APPLICATION OF A REDUCED-ORDER MODEL TO TRANSMISSION LINE ANALYSIS

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INTRODUCTION

Systems with transmission lines and lumped linear elements are commonly used for modeling interconnections on integrated circuits, printed circuit boards, and multichip modules. Conventionally, the transmission lines are characterized by distributed parameters determined on the basis of a quasi-TEM approximation. With rapid increase in operating frequencies, distributed parameter models for interconnects are no longer adequate and electromagnetic field models that take into account all possible field components are needed to accurately analyze signal propagation. However, the solution of electromagnetic problems based on these more complex models using traditional techniques generally involves high computational cost. To circumvent this difficulty, a reduced-order model technique employing the asymptotic waveform evaluation (AWE) [1],[2] has recently been developed. The AWE is a moment-matching method where the s-domain transfer function of a linear system is approximated by using a reduced-order model function containing only a relative small number of dominant poles and residues. The resultant reduced-order model can be used to compute the time-domain and frequency-domain responses of the system in an efficient manner. In this paper, we utilize a reduced-order model technique to compute the transient response of hybrid systems containing both transmission lines and lumped linear elements. By taking into account the effect of the lumped elements into Maxwell's equations, we present a frequency-domain analysis of the entire hybrid system comprising lumped elements and transmission lines by using an electromagnetic field model, instead of treating various components separately by different techniques. This way, a global solution for hybrid systems can be obtained efficiently. Numerical examples of the proposed method are presented.

FORMULATION

In this paper, the Maxwell time-dependent equations are discretized only with respect to the spatial coordinates by employing Yee's lattice, for instance, while the time variable is maintained continuous. Furthermore, the presence of linear circuit elements can be accounted for in Maxwell's equations by adopting lumped-circuit models, initially developed for the FDTD algorithm [3],[4]. A z-directed lumped resistor \( R \) located at a point where the electric field intensity is \( E_z|_{i,j,k} \) can be represented by completing the corresponding Maxwell equation in the form

\[
(\nabla \times \mathbf{H}_{i,j,k}) \cdot \mathbf{a}_z = J_{cz}|_{i,j,k} + \frac{\partial D_z|_{i,j,k}}{\partial t} + J_{Lz}|_{i,j,k}
\]  

(1)
where \( J_{z_{i,j,k}} \) is the source current density due to the resistor and is related to \( E_{z_{i,j,k}} \) by

\[
J_{z_{i,j,k}} = \frac{\Delta z E_{z_{i,j,k}}}{R \Delta x \Delta y}
\]  

(2)

with \( \Delta x, \Delta y \) and \( \Delta z \) being the space increments in \( x, y \) and \( z \) direction, respectively. Applying the Laplace transform with respect to time to the space-discretized Maxwell's equations yields the \( s \)-domain matrix equation

\[
A(s)X(s) = B(s)
\]  

(3)

where \( X(s) \) is a vector containing all the field components at all spatial nodes, \( B(s) \) is a vector containing the contribution of external sources, and \( A(s) \) is a square matrix generated by finite difference space-discretization.

To obtain an approximate solution, \( X(s) \) in (3) is expanded in Taylor series about \( s_0 \) in the form

\[
X(s) = \sum_{n=0}^{\infty} M_n (s - s_0)^n
\]  

(4)

where \( M_n \) is the \( n \)-th vector coefficient (\( n \)-th moment) of the Taylor's expansion and is given as

\[
M_n = \frac{\partial^n}{\partial s^n} \left[ [A(s)]^{-1} B(s) \right]_{s=s_0}
\]  

(5)

In the AWE, all the moment vectors are generated by a recursive procedure:

\[
A(s_0) M_n = \frac{\partial^n B(s)}{\partial s^n} \bigg|_{s=s_0} - \sum_{m=0}^{n-1} M_m \frac{\partial^{n-m} A(s)}{\partial s^{n-m}} \bigg|_{s=s_0} , \quad n \geq 1
\]  

(6)

with

\[
A(s_0) M_0 = B(s_0)
\]  

(7)

One can see that the generation of all moment vectors requires only one \( LU \) factorization and each vector \( M_n \) \((n \geq 1)\) can be obtained by performing only one forward-backward substitution. This makes the AWE technique superior over conventional techniques such as finite element
method, where the solution of a matrix equation has to be repeated at each frequency. The reduced-order model solution of the desired field component $X_i(s)$ is obtained in the form of a rational function Padé approximation (8),

$$X_i(s) = \frac{\sum_{j=0}^{q-1} b_j (s-s_0)^j}{1 + \sum_{j=1}^{q} a_j (s-s_0)^j} \quad (8)$$

with the coefficients $a_j$ and $b_j$ determined by matching the first $2q$ moments calculated from (6)-(7) with the corresponding Taylor coefficients of (8). The time-domain response can be obtained by using a pole-residue representation of (8).

The computation domain is truncated by applying absorbing boundary conditions at appropriate boundaries. Based on the time-domain Mur's first-order absorbing boundary conditions, the $s$-domain absorbing boundary conditions suitable to be used in conjunction with the reduced-order model technique are formulated and implemented in this paper. For example, for a boundary section normal to $a_y$, the $s$-domain absorbing boundary condition is expressed as

$$s\Delta y (\hat{E}_0 + \hat{E}_1) + 2c \left( \hat{E}_0 - \hat{E}_1 \right) = \Delta y \left( E_0|_{t=0} + E_1|_{t=0} \right) \quad (9)$$

where $\hat{E}_0$ and $\hat{E}_1$ represent the $s$-domain tangential electric field components on the boundary and at one node from the boundary, respectively.

**NUMERICAL EXAMPLES**

To demonstrate the capability of the reduced-order model technique, a parallel-plate transmission line connected to different linear lumped elements was analyzed and time-domain responses were computed and compared with results calculated by the FDTD method. Fig. 1 shows the transmission line configuration [3]. In all the calculations, the cell size used is $1cm \times 1cm$, the total number of mesh cells of the transmission line is $200 \times 2$, and a lumped voltage source $V(t) = V_0 \sin(\omega t)$, with a frequency of $200MHz$, was used as excitation. The plates of the

![Fig. 1. Cross section of a parallel-plate transmission line connected to lumped elements.](image)
transmission line are separated by 2 cells and the s-domain absorbing boundary conditions were imposed at the both ends of the transmission line to minimize the reflection due to mesh truncation. Even though the transmission line structure chosen for analysis is simple in geometry, the proposed approach is general and can be applied to other complex configurations. Fig. 2 shows the current through a resistive load as calculated by using the proposed approach and by the FDTD method. For an inductor load, a comparison of results obtained by the new approach and by the FDTD method is shown in Fig. 3. In both examples, the results computed by the proposed approach are in full agreement with those calculated by the FDTD method.

CONCLUSION

An s-domain reduced-order model method is proposed for the solution of hybrid systems comprising transmission lines and lumped linear elements. Both transmission lines and lumped linear elements are modeled electromagnetically. Reduced-order models can be generated at a low cost and provide an efficient tool for characterizing the electromagnetic behavior of large linear systems. They are also useful in the simulation of non-linear systems consisting mainly of linear subsystems once the latter are replaced by reduced-order models.

REFERENCES


