A new approach to microwave imaging of dielectric bodies is presented. The proposed procedure is based on the stochastic inverse of a linear system matrix derived from the coupled nonlinear integral equations used in microwave imaging by employing an equivalent current density. The prior knowledge required in this method is provided by a first estimate of the reconstructed values obtained by applying the Tikhonov regularization. It is found that, by the proposed method, the dielectric permittivities of two-dimensional bodies can be reconstructed with very high accuracy, in a reduced number of iterations, even for the measured field data containing a high level of noise.

IMPLEMENTATION OF STOCHASTIC INVERSION

Consider a two-dimensional dielectric body illuminated by a TM incident wave. The electric field intensity scattered by the body can be expressed as

\[ E_s(x, y) = \int_{D} J_e(x', y') G(x, y; x', y') dx' dy' \]  

where the integration is over the investigation domain, \( G(x, y; x', y') \) is the Green function for an unbounded homogeneous space, \( E_s \) is the scattered field intensity, and \( J_e \) is an equivalent current density related at any point inside the body to the local total electric field intensity by

\[ J_e(x, y) = j(\omega \mu_0)^{-1}[k^2(x, y) - k_0^2]E_s(x, y) \]  

with \( k \) and \( k_0 \) being the wave numbers inside and outside the dielectric body, respectively. In electromagnetic imaging problems the scattered field is measured at a finite number of points in the observation domain. The integral equation (1) can, in principle, be solved by the application of the moment method, but this inverse problem is, in general, ill-posed. Considering the errors and the noise in the scattered field, the matrix equation associated to (1) can be written as
\[ [E^t] = [G][J_e] + [N] \]

where \([E^t]\) is a vector of a dimension equal to the number of detectors outside the investigation domain, \([G]\) is a rectangular matrix, \([J_e]\) is a vector of a dimension equal to the number of cells used to discretize the body, and \([N]\) a vector of the same dimension as \([E^t]\), indicating the errors and the noise in the scattered field. Once the equivalent current \([J_e]\) is determined, the total electric field inside the scatterer is obtained by the superposition of the incident field and the "scattered" field computed from (1) and, then, the permittivity at any point within the dielectric body is obtained from (2) through \(k(x, y)\).

In this paper, we solve (3) with the help of the stochastic inversion. The algorithm is constructed based upon statistical considerations, with \([J_e]\) and \([N]\) simulated by stochastic or random processes. Under the assumption that the random variables in \([J_e]\) and \([N]\) are uncorrelated, the reconstruction algorithm employs the following expression for evaluating \([J_e]\)

\[ [J_e]_r = [R_J][G]^H([G][R_J][G]^H + [R_N])^{-1}[E^t] \]  

(4)

where \([R_J]\) is the correlation matrix of \([J_e]\), \([R_N]\) is the correlation matrix of \([N]\), and \(H\) denotes the conjugate transpose of a matrix. In our problem, the additive noise can be assumed to be uncorrelated and isotropic, and thus \([R_N]\) is a diagonal matrix

\[ R_{N_{ij}} = \sigma_\Delta^2 \delta_{ij} \]  

(5)

with \(\sigma_\Delta^2\) being available from a prior knowledge and \(\delta_{ij}\) being the Kronecker delta symbol. Since the representative random vector of the \([J_e]\) is assumed to be uncorrelated, its correlation matrix \([R_J]\) is approximated to be [1]

\[ R_{J_{ij}} = |J_{ei}|^2 \delta_{ij} \]  

(6)

where, in our case, no a priori information is available about the distribution of the equivalent current. In this paper, we use the Tikhonov regularization technique to obtain a first estimate of \([J_e]\) [2],[3], which gives the diagonal matrix elements \(J_{ei}\) in (6). The subsequent evaluations of \([J_e]\) are obtained from (4), with (5) and with \([R_J]\) determined in terms of the latest values of the current density. The iteration process continues until a stable solution is obtained.

**NUMERICAL SIMULATIONS**

Consider an investigation domain of a square cross section of \(5/3\lambda_0\) to each side, illuminated by a TM plane electromagnetic wave with an electric field intensity
of unit amplitude, where $\lambda_0$ is the wavelength of the incident wave. 25 cells are used to discretize the domain and 25 equally spaced detectors are located on a concentric circular loop of diameter $3\lambda_0$. The electric field values measured by the detectors are provided by direct scattering computation [4], and the presence of noise in the scattered field is simulated by adding to the real and imaginary parts of the field values two independent sequences of Gaussian random variables of zero mean value.

Fig. 1(a) presents the original image with two scatterers of relative permittivity $\varepsilon_r = 3$ occupying two cells in the investigation domain. Fig. 1(b) shows the image developed from the reconstructed dielectric permittivities for a signal-to-noise ratio (S/N) of 20dB in only three iterations. Fig. 2 gives the number of iterations versus the relative mean square error (MSE) which is defined as

$$
\delta_\varepsilon = \sqrt{\frac{\sum_{k=1}^{q} (\varepsilon_{r_k} - \hat{\varepsilon}_{r_k})^2}{q}}^{1/2}
$$

where $\varepsilon_{r_k}$ and $\hat{\varepsilon}_{r_k}$ stand for the values of the original relative permittivity and of its reconstructed value in the k-th cell, respectively, with $q$ being the total number of cells. Only two iterations are needed to reach practically a stable value of permittivity. For S/N=20dB the MSE stays at 1.12% after three iterations, while in the 40dB case it drops to 0.1% after the same number of iterations. The tests performed so far show that the images developed by the proposed method are of a very high resolution even for the scattered field containing a high level of noise (10 percent uncertainties in the measured scattered field values) after a relatively small number of iterations.

REFERENCES

Fig. 1. Two dielectric cylinders of square cross section: (a) original image; (b) microwave image for \( S/N=20\text{dB} \), after 3 iterations.

Fig. 2. Relative mean square error versus the number of iterations.