NUMERICAL IMPLEMENTATION OF AN EFFICIENT RECURSIVE ALGORITHM FOR ELECTROMAGNETIC WAVE SCATTERING BY COMPLEX OBJECTS

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Abstract — The numerical implementation of an efficient single source surface integral equation algorithm for the problem of electromagnetic wave scattering by complex structures is discussed. Its salient features include the invariance of an equivalent surface representation and the recursive region-by-region solution algorithm.

INTRODUCTION

Recently, a novel reduction procedure has been derived for the electromagnetic wave scattering analysis of complex objects formed as a mixture of electric conductors and inhomogeneous dielectric material [1], [2]. In this scheme, the anatomy of the scattering obstacle is organized into a hierarchical, multiply-nested structure to facilitate an efficient analysis by recursive application of a single source surface integral equation. Ultimately, the wave scattering problem is expressed in terms of only a single electric current density distributed over the outermost surface of the body through the derivation of exclusive operators.

In this paper, we discuss how the most significant aspects of the reduction formulation — namely the invariance of the exclusive operators and the recursive nature of the algorithm — are exploited in a numerical code which minimizes both the computational effort and the storage requirements. After a brief summary of the reduction algorithm, we discuss the special "nested-body" data type and outline the structure of the recursive function "reduce-body" which automatically orders the recursive reduction procedure. Finally, we step through a sample problem which demonstrates the salient features of the algorithm.

SUMMARY OF THE REDUCTION ALGORITHM

Consider a complex cylindrical body formed by embedding a set of arbitrary cylindrical inclusions within an inhomogeneous dielectric medium that is partially covered by a perfect electric conducting (p.e.c.) shell. The opening in the p.e.c. shell, which forms a boundary between the inner dielectric embedding medium and the exterior region, is referred to as the aperture. Let the inclusions be composed in a similar manner. If we assume that the actual fields tangent to the aperture of each inclusion can be expressed in terms of a single electric surface current by way of a pair of exclusive operators, then it is a simple matter to derive the appropriate exclusive operators to express the fields tangent to the outermost aperture [1], [2]. This recursive reduction of a complex body to an equivalent surface representation proceeds as follows.

Let the true fields in the embedding medium of a host body $V_a$ be generated by a single layer of electric current distributed over the aperture of this body. Next, replace the inhomogeneous embedding medium with a homogeneous material and an equivalent volume density of electric polarization current, and let the actual fields tangent to the aperture of each inclusion be expressed in terms of an electric current density distributed on that same aperture as $E_i = E_i^{ex} J_i$, $H_i = H_i^{ex} J_i$ where $E_i$ and $H_i$ are column vectors of the electric and magnetic fields intensities tangent to each discrete patch of the aperture of the $i^{th}$ inclusion, and $E_i^{ex}$ and $H_i^{ex}$ are the so-called exclusive operator matrices derived for the $i^{th}$ inclusion. For bookkeeping reasons, we apply the generic designation of "scattering feature" to each inclusion aperture, to the outer p.e.c. surface of each inclusion, to the inner p.e.c. surface of the host body, and to the volume density of polarization current in the embedding medium. Enforcing the electromagnetic boundary conditions on each interface (excluding the host aperture) and taking into account the polarization current density, we form the matrix equation.
where $A$ is an $n_t \times n_t$ square matrix, $B$ is an $n_t \times n_a$ rectangular matrix, $n_t$ is the total number of unknowns on all $n$ scattering features, $n_a$ is the number of unknowns associated with the host aperture, $J_f = [J_1, J_2, \ldots, J_n]^T$ is the column vector of the feature currents, and $J_a$ is the column vector of the host aperture current. Solving (1) for $J_f$ in terms of $J_a$ yields the source mapping $J_q = L_q J_a$, $q = 1, 2, \ldots, n$, which permits the expression of the fields tangent to the host aperture exclusively in terms of the host aperture current, i.e. $E_a = E^{xx}_a J_a$, $H_a = H^{xx}_a J_a$. The integral operators $E^{xx}_a$ and $H^{xx}_a$ are the exclusive operator matrices for the host aperture and are defined recursively as

$$E^{xx}_a = E^{xx}_a + \sum_{q=1}^{n} E^{xx}_q L_q, \quad H^{xx}_a = H^{xx}_a + \sum_{q=1}^{n} H^{xx}_q L_q,$$

where $E^{xx}_a$ and $E^{xx}_q$ are rectangular matrices that operate on the electric current densities on the host aperture and on the $q^{th}$ scattering feature, respectively, to give the electric field intensities tangent to the host aperture. Similarly, $H^{xx}_a$ and $H^{xx}_q$ yield the magnetic field intensities tangent to the host aperture. By way of (1) and (2), $E^{xx}_a$ and $H^{xx}_a$ are obtained recursively in terms of the exclusive operators for each inclusion there in. Furthermore, $E^{xx}_a$ and $H^{xx}_a$ are derived in a manner that is independent of material and sources outside of the host body and are therefore invariant under rotation and translation. In general, the scattering problem may consist of a system of such complex bodies. After all bodies have been so reduced, a free space boundary value problem is solved for the single electric current density on the outermost surface of each body in terms of a known incident field.

### NUMERICAL IMPLEMENTATION

The two key aspects of the above algorithm are the recursive nature of the solution, and the invariance of the exclusive operators and source mappings. These aspects are exploited in a numerical code by way of a recursive function call that operates on a specialized data structure explained as follows.

#### The Data Structure

Each complex body (i.e. every inclusion as well as the final host body) is declared as a nested-body data type. Briefly stated, this data type consists of an identifying index, the permittivity and permeability of the homogeneous portion of the embedding medium, pointers to four arrays that provide a geometric description of the body, pointers to the three complex matrices derived during the reduction procedure, pointers to two lists, and miscellaneous counters and flags. These arrays, lists and matrices are detailed as follows. The local geometry of the body is described by three arrays of surface elements (defining the aperture, inner p.e.c. surface and outer p.e.c. surface) and by an array of polarization cells that define the volumetric inhomogeneities in the embedding medium (if the embedding medium is homogeneous, then the latter array is not required). Each nested-body data type contains an inclusion list that consists of the identifying index of each inclusion together with its location and orientation with respect to the local coordinate system of the host body; a second list, the feature list, is internally generated during the reduction process and catalogs the nature of each specific scattering feature as well as the identifying index of the body on which it resides. The feature list points to the apertures and to the outer p.e.c. surfaces of each inclusion, to the array of volume polarization elements representing the inhomogeneity in the embedding medium, and to the inner p.e.c. surface of the host. The nested-body data type also contains pointers to the exclusive operator matrices $E^{xx}_a$ and $H^{xx}_a$, and to the union of the source mapping matrices $L = [L_1, L_2, \ldots, L_n]^T$. Note that memory for these arrays is only allocated as required during the reduction process and de-allocated as soon as the respective reduction is performed. Lastly, this data type holds pointers to the arrays of electric current densities assumed on the aperture, on the inner p.e.c. surface, on the outer p.e.c. surface, and throughout the polarization cells; however, these currents are never determined during the course of the reduction algorithm — they are only determined if we require the actual fields in a particular region.
**Recursive Function for the Reduction of the Complex Body**

A body is said to be reduced when its exclusive operators have been obtained. The *reduce-body* function begins by checking if the body to be reduced is in fact just a copy of another body; if so, then the exclusive operators are simply copied from the original body; if not, then the reduction process begins in earnest. The *reduce-body* function checks the *inclusion list* to confirm that each inclusion has itself been reduced (i.e. possesses exclusive operators); if not, then the *reduce-body* function is called, recursively, for that particular inclusion. In this manner, layers of such function calls are generated, reaching down from the outermost surface to the innermost cell. Once all inclusions have been reduced, they are positioned and orientated as prescribed in the *inclusion list* and the *feature list* is generated (for the inclusions placed in direct contact with each other or with the surrounding host surface a special treatment of “coincident” surfaces is applied [1]). Memory is allocated for the coefficient matrix $A$ of (1) and for the source mapping matrices $L_q$, $q = 1, 2, \ldots, n$. After solving (1) for the source mapping matrices, the memory allocated for the matrix $A$ is freed and we proceed to construct the exclusive operator matrices (2). Finally, having thus reduced the body, we free the memory that had been allocated for the source mapping matrices $L_q$, $q = 1, 2, \ldots, n$, and for the exclusive operator matrices of each inclusion. If requested, these matrices may be archived for later retrieval — for example, if we require the fields at interior points or if the scattering problem is to be repeated with only minor modifications such as in material optimization problems. Through this recursive region-by-region approach, large systems such as (1) are restricted in size to the number of unknowns on just the scattering features of a specific host body. Thus, for cylindrical obstacles with a high degree of inner detail described in a total of $N$ unknown quantities, the computational effort increases as $O(N^{1.5})$, while memory requirements increase as $O(N)$ [1].

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**Fig. 1.** TM illumination of a non-magnetic elliptical cylinder partially covered by a p.e.c. foil and containing three identical inclusions. Each inclusion is a circular cylinder partially covered with a p.e.c. foil and containing a square cylinder of linearly graded dielectric material. A semi-circular dielectric cap covers the main aperture. $\lambda_0$ is the free space wavelength.
Numerical Example
Consider the complex cylinder $V_a$ shown in a cross section in Fig. 1. The data for $V_a$ includes the inner and outer walls of a p.e.c. foil (each discretized into 64 curved segments), an aperture (discretized into 8 curved segments), the permittivity and permeability of a homogeneous embedding medium, and the location and orientation of three inclusions, namely $V_1$, $V_2$, and $V_3$. When the reduce-body function is called for $V_a$, it first checks to see if the inclusion $V_1$ has been reduced: it has not, so the reduce-body function is called for $V_1$. The data for $V_1$ includes the inner and outer walls of a p.e.c. foil (each discretized into 24 curved segments), an aperture (discretized into 8 curved segments), the permittivity and permeability of the homogeneous portion of the embedding medium, as well as a square cylindrical region in which the permittivity varies linearly from $4\varepsilon_0$ to $8\varepsilon_0$ (discretized into 25 square patches). Since no inclusions are listed within $V_1$, the reduce-body function calculates the exclusive operator matrices for the aperture of $V_1$ in terms of the linearly graded dielectric region and of the inner p.e.c. wall of $V_1$. Returning to $V_a$, the reduce-body function notices that $V_2$ and $V_3$ are simply rotated and translated copies of $V_1$, so the exclusive operator matrices calculated for $V_1$ are simply copied to $V_2$ and to $V_3$, without repeating the associated calculations. The exclusive operator matrices $E_a^{\text{ex}}$ and $H_a^{\text{ex}}$ for the aperture of $V_a$ are obtained in terms of the inner p.e.c. wall of $V_a$ and of the apertures and outer p.e.c. surfaces of the inclusions $V_1$, $V_2$, and $V_3$. Having reduced the content of $V_a$ to the exclusive operators formulated over its aperture, we may add the dielectric cap (as shown in Fig. 1) without further consideration of these reduced bodies. Furthermore, rotation or translation of any of the inclusions $V_1$, $V_2$, $V_3$ entails only the recalculation of the exclusive operator matrices $E_a^{\text{ex}}$ and $H_a^{\text{ex}}$ via (1) and (2). The bistatic transverse magnetic (TM) RCS of this test object is plotted in Fig. 2. The effect of rotating the aperture of the centre inclusion, $V_2$, towards or away from the main aperture is to increase or decrease, respectively, the backscattered echo while leaving the forward scattered RCS unchanged. The presence of the dielectric cap serves to partially mute the effect of rotating $V_2$. The fields in any interior region are recoverable through a fast backward recursion involving the archived source mapping matrices.

CONCLUSION
An efficient formulation of the problem of wave scattering by complex cylindrical objects consisting of inhomogeneous dielectric material and p.e.c. shells has been implemented in a numerical code. The most significant features of the formulation, namely the invariance of the so-called exclusive operators and the recursive nature of the algorithm are accounted for through a recursive function call that operates on a specialized data structure. When applied to scattering obstacles with a high degree of inner details described by $N$ unknown quantities, the computational effort increases as $O(N^{1.5})$, while the memory requirements increase as $O(N)$.

REFERENCES