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A new method for the finite-element analysis of unipolar DC Corona ionized fields.

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ABSTRACT

Solution of the DC corona ionized field problem is, in general, extremely difficult because of the nonlinearity of the differential equations describing it. Furthermore, except for the potential, the boundary conditions at the coronating electrodes are not known. Various simplifying assumptions have been employed to determine corona power loss and field quantities for practical transmission line configurations. In this paper, a new method based on a finite-element iterative algorithm, without invoking simplifying assumptions used so far, is proposed. It consists of a simple and realistic modelling of the initial space charge distribution, and of a new, efficient updating criterion. The method is applied to the unipolar DC field in a cylinder-to-plane configuration and the numerical results agree well with experimental data. Comparison with previous methods shows that the proposed method is highly efficient and more accurate, yielding also the correct distribution of the electric field intensity over the coronating conductor surface.
The corona performance of high voltage direct current (HVDC) transmission lines is one of the critical criteria in their design due to associated, undesirable effects, such as electromagnetic interference and audible noise, and to supplementary power losses. The presence of ionic charges in the interelectrode region modifies substantially the electrostatic field and, consequently, has an important environmental impact [1, 2].

First attempts to solve the nonlinear partial differential equations describing the corona ionized fields were based on the Deutsch assumption [3], which states that the presence of the space charge affects only the magnitude and not the direction of the electric field. As expected, the corresponding methods of analysis proposed [4, 5] yield acceptable results only at low corona levels. Recently, the Deutsch assumption has been waived in the analysis of corona ionized fields from HVDC transmission lines by applying iteratively the finite element method (FEM). Most of the techniques proposed [6,7] are constructed on the basis of Kapitza's assumption [10] that the magnitude of the electric field intensity at the coronating conductor surface remains constant at its onset value regardless of the level of the applied voltage. The validity of these assumptions, as applied to the analysis of HVDC transmission lines, has been questioned in literature [11-14]. It was claimed that they could have a significant effect on the magnitude and spatial distribution of the ionic space charge, electric field and current density in the close vicinity of these lines [11-13]. In fact, the electric field intensity at the coronating conductor surfaces does depend on the applied voltage [15, 16] and the field quantities in the interelectrode region, at a given voltage, are significantly influenced by the field distribution around the conductors in corone. On the other hand, the experience of the authors of this paper shows that finite-element techniques based on an imposed, constant conductor surface potential gradient yield results of a limited accuracy, which require, in general, a huge amount of computation.

The method proposed here employs a novel finite-element based iterative algorithm, which uses isoparametric finite elements and a new updating criterion, in order to achieve accurate values of the nodal potentials with a substantially reduced computational effort. Following an iterative procedure, with an approximate initial volume charge distribution corresponding to the corona onset electric field intensity, it finally yields all the field quantities, including the actual potential gradient variation around the coronating conductor surface. The method is illustrated for the case of the unipolar DC corona field of a circular cylindrical conductor parallel to the ground plane.

**FEM FIELD ANALYSIS**

The general equations governing the steady-state unipolar DC corona field represent the Gauss's law, the relationship between the current density and field intensity and the current continuity equation, respectively:

\[ \nabla E = \frac{\rho}{\varepsilon_0} \]  
\[ J = \kappa \varepsilon_0 E \]  
\[ \nabla J = 0 \]

where the electric field intensity can be derived from a scalar potential \( \phi \).

\[ E = -\nabla \phi \]

\( \varepsilon_0 \) is the permittivity of free space and \( \kappa \) the ionic mobility, which is assumed to be constant. From equations (1) and (4),

\[ \phi(\text{VSS}) = \frac{\rho_0}{\varepsilon_0} \]

and from equations (2) - (4),

\[ \phi(\text{COM}) = 0 \]

with the volume charge density \( \rho \) being an unknown function of point. Equations (5) and (6) are equivalent to equations (1)-(4) and their simultaneous solution for \( \phi \) and \( \rho \) provides the solution to the corona problem.

The following boundary conditions are used:

- the potential on the coronating conductor is given,

\[ \phi = \phi_0 \]

- the potential on the ground plane

\[ \phi = 0 \]

- since for transmission line configurations the domain is unbounded, an artificial boundary is placed sufficiently far away from conductors, with the potential on that boundary assumed to be approximately equal to the corresponding electrostatic value,

\[ \phi = \phi_{es} \]

The boundary conditions (7)-(9) are not sufficient for obtaining a unique solution to the problem. As shown below, in the proposed method only an initial distribution of charge within the interelectrode region, which is sufficiently close to that corresponding to the corona onset value of the conductor potential gradient, is needed when starting the iterative procedure, without being necessary to specify any other boundary conditions. The algorithm will modify this initial distribution at each iteration, yielding finally the solution to the problem.

The coupled partial differential equations (5) and (6) are solved iteratively by implementing a standard two-dimensional FEM technique [17, 18], with isoparametric quadratic elements.

A semi-automatic mesh generation routine was developed and used for the problem region. Initially, a number of equidistant nodes is chosen on the coronating conductor contour. Starting from one conductor node, the subsequent nodes are placed approximately on the same electrostatic field line. The second node is removed from the first one by a distance equal to \( r_{A} \), where \( r \) is the conductor radius and \( r_{A} \) is the angle subtended at the conductor centre by a chord joining two consecutive conductor nodes. Any subsequent node is at a distance equal to \( r_{A} r_{1} \) from the previous one, where \( r_{1} \) is the radial distance from the latter node to the conductor centre and \( r_{1} \) is a parameter that controls the mesh growth pattern. A value of \( r_{1}=1.5 \) was found to result, generally, in good element shapes throughout the whole mesh, for problems relative to transmission line configurations.

The initial distribution of charge density \( \rho \) at the finite-element nodes is obtained by exploiting the way in which the mesh is constructed. As mentioned above, the mesh is generated by tracing, approximately, the electrostatic field lines starting from the conductor surface towards the outer boundary. The length of each field line is considered to be equal to the outer radius of a coaxial cylinder system whose inner radius is the conductor radius. With the potential on the inner conductor taken equal to the approximate corona onset value given by the empirical Peck's formula [19], the known analytical solution for the coaxial cylinder geometry [20] is then applied to obtain the initial space charge distribution along that line. This calculation is repeated for all the field lines in order to obtain the charge distribution

\[ \rho(x, y) = \rho_{0} \]
at all the nodes. Such a procedure for determining the initial charge distribution is much simpler and easier to implement than previous techniques [21, 9].

The proposed algorithm is based on the simultaneous solution of equations (5) and (6), iteratively. Each equation is solved separately by applying the FEM, with the boundary conditions mentioned before. The algorithm begins by using the initial distribution of space charge density \( \rho \). Since this distribution is not the one corresponding to the actual configuration, the solution \( \phi \) from equation (5) and \( \rho \) from equation (6) will differ. The charge distribution is then updated at each node according to the following criterion:

\[
\rho_{\text{new}} = f(E_{1c}, E_{2c}, \rho_{\text{old}}) + g(\phi_1, \phi_2, V, \rho_{\text{old}})
\]  

(10)

where \( \rho_{\text{new}} \) is the updated volume charge density at the node, \( \rho_{\text{old}} \) is the charge density at the same node used in the previous iteration, \( E_{1c} \) and \( E_{2c} \) are the electric field intensities determined from equations (5) and (6), respectively, at the node on the conductor surface lying on the same field line as the node considered, and \( \rho_c \) is the volume charge density at this node on the conductor surface. For the unipolar DC field case, the functions \( f \) and \( g \) are chosen to be

\[
f = \rho_c \left[ \frac{E_{1c}}{E_{av}} \right] \left[ \frac{E_{2c}}{E_{av}} \right] b
\]

(11)

and

\[
g = c \rho_{\text{old}} \frac{\phi_1 - \phi_2}{\phi_{av}} \exp \left[ \frac{V - \phi_{av}}{dV} \right]
\]

(12)

in which \( V \) is the applied voltage, \( \phi_{av} \) is the arithmetic average of \( \phi_1 \) and \( \phi_2 \), \( E_{av} \) is the arithmetic average of \( E_{1c} \) and \( E_{2c} \), and \( a, b, c, d \) are constants determined so that the convergence of the iterative process is as rapid as possible. The updating criterion (10)-(12) reflects the fact that the space charge distribution is affected not by the conductor surface potential gradient alone, or by the local potential alone, but it is actually affected by their combined effect. This is an improvement over previous updating criteria [7, 9], where only the effect of the corona onset potential gradient alone, or that of the local potential alone, were taken into account.

Following the first iteration, which uses the initial distribution, \( \rho \), the FEM solution of equations (5) and (6) is obtained again by using the new distribution \( \rho_{\text{new}} \). The iterative process is performed by continuously inserting into the updating criterion (10)-(12) the most recent values of all the corresponding quantities. In order to ensure an accurate solution for all the field quantities, this process is continued until the following conditions are satisfied simultaneously at all the nodes except those lying on the artificial boundary:

\[
|\phi_1 - \phi_{av}| < \tau_1 \phi_{av},
\]

(13)

\[
|\rho_{\text{new}} - \rho_{\text{old}}| < \tau_2 \rho_{\text{new}},
\]

(14)

\[
|E_{1c} - E_{av}| < \tau_3 E_{av},
\]

(15)

in which \( \tau_1, \tau_2 \) and \( \tau_3 \) are small deviations specified in terms of the desired accuracy.

It should be noted that the updating formulas (10)-(12) do not contain an imposed value for the electric field intensity on the coronating conductor surface, therefore the presented algorithm does not rely on Kaptsov's assumption. Only the corona onset value of the conductor potential gradient is necessary for determining the initial space charge distribution, which is used for the first iteration. In all the subsequent iterations the field intensity on the surface of the coronating conductors is let free to "float" such that all the field quantities will satisfy the problem equations with a desired accuracy at all the nodes. This "relaxation" procedure yields, finally, the actual variation of the electric field intensity over the coronating conductor surface. As shown in the following section, the theoretical results agree well with existing experimental data.
For calculating the initial volume charge distribution a value of 50.84 kV/cm was used for the corona onset potential gradient on the cylinder surface [19]. To achieve a good rate of convergence for the iterative algorithm, the numerical values for the constants in formulas (10)-(12) were chosen as follows: $a=1$, $b=0.8$, $c=0.01$, $d=0.21$. Highly accurate results were obtained by imposing the following relative errors in equations (13)-(15): $\epsilon_1=0.025$, $\epsilon_2=0.01$, $\epsilon_3=0.015$. 

Fig. 3 Distributions over the ground plane for $V=200$ kV:
--- Computed results by presented method
--- Computed results with Kaptsov's assumption
AAA Experimental values [22]

Fig. 4 Distributions around the conductor periphery for $V=200$ kV
The convergence of the proposed iterative algorithm is illustrated in Figures 5, 6 and 7. In Figures 3b and 3c computed results by the proposed method are compared with those obtained by using Kapteyn's assumption, for the same mesh and the same number of iterations, and with experimental data [22]. The results obtained by the method presented in this paper are in better agreement with experimental values. Furthermore, as shown in Figure 4b, the potential gradient is not constant over the coronating conductor periphery, presenting a variation of about 4%, with the minimum value being at the bottom of the conductor. On the other hand, Figures 4b and 5 show that the average value of the electric field intensity at the surface of the conductor in corona drops by about 9% from its onset value, which is in agreement with available experimental observations [15, 16].

The variation of charge density, electric field intensity and current density over the ground plane and around the conductor periphery is presented in Figures 3 and 4, respectively. In Figures 3b and 3c computed results by the proposed method are compared with those obtained by using Kapteyn's assumption, for the same mesh and the same number of iterations, and with experimental data [22]. The results obtained by the method presented in this paper are in better agreement with experimental values. Furthermore, as shown in Figure 4b, the potential gradient is not constant over the coronating conductor periphery, presenting a variation of about 4%, with the minimum value being at the bottom of the conductor. On the other hand, Figures 4b and 5 show that the average value of the electric field intensity at the surface of the conductor in corona drops by about 9% from its onset value, which is in agreement with available experimental observations [15, 16].

The convergence of the proposed iterative algorithm is illustrated in Figures 5, 6 and 7.

![Fig. 5 Average electric field around the conductor surface in terms of the number of iterations](image)

![Fig. 6 Maximum percentage deviation in nodal potentials versus the number of iterations](image)

CONCLUSIONS

A new method for the finite-element analysis of unipolar DC corona ionized fields has been developed. The method does not rely on simplifying assumptions used in methods proposed so far. It employs a simple procedure for determining the initial charge distribution within the interelectrode region and an iterative algorithm based on a new updating criterion. The "relaxation" of the potential gradient on the conductor surface from its corona onset value yields a relatively rapid convergence of the iterative process and also a distribution of the corona conducting potential gradient in good agreement with experimental data. Comparison with previous techniques shows that the method presented in this paper is more accurate and computationally much more efficient. The authors intend to extend this general method to the more practical case of bundled conductors.

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REFERENCES


