A NEW DIRECT METHOD FOR SOLVING SYSTEMS OF LINEAR ALGEBRAIC EQUATIONS

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Abstract. The direct method presented in this paper has some advantageous features when applied to practical problems, such as various image reconstruction devices and linear optimization techniques. The complexity of the method is identical to that in the Gauss elimination method, with the same number of multiplications/divisions and of additions/subtractions for a complete solution, but it is more efficient for special types of computations involved in the analysis of these problems. The solution is obtained by constructing a homogeneous equation satisfied by the sets of coefficients in all the equations of the system. As a consequence, the entries of the intermediate column vectors computed at each main step of the algorithm are solutions of subsystems whose coefficients are those in the original equations. This makes it possible to monitor and improve the accuracy during the solution process, which is important when the accuracy refinement cannot be performed at the end of the process (e.g. for not so well-conditioned systems). Only one original equation is used at each step, without changing the rest of the equations; there is no backward substitution operation, and the corresponding storage space that is not needed any more as the computation proceeds is greater than in the classical Gauss elimination method. It is shown how to apply the method when repeated changes of some columns or rows occur in the basic system matrix in order to reduce substantially the required amount of computation. For instance, when one equation is changed repeatedly, as in optimization or image reconstruction problems, the solution of each new system requires only $6n - 5$ arithmetic operations in the method presented as compared with about $2n^2$ operations in the Gauss elimination method.

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