SINGLE SURFACE INTEGRAL EQUATION FOR WAVE SCATTERING BY MANY-BODY SYSTEMS

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ABSTRACT
Analysis of wave scattering by a system of parallel, homogeneous dielectric cylinders of arbitrary cross section is performed by means of an uncoupled surface integral equation satisfied by a single source which is distributed on the cylinder surfaces. This integral equation is constructed by replacing the original problem by an equivalent problem in a manner which is different from that used in the classical equivalence theorem. The physical fields are expressed simply in terms of the single source function. Numerical results demonstrate the efficiency of the single-source surface integral equation method with respect to that of the volume integral equation method.

INTRODUCTION
Construction of new surface integral equations for wave scattering problems is motivated by the possibility of substantially increasing the computational efficiency of the associated algorithms as compared to that corresponding to volume integral equation methods. In the latter methods all media are replaced by free-space and by an equivalent distribution of polarization current [1]. This permits the modeling of complicated nonhomogeneous objects, but at the expense of creating a large number of unknowns. Surface integral equation methods reduce the dimensionality of the problem by using only surface distributions of electric and magnetic currents on the interfaces between homogeneous regions [2]. However, typical surface integral equation methods become cumbersome due to the coupling of the equations which are expressed in terms of both electric and magnetic current. In such methods, the fields within a homogeneous region are expressed only in terms of the electric and magnetic currents on the enclosing surface. These currents must therefore represent all the complexity of the surrounding media; this is an unnecessary modeling burden.

Recently, a single surface integral equation was constructed for wave scattering by a single simply-connected homogeneous dielectric [3]. In the present paper, the single surface integral equation is rederived for application to the problem of two-dimensional wave scattering by an arbitrary number of distinct homogeneous dielectric cylinders. The field within each cylinder is calculated by using a single source which is distributed on all material interfaces in the many-body system. In this sense, the solution differs greatly from that obtained by coupled surface integral equation methods and is reminiscent of surface integral formulas in electrostatics where all media are replaced by free-space with a source distribution over interfaces. Due to the usage of a single source function to express the fields everywhere, with no coupling of the integral equation, it is expected that the application of the presented method to many-body scattering problems yield an increased efficiency and numerical stability with respect to existing methods.

FORMULATION
Consider the transverse magnetic ($H_z = 0$) plane wave scattering by an arbitrary number of dielectric cylinders, each having a distinct cross section, permittivity and permeability, as shown in Fig. 1. A time dependence of $e^{j\omega t}$ is assumed and suppressed. The resulting field has only an $E_z$ component which obeys a homogeneous Helmholtz equation in each subregion $V_i$,

$$\left( \nabla^2 + k_i^2 \right) E_z(r) = 0, \quad r \in V_i$$  \hspace{1cm} (1)

where $k_i = \omega \sqrt{\varepsilon_i \mu_i}$ is the medium wave number, with $\varepsilon_i$ and $\mu_i$ being the permittivity and permeability of $V_i$, respectively. The field and its normal derivative are subject to the boundary conditions

$$\left( E_z(r) \right)_+ = \left( E_z(r) \right)_-, \quad r \in S_i$$  \hspace{1cm} (2)

$$\left( \frac{\partial}{\partial n} E_z(r) \right)_+ = \frac{H_i}{H_0} \left( \frac{\partial}{\partial n} E_z(r) \right)_-, \quad r \in S_i$$  \hspace{1cm} (3)
where $S_i$ is the surface separating subregion $V_i$ from the surrounding free-space region $V_0$, with a unit normal $\mathbf{n}$ directed from the free-space (-) to the dielectric (+) side. In this many-body problem, the dielectric subregion $V_i$ may be simply-connected or multiply-connected, it may be a compound region consisting of several separate cylinders of common material, it may also surround other dielectric bodies provided that they are separated by free-space. The electric field in the free-space region is expressed in terms of the incident and scattered fields as

$$E_z(r) = E_z^{inc}(r) + E_z^s(r), \quad r \in V_0$$

(4)

Construction of a single surface integral equation for wave scattering by this many-body system is based on the integral expression of a wave function $\mathcal{E}$ in a homogeneous region in terms of step functions defined on its surfaces of discontinuity $\Sigma$. If $\mathcal{E}$ and its normal derivative present the jumps $\phi = (\mathcal{E})_+ - (\mathcal{E})_-$ and $\eta = (\partial \mathcal{E}/\partial n)_+ - (\partial \mathcal{E}/\partial n)_-$ across $\Sigma$, then $\mathcal{E}$ can be expressed in terms of the source layers $\phi$ and $\eta$ in the form

$$\mathcal{E}(r) = G[\eta] + N[\phi], \quad r \notin \Sigma$$

(5)

where the integral operators $G$ and $N$ are defined for two-dimensional problems as

$$G[\eta] = \frac{i}{4} \int_{\Sigma} \eta(r') H_0^{(2)}(kR) d\ell$$

$$N[\phi] = \frac{i}{4} \int_{\Sigma} \phi(r') H_1^{(2)}(kR) \frac{\mathbf{n}' \cdot \mathbf{R}}{R} d\ell$$

(6) \hspace{1cm} (7)

where $\mathbf{R} = r - r'$ is the relative position vector, $R = |\mathbf{R}|$, $H_0^{(2)}$ and $H_1^{(2)}$ are the Hankel functions of the second kind and order 0 and 1, respectively, $k$ is the medium wave number, $j = \sqrt{-1}$, $\{\cdot\}$ indicates the field source in the respective operators, and the integrals are performed over the cross sectional contour of $\Sigma$ (with the principal value of the integral in (7)). In the equivalence theorem terminology, the field discontinuities $\phi(r') = (\mathcal{E} \times \mathbf{n})' \cdot \mathbf{M}(r')$ and $\eta(r') = j\omega \mu \mathbf{J}(r')$, where $\mathbf{M} = (\mathcal{E} - \mathcal{E}_0) \times \mathbf{n}$ and $\mathbf{J} = \mathbf{n}' \times (\mathbf{H}_0 - \mathbf{H}_s)$ are fictitious distributions of magnetic and electric current, respectively. The values of $\mathcal{E}$ and its normal derivative on either side of $\Sigma$ are

$$(\mathcal{E}(r))_\pm = \pm \frac{1}{2} \phi + \mathcal{E}(r), \quad r \in (\Sigma)_\pm$$

$$\left(\frac{\partial}{\partial n}\mathcal{E}(r)\right)_\pm = \pm \frac{1}{2} \eta + \frac{\partial}{\partial n}\mathcal{E}(r), \quad r \in (\Sigma)_\pm$$

(8) \hspace{1cm} (9)

where the last term is computed from (5).

For our problem, we particularize the field $\mathcal{E}$ to a field $\mathcal{E}_0$ which is continuous across all body surfaces, with step discontinuities only in its normal derivative across these surfaces, and is equal to the true scattered field in the free-space region $V_0$. The fact that $\mathcal{E}_0$ exists throughout the entire problem space is of prime importance to the decoupling of the surface integral equations for the many-body problem. We also define a particular field $\mathcal{E}_i$, which is equal to the true total field in the subregion $V_i$ and vanishes outside $V_i$. Each particular field $\mathcal{E}_i$ is expressed as an integral in terms of its discontinuities and is made to vanish on $(S_i)$. Boundary conditions (2) and (3) are used to express the discontinuities of $\mathcal{E}_i$ in terms of the incident field and $\mathcal{E}_0$. When $\mathcal{E}_0$ and its derivatives are then expressed as in (8) and (9), a single integral equation is established for the source distribution $\eta_0$ (i.e. the jump in the normal derivative of $\mathcal{E}_0$),

$$\frac{1}{2}G_0[\eta_0] - N_i[G_0[\eta_0]] + \frac{\mu_i}{\mu_0} \left\{ \frac{1}{2}G_0[\eta_0] - G_i[G_0[\eta_0]] \right\} = -\frac{1}{2}E_z^{inc} + N_i[E_z^{inc}] + \frac{\mu_i}{\mu_0} \left\{ \frac{\partial E_z^{inc}}{\partial n} \right\}$$

(10)

for $r \in S_i$ and for all $i$'s, where $G_0[\eta_0]$ is the normal derivative of $G_0[\eta_0]$, and

$$G_0[\eta_0] = -\frac{i}{4} \int_{\Sigma} \eta_0(r') H_1^{(2)}(k_0R) \frac{\mathbf{R}}{R} d\ell$$

(11)

The operator subscripts refer to both the wave number, $k = k_0$ or $k = k_i$, and the respective surfaces of discontinuity, $\Sigma = S_i$ for subscript $i$ and $\Sigma = S_1 \cup S_2 \cup \ldots$ for subscript 0. Equation (10) has the same form as the integral equation in [3]. It contains a single source function $\eta_0$, defined for all surfaces in the problem space, with no coupling due to individual body surfaces.
Once $\eta_0$ is determined, $\mathcal{E}_0$ and $\mathcal{E}_i$ are computed from (see (5))

$$\mathcal{E}_0 = E_0^z(r) = G_0(\eta_0), \quad r \in V_0$$

$$\mathcal{E}_i = E_i^z(r) = G_i(\eta_i) + N_i(\phi_i), \quad r \in V_i$$

(12)

(13)

where $\eta_i$ and $\phi_i$ are simply expressed as

$$\eta_i = \frac{\mu_i}{\mu_0} \left( -\frac{1}{2} \eta_0 + G_0(\eta_0) + \frac{\partial E_{inc}}{\partial n} \right)$$

(14)

$$\phi_i = G_0(\eta_0) + E_{inc}^z$$

(15)

The single surface integral equation for the scattering of a transverse electric plane wave by the many-body system is constructed in a similar manner.

EXAMPLE CALCULATION

As a test example, the radar cross section (RCS) of the parallel dielectric cylinders shown in Fig. 2 is computed by the many-body single surface integral equation method and the results are compared to those obtained by the volume integral equation method [1]. Both programs were run on a HP APPOLLO 9000/735 computer. The number of unknowns was successively doubled until the results were reasonably accurate (0.063% error in the forward scattering cross section). The radar cross section as a function of the scattering angle is plotted in Fig. 2. The increased efficiency of the single surface integral equation method (7.4 s versus 104.3 s of CPU time for this example) becomes even more pronounced for larger targets.

CONCLUSION

A single surface integral equation is presented and applied for the first time to the solution of wave scattering problems by a many-body system of dielectric cylinders. The fundamental fact used in this paper is that the particular field $\mathcal{E}_0$ exists throughout the entire problem space and may therefore be used to formulate the many-body problem in terms of a single source distribution, with no coupled equations. The presented formulation can be extended to the problem of wave scattering by multiply-connected bodies and also by heterogeneous bodies through the proper handling of the corresponding singularities. The computational efficiency of this method has been demonstrated by comparison with the volume integral equation method. A reduction in CPU time by more than a factor of 14 was realized for the selected example. The numerical stability of the method was found to be excellent.

REFERENCES

