NON-SINUSOIDAL LOW FREQUENCY SHIELDING

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Abstract – The aim is to introduce shielding of non-sinusoidal waveforms through a basic shielding structure allowing a one-dimensional field model. By providing analytical solutions, it is hoped that this novel approach will contribute to a better understanding of shielding problems raised by power electronics circuitry where discontinuous waveforms are typical. The approach is based on the eigenfunction expansion of the field. This approach represents an alternative to the classical harmonic analysis of time periodic waveforms. Quasi-stationary magnetic fields are assumed.

I. INTRODUCTION

Low frequency (LF) shielding is essential when unwanted magnetic fields of power frequency may cause disturbances on sensitive equipment or possible health hazards. A recent survey paper [1] has outlined the significant role played by line-connected power electronics equipment in causing electromagnetic pollution that can propagate by conduction on the power lines. It is well known that power electronics raise numerous problems in EMC [2].

Since LF shielding is based on the quasi-stationary field assumption it is often referred to as extremely low frequency (ELF) shielding. For time-harmonic electromagnetic fields, LF or ELF shielding is well understood. The aim of this paper is to introduce the eigenfunction approach in LF shielding theory of non-sinusoidal waveforms. Teaching of steady and time-harmonic shielding theory is thus completed with the non-sinusoidal one.

Similarly to the classical LF time-harmonic analysis, the non-sinusoidal analysis reported here is based on the assumption that the magnetic field is quasi-stationary. This assumption has been used to derive analytical expressions for magnetic shielding effectiveness of basic shielding structures such as spheres, cylinders, and infinitely large plates since the inception of EMC concepts in 1930s. Based on the same quasi-stationary assumption, recent investigations in LF shielding are devoted to the analytical and numerical analysis of shielding behavior for different combinations of geometrical and material properties [3], extension of the moment method [4] and shielding by cylindrical structures of arbitrary cross sections [5].

II. EIGENFUNCTION EXPANSION METHOD

II.1 Shielding problem

To illustrate this approach we consider a basic shielding structure for which an analytical solution can be derived. The shield is an infinitely long circular cylinder of thickness much smaller than its radius (Figure 1). The interior current free region is to be shielded from a homogeneous axially directed external magnetic field.

![Figure 1 Cylindrical shielding structure.](image1)

The external magnetic field \( H_1(t) \) is periodic non-sinusoidal with repetition period \( T \). We are particularly interested in the shielding of periodic sequences of trapezoidal pulses. An alternating sequence of such trapezoidal pulses is shown in Figure 2.

![Figure 2 Alternating sequence of trapezoidal pulses.](image2)
The trapezoidal pulse has the rise time $T_r$, fall time $T_f$, and plateau duration $T_p$. The pause between two successive pulses is $T_o$.

![Figure 3](image)

**Figure 3** Periodic sequences of rectangular and triangular pulses.

Of particular interest are periodic sequences of rectangular and triangular pulses (Figure 3). Various different periodic waveforms result from combining these sequences, e.g. the alternating trapezoidal waveform shown in Figure 2.

### II.2 Statement of the boundary value problem

Under the quasi stationary field assumption, the boundary value problem (BVP) for the magnetic vector potential $A$ is given by

$$\begin{align*}
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial A}{\partial r} \right) &= \mu \sigma \frac{\partial A}{\partial t}, \quad a < r < a + d \\
\frac{\partial A}{\partial r} &= \mu H_1(t), \quad r = a + d \\
A + \frac{\mu \sigma}{2 \mu d} \frac{\partial A}{\partial r} &= 0, \quad r = a
\end{align*}$$

(1) (2) (3)

Here $d$ is the thickness of the shield and $a$, its interior radius. The conductivity of the cylindrical shield is $\sigma$, the permeability $\mu = \mu_0 \mu_r$. Recall that $H_1(t)$ is the time-periodic non-sinusoidal magnetic field uniformly distributed in the region $r > a + d$, hence boundary condition (2). Recall also that in current free regions, such the shielded region, the magnetic field has to be a function of time alone, hence $H(r, t) = H_2(t)$, $r < a$. Boundary condition (3) follows straightforwardly by making use of the Faraday's law of induction.

Neglecting the curvature radius, using Cartesian coordinates, the BVP reduces to

$$\frac{\partial^2 A}{\partial x^2} = \mu \sigma \frac{\partial A}{\partial t}, \quad 0 < x < d$$

(4)

subject to the boundary conditions

$$\begin{align*}
- \frac{\partial}{\partial x} A(x, t) &= \mu H_1, \quad x = 0 \\
(m d) \frac{\partial}{\partial x} A(x, t) &= A(x, t), \quad x = d
\end{align*}$$

(5) (6)

where

$$m = \frac{\mu_0 a}{2 \mu d}$$

(7)

To obtain eigenvalues and eigenfunctions, the homogeneous version of the BVP (4) - (6) has to be solved for an initial condition that corresponds to a uniform magnetic field $H_0$ in the shield. The magnetic vector potential of a uniform magnetic field is given by

$$A(x, t) = \mu H_0 x + A_0, \quad t = 0$$

(8)

where $A_0$ is an arbitrary constant.

Separation of variables leads to a standard Sturm-Liouville system

$$\begin{align*}
\frac{d^2 N}{dx^2} &= \lambda^2 \mu_0 N, \quad 0 < x < d \\
N'(\lambda,0) &= 0 \\
mdN'(\lambda, d) + N(\lambda, d) &= 0
\end{align*}$$

(9) (10) (11)

The eigenvalues satisfy the eigenvalue equation

$$\tan(\lambda d) = \frac{1}{md\lambda}$$

(12)

Corresponding to the eigenvalues $\lambda_k$, $k=1,2,\ldots$ are the trigonometric eigenfunctions

$$N_k(x) = \frac{1}{M_k} \cos(\lambda_k x)$$

(13)

$$M_k = \frac{(m \lambda_k d)^2 + m + 1}{(m \lambda_k d)^2 + 1}$$

(14)

Using $\xi_k = \lambda_k d$, (13) and (14) transform to

$$N_k(x) = \frac{1}{M_k} \cos(\xi_k \frac{x}{d})$$

(15)

$$M_k = \frac{(m \xi_k)^2 + m + 1}{(m \xi_k)^2 + 1}$$

(16)

### II.3 Eigenfunction expansion

We note that
\[ A_k(x,t) = N_k(x) \exp\left(-\xi_k^2 \frac{t}{\tau}\right) \]  

(16)

verifies the homogeneous version of the BVP (4)-(6). Here \( \tau = \mu ad^2 \) is the penetration time constant into the shield. Therefore the eigenfunction expansion of the magnetic vector potential can be expressed as

\[ A(x,t) = \sum_{k=1}^{\infty} C_k N_k(x) \exp\left(-\xi_k^2 \frac{t}{\tau}\right) \]

(17)

where the set of arbitrary constants \( C_k \) results from the initial condition (8). Finally we get

\[ A(x,t) = -\mu H_0 d^2 \times \]

\[ \sum_{k=1}^{\infty} \frac{1}{\xi_k^2 M_k^2} \cos(\xi_k x/d) \exp\left(-\xi_k^2 \frac{t}{\tau}\right) \]

(18)

The magnetic field strength follows straightforwardly:

\[ H(x,t) = H_0 d \times \]

\[ \sum_{k=1}^{\infty} \frac{1}{\xi_k M_k^2} \sin(\xi_k x/d) \exp\left(-\xi_k^2 \frac{t}{\tau}\right) \]

(19)

Recall that (19) represents the relaxation of a steady magnetic field uniformly distributed in the shield. Based on this eigenfunction expansion periodic non-sinusoidal solutions can be derived.

### III. SHIELDING OF PERIODIC SEQUENCE OF RECTANGULAR PULSES

Based on the eigenfunction expansion (17) we can build up the solution for the periodic sequence of positive rectangular pulses of duration \( T \), period \( T \) and peak value \( H_0 \). We impose continuity conditions at the end points of the two time intervals. The solution for the magnetic field strength is given by

\[ H(x,t) = \begin{cases} 
1-H^I(x,t), & 0 \leq t < T_1 \\
H^II(x,t), & T_1 \leq t < T 
\end{cases} \]

(20)

where

\[ \frac{H^I(x,t)}{H_m} = \sum_{k=1}^{\infty} G_k^I \sin(\xi_k x/d) \exp\left(-\xi_k^2 \frac{t}{\tau}\right) \]

(21)

\[ \frac{H^II(x,t)}{H_m} = \sum_{k=1}^{\infty} G_k^II \sin(\xi_k x/d) \exp\left(-\xi_k^2 \frac{t-T_1}{\tau}\right) \]

(22)

\[ G_k^I = 2 \frac{1 + m^2 \xi_k^2}{\xi_k} \frac{1 - \exp(-\xi_k^2(T-T_1)/\tau)}{m + 1 + m^2 \xi_k^2} \]

(23)

\[ G_k^II = 2 \frac{1 + m^2 \xi_k^2}{\xi_k} \frac{1 - \exp(-\xi_k^2 T_1/\tau)}{m + 1 + m^2 \xi_k^2} \]

(24)

In a similar manner, we obtain the solution for a periodic sequence of triangular pulses. Using superposition, we obtain the solution for any piecewise waveform, approximated by linear segments. To be more specific, in the next Section we consider a symmetrical trapezoidal waveform.

### IV. SHIELDING OF SYMMETRICAL TRAPEZOIDAL WAVEFORMS

To illustrate the approach, we consider an alternating trapezoidal waveform of frequency \( f = 1/T \). The rise, fall and plateau time duration are 0.05, 0.1 and 0.3 of the half period, respectively. The normalized shielded magnetic field as a function of normalized time for different thickness values, frequencies and materials are shown in Figures 4, 5, and 6, respectively. The shielded magnetic field is normalized to the peak value of the external magnetic to be shielded. The time variable is normalized to the period.

![Figure 4 Waveforms of the shielded magnetic field; thickness of the copper shield is indicated (f=200 Hz).](image)

![Figure 5 Waveforms of the shielded magnetic field, for different frequencies (d=7 mm).](image)

The shielding effectiveness defined as the ratio of the r.m.s. values of the shielded magnetic field and the magnetic field to be shielded in shown in Figure 7.
Cylindrical shields allow both quantitative and qualitative illustrations in LF shielding. The magnetic field to be shielded can be created by long solenoids carrying suitable currents to give the desired waveform. For 2D-field analysis, the magnetic field in shielded regions has to be a function of time alone. In fact, from curl $\mathbf{H} = 0$, in current free regions, there results

$$\nabla \times (H_u_z) = (\nabla H) \times u_z = 0$$ (25)

which proves this assertion. Accordingly, the magnetic field in shielded regions is uniformly distributed and varies instantaneously in all points. For the cylindrical shield, the magnetic field in the shielded region $r<a$ is the response of the shield to the external magnetic field. The response provided by (20) corresponds to the periodic sequence of rectangular pulses shown in Figure 3. The response is described by its restriction to the first period. In the next periods the waveform repeats itself. This periodic response reduces to the step response when both the period and duration of rectangular pulse approach infinity. Similarly, the response to the periodic sequence of triangular pulses, shown in Figure 3, reduces to the ramp response. We may notice that eigenfunction expansion method leads to series that are similar to those describing transient responses. We may notice also that, if we restrict the evaluation of the response, for example, to the first three eigenvalues, we can model the conducting shield by an equivalent circuit of order three. Then the periodic solution is the closed form solution of the corresponding Fourier series.

VI. CONCLUSIONS

Low frequency shielding of non-sinusoidal magnetic fields has been analyzed by means of eigenfunction expansion method. The method has been illustrated for one of the basic shielding configuration. Compared with the classical method based on the time-Fourier series of non-sinusoidal waveforms, the eigenfunction expansion method leads to a different series solution, similar to the one obtained in transient analysis. These series shows an accelerated convergence for waveforms and frequencies of interest in power electronics. The method is particularly suitable for non-sinusoidal linear piecewise waveforms. The analytical results included are useful in a shielding test arrangement as well as in teaching shielding theory.

VII. REFERENCES


