On the Corona Voltage-Current Characteristic of Unipolar HVDC Transmission Lines

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Abstract

Simplified analytical analysis of ionized fields associated with HVDC transmission lines is possible if it is based on Deutsch's assumption. The resulting voltage-current (V-I) characteristic may be conveniently used to calculate the power losses of a HVDC transmission line. However, such an approach has been criticized because Deutsch's assumption is considered invalid for the line-plane geometry. This paper verifies the validity of the V-I characteristic thus obtained by comparing it with numerical results which are generated using a new method developed by the authors, in which Deutsch's assumption is not employed.

Introduction

In a HVDC transmission line, space charges created by corona discharges around the energized line conductor flow in the entire interelectrode space. The resulting corona currents cause power losses and environmental concerns at ground level. In order to evaluate these effects, one must solve the electric field in the presence of the flowing space charge (i.e., ionized field).

The difficulty arises due to the nonlinearity of the ionized field. Since the 1930's, considerable efforts have been made to seek an effective solution method, both analytically and numerically. Early research mainly dealt with analytical methods based on Deutsch's assumption which states that the presence of space charge only affects the magnitude of the field but not its direction. Popkov [1] used the above assumption to determine the corona current in a line-plane geometry as an implicit function of the applied voltage. The formula yields a V-I characteristic whose form is similar to that of a coaxial cylindrical geometry, but an empirical constant \( P \) is included to take into account the effect of the nonuniform distribution of the corona current density at ground level. Unfortunately, there is no theoretical guideline for the determination of the empirical constant. A rigorous expression of the V-I characteristic for a line-plane geometry in the absence of wind has been derived by Ciric et al. [2]. This formula, unlike Popkov's, does not involve any empirical constant. The only possible source of error in this method results from using Deutsch's assumption. Some authors [3,4] have argued that Deutsch's assumption is not valid for a line-plane geometry even in the absence of wind. Their studies show that the flux lines of an ionized field do deviate from those of the corresponding charge-free field.

Since an analytical expression of the V-I characteristic is a convenient form to use in the design of a HVDC transmission line, it is of practical importance to verify the validity of Deutsch's assumption in the absence of wind. In this paper, results generated by using a new numerical method, developed by the authors [5,6,7], are compared with those obtained by employing the analytical method in [2]. The numerical method does not resort to Deutsch's assumption.

Modelling of Unipolar Ionized Field

A unipolar ionized field is described by the following equations:

\[
\nabla \cdot E = \frac{\rho}{\varepsilon_0} \quad (1)
\]
\[
\nabla \cdot j = 0 \quad (2)
\]
\[
\rho = k p E \quad (3)
\]
\[
E = -\nabla u \quad (4)
\]

where \( E \) is the electric field intensity, \( \rho \) the space charge density, \( j \) the corona current density, \( u \) the electric potential, \( \varepsilon_0 \) the permittivity of free space, and \( k \) the ionic mobility (assumed to be a constant). Eliminating the vectors from the above equations yields

\[
\nabla^2 u = -\frac{\rho}{\varepsilon_0} \quad (5)
\]
\[
\nabla \cdot (k p \nabla u) = 0 \quad (6)
\]

The boundary conditions are

\[
\begin{align*}
\nabla^2 u &= -\frac{\rho}{\varepsilon_0} \\
\n\nabla \cdot (k p \nabla u) &= 0
\end{align*}
\]

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\[
\begin{align*}
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\end{align*}
\]

\[
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\rho &= k p E \\
\nE &= E_0
\end{align*}
\]
where \( V \) is the applied voltage, and \( E_0 \) is the corona onset field which may be easily found using Kaptozov's assumption, i.e., the electric field at the conductor surface in corona remains at its corona onset level.

### Ciric-Kuffel's Formula — Analytical Method

In order to obtain the V-I characteristic from the above boundary value problem analytically, Deutsch's assumption is employed and the following V-I characteristic \([2]\) is obtained:

\[
\frac{V}{V_0} = \frac{2}{J_3} \int_0^{\xi_1} \left( \frac{1}{1 + \frac{2Y_e}{\sqrt{3}C}} \right) f(\xi) \, d\xi \tag{9}
\]

with

\[
f(\xi) = \frac{\sqrt{3} + \tanh \xi}{\sqrt{3} - \tanh \xi} \tag{10}
\]

\[
\xi_1 = \ln \left[ \frac{H/r_0 + \sqrt{(H/r_0)^2 - 1}}{\sqrt{1 - (r_0/H)^2}} \right] \tag{11}
\]

\[
Y_e = \frac{C}{2\pi r_0^2} \left[ H / (E_0 r_0) \right]^2 \tag{12}
\]

\[
C = \frac{2}{\sqrt{3}} \ln \frac{\sqrt{3} + \sqrt{1 - (r_0/H)^2}}{\sqrt{3} - \sqrt{1 - (r_0/H)^2}} \tag{13}
\]

where \( V_0 \) is the corona onset voltage, \( r_0 \) the conductor radius, and \( H \) the conductor height.

For practical HVDC transmission lines, \( r_0/H \ll 1 \), and therefore equations (11) and (13) may be simplified to \( \xi_1 = \ln \left( 2H/r_0 \right) \) and \( C = 1.520692 \) with resulting negligible error.

Equation (9) can be easily evaluated using a numerical integration method. Several simplified forms of equation (9) have been derived in \([2]\).

### Numerical Method

A new numerical method for solving the ionized field of unipolar HVDC transmission lines has recently been developed by the authors. The detailed formulation and numerical tests on the new method may be found in \([5,6,7]\). In this method, Deutsch's assumption is waived. The discretization of Poisson's equation (5) and the current continuity equation (6) is performed by the Galerkin finite element method (FEM). Linear triangular elements are used. In order to apply FEM, an artificial boundary is assumed to truncate the solution domain. The boundary condition at the artificial boundary is determined from the corresponding charge-free field. After the solution domain is approximated by a finite element mesh with \( n_e \) elements and \( n_p \) nodes, Poisson's equation and the current continuity equation are discretized into a nonlinear system of equations with the nodal values of the electric potential and the charge density as unknowns. The system of equations is solved by transforming it into a nonlinear optimization problem in which the boundary conditions are included. The modified Gaussian algorithm is applied to solve the optimization problem. The electric field intensity at the boundary is found from the finite element equations rather than from the derivatives of the function approximating the electric potential.

### Numerical Investigation

#### Line-Plane Geometry

Hara's test model \([8]\) is used. In this model, the line conductor is a smooth cylinder with radius \( r_0 = 0.25 \) cm at height \( H = 2 \) m. The corona onset field, \( E_0 \), in kV/cm is determined from Peek's law

\[
E_0 = 30(1 + 0.301 \cdot \frac{r_0}{H}) \tag{14}
\]

In reference \([8]\), the ionic mobility is found to be \( 1.4 \times 10^{-4} \) m\(^2\)/N.s. This value has been used in this paper.

#### Application of Ciric-Kuffel's Formula — Analytical Method

In this paper, equation (9) is used rather than its simplified form. The integration involved is performed using Gaussian numerical integration method, by means of which a sufficiently high accuracy can be achieved.

#### Application of Numerical Method

The artificial boundary is located at a distance \( 3H \) above and \( 4H \) laterally away from the conductor. The truncated solution domain is discretized using an automatic mesh generation program \([9]\). Figure 1 shows the created meshes with different numbers of nodes. Figure 2 shows the effect of the number of nodes on the calculated corona current. It is observed that the calculated corona current decreases as the number of nodes is increased. When the number of nodes exceeds 1500, the calculated corona current no longer decreases signifi-
cantly with \( n_P \), which indicates that the calculated corona current is sufficiently close to its exact value. Also, Figure 2 shows that for a limited number of nodes, the corona current obtained exceeds its exact value.

\[
\begin{align*}
n_P &= 376 \\
n_P &= 1458 \\
n_P &= 494 \\
n_P &= 1868 \\
n_P &= 721 \\
n_P &= 2273 \\
n_P &= 1088 \\
n_P &= 2817
\end{align*}
\]

Figure 1. Discretization of solution domain with different numbers of nodes.

\[
\begin{align*}
\text{Corona Current (\( \mu A/m \))} & \\
n_P &
\end{align*}
\]

Figure 2. Dependence of the corona current on the number of nodes in the numerical method. \( H = 2 \text{ m}; \ r_0 = 0.25 \text{ cm}; \ V/V_0 = 3 \).

Comparison of Calculated Results

Figure 3 compares the results generated by the numerical and analytical methods with experimental data [8]. It is seen that the analytical method yields results larger than both the numerical and the measured values. The numerical results are closer to the measured values.

Figure 4 shows the V-I characteristics for model line-plane geometries with different ratios of \( H/r_0 \), obtained by using Ciric-Kuffel's formula and the numerical method. In all cases, the analytical results are larger than the numerical results. The difference between the analytical and the numerical results is less than 10% for the range of \( H/r_0 \) considered and for a practical range of \( V/V_0 \). It should be mentioned that the difference does not increase with the ratio \( V/V_0 \).

\[
\begin{align*}
\text{Corona Current (\( \mu A/m \))} & \\
V/V_0 &
\end{align*}
\]

Figure 3. V-I curves for Hara's line-plane geometry obtained by different methods. \( H = 2 \text{ m}; \ r_0 = 0.25 \text{ cm}; n_P = 1868 \).

\[
\begin{align*}
\text{Corona Current (\( \mu A/m \))} & \\
V/V_0 &
\end{align*}
\]

Figure 4. V-I curves for model line-plane geometries of different conductor radii. \( H = 2 \text{ m} \).

In order to compare the results yielded by the analytical and numerical methods when applied to a practical line-plane geometry, the dimensions in Hara's model were multiplied by a factor of 5, yielding \( H = 10 \text{ m} \) and \( r_0 = 1.25 \text{ cm} \). The results are shown in Figure 5. Once again the analytical method gives a result larger than that obtained by using the numerical method; the difference is of the same order as that obtained using the model line-plane configuration.
Ciric-Kuffel's results

Numerical results

Figure 5. V-I curves for a line-plane geometry with practical dimensions. $H = 10$ m; $r_0 = 1.25$ cm.

Since the numerically obtained V-I characteristic is sufficiently close to the exact one and lies above it, the accuracy of the Ciric-Kuffel analytical method is of the order of 10%.

**Concluding Remarks**

Analytical and numerical methods have been compared to investigate the V-I characteristics of line-plane geometries with height to conductor radius ($H/r_0$) ratios in the range 400 - 1200. The analytical method utilizes Deutsch's assumption, but the numerical method does not. The results show that the error in the V-I characteristic, in the absence of wind, obtained by use of the analytical method in [2] is of the order of 10%. This error is solely due to the use of Deutsch's assumption. From an engineering viewpoint, it seems to be acceptable.

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**References**


