Performance of Surface Impedance Integral Equation for Quasistationary Field Analysis in Axisymmetric Systems

Himal C. Jayatilaka and Ioan R. Ciric, *Fellow, IEEE*
Department of Electrical and Computer Engineering
The University of Manitoba, Winnipeg, MB R3T 5V6, Canada
e-mail: himal@ee.umanitoba.ca

Abstract – Surface integral equations satisfied by the induced current density are formulated for axisymmetric solid conductors by applying the surface impedance model. Their performance is investigated employing prolate and oblate conducting spheroids with a large range of geometric parameters. Numerical results generated are compared with available analytical results and with experimental data.

INTRODUCTION

Boundary integral equations are used for solving approximately a wide range of electromagnetic field problems. They have the advantage of requiring less computation than methods based on the discretization of the entire conducting region. The perfect conductor model is commonly used in the formulation of surface integral equations for solid conductors, especially at high frequencies. The validity of this model has been analyzed in [1] for spheroids of various axial ratios by comparison with experimental results.

In this paper, we formulate surface integral equations for axisymmetric conductors in the presence of quasistationary magnetic fields using both the surface impedance and the perfect conductor models. These integral equations are solved numerically for the unknown surface current density by applying a point matching procedure [2]. Power losses and forces are derived from the induced current and computed results are compared with available measured data. The minimum number of necessary unknowns for a desired accuracy is determined for various prolate and oblate spheroidal conductors.

INTEGRAL EQUATION FORMULATION

Consider an arbitrarily shaped axisymmetric good conductor, as depicted in Fig. 1, in the presence of a quasistationary magnetic field produced by coaxial turns carrying sinusoidal with time currents of same frequency. At sufficiently small depths of penetration, the electromagnetic field can be analyzed by determining the equivalent surface current density \( J_s \) which has an azimuthal \( \phi \)-direction. An integral equation satisfied by \( J_s \) is constructed by imposing the condition that the tangential electric field intensity at the conductor surface \( S \),

\[
E_\phi = -\frac{j\omega \mu_0}{4\pi} \left[ u_x \cdot \left( \frac{J_s(r')}{R} \right) \, ds' + \sum_{k=1}^{N} I_k u_\phi \cdot \frac{dl'}{C_k} \right]
\]

is related to the tangential magnetic field intensity,

\[
H_t = \frac{1}{4\pi} \left[ u_x \cdot \left( \frac{J_s(r')}{R^2} \, ds' + \sum_{k=1}^{N} I_k u_\phi \cdot \frac{dl'}{C_k} \right) \right]
\]

through the surface impedance \( Z_s \).

Here, \( R = r - r' \), with \( r \) and \( r' \) being the position vectors of the observation point and the source point, respectively, \( dl' \) is the vector length element in the direction of the current along the respective inducing turn \( C_k \), \( N \) is the total number of current-carrying turns, \( u_x \) and \( u_\phi \) are unit vectors along the
generator curve \( C \) of the conductor and along the azimuthal direction, respectively, \( I_i \) is the current carried by the \( k \)-th turn, and \( Z_i=R_i/(1+\omega R_i) \), with \( R_i=(\omega \mu_b/(2 \pi))^{1/2} \) and \( j=\sqrt{-1} \). \( \omega \), \( \mu_b \) and \( \sigma \) being the angular frequency, the permeability and the conductivity of the conductor, respectively. The formulation for the perfect conductor model is obtained by taking \( Z_i=0 \).

The surface integrals in (1) and (2) are taken in principal values and the singularities in the integrands are evaluated by considering separately the contributions of each rectangular self-patch of dimensions \( g \times h \) to the fields at its center, namely

\[
E'_\phi = \frac{j \omega \mu_0 J_i}{\pi} \left[ g \ln \left( \frac{h}{g} \sqrt{1 + \frac{h^2}{g^2}} \right) + h \ln \left( \frac{h}{h} \sqrt{1 + \frac{h^2}{h^2}} \right) \right]
\]

and

\[
H'_\phi = -\frac{J_i}{2}
\]

Numerical Results and Conclusion

To illustrate the performance of the surface integral equation in (3), we consider an Aluminum \((\sigma = 3.77 \times 10^7 \text{ S/m})\) prolate spheroid of major and minor semi-axes \( a_0 \) and \( b_0 \), respectively, in the presence of three coaxial turns connected in series and carrying a current \( I_0 \). The three turns are placed on the surface of a cone of opening \( 2\theta \), as shown in Fig. 1, the distance between the plane of the lower turn and the spheroid center being \( d_1 \). Due to the axisymmetry of the system, the induced current density has a component only in the \( \phi \)-direction. The conductor surface is discretized into a number of \( M \) coaxial rings and their vector potential and magnetic field intensity are computed by employing Biot-Savart formulas. Each ring and each external current-carrying turn is divided into a number of elements to evaluate their vector potential and magnetic field intensity. The current density of each ring is considered to be constant and concentrated on the centerline of the ring, except wherever the source point coincides with the observation point, when the contributions in (4), (5) are used. The integral equation is finally reduced to a system of \( M \) linear equations with \( M \) unknowns, which are the current densities of the rings. The force acting upon the spheroid is evaluated by computing the force upon the inducing turns, which is the same in magnitude but opposite in direction. The accuracy of the numerical results for the forces and for the power loss in the conductor is increased to a desired level by increasing the number of elements on the conductor surface and by comparison with available analytical results and experimental data. The force normalized to \( \mu_0 J_0^2 \) for two frequencies and various geometric parameters is plotted in Figs. 2 and 3 as a function of \( d_1/b_1 \). A good agreement with experimental data is achieved in the cases considered when using a number of about forty rings and one hundred elements on each ring. The improvement brought by the surface impedance model over the perfect conductor one, especially at smaller distances between the inducing turns and the induced body, is shown in Fig. 3.

Fig. 2. Normalized force versus \( d_1/b_1 \) for an Aluminum conducting spheroid at 8 KHz, with \( N=3 \), \( \tan \beta=0.4 \), \( d/b_1=0.25 \), \( b_0/b_1=0.5 \) and \( b_0=2 \text{ cm} \): (I) \( b_0/a_0=0.6 \); (II) \( b_0/a_0=0.4 \).

Fig. 3. Normalized force versus \( d_1/b_1 \) for an Aluminum conducting spheroid at 2 KHz, with \( N=3 \), \( \tan \beta=0.4 \), \( d/b_1=0.25 \), \( b_0/b_1=0.75 \), \( b_0/a_0=0.6 \) and \( b_0=2 \text{ cm} \).

References
