Formulation of reduced surface integral equations for the electromagnetic wave scattering from three-dimensional layered dielectric bodies

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[1] A reduction procedure is developed for an arbitrarily shaped layered dielectric body using for each interface a single unknown function to which the classical surface electric and magnetic currents are related by some surface operators. These operators and single functions are determined recursively from one interface to the next. This allows us to derive the field everywhere from the solution of a surface integral equation in only one vector function relative to only the interface between the layered body and the source region. Since the reduction operators are independent of the structure of the outside region and of the given field source, and also invariant under translation and rotation, the analysis of the three-dimensional electromagnetic wave scattering and propagation for systems of multilayered or/and multiply nested dielectric bodies based on reduced single integral equations is substantially more efficient than that based on existing coupled integral equation formulations using electric and magnetic currents on all the interfaces, especially for configurations with identical such bodies arbitrarily located and oriented with respect to each other.

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1. Introduction

[2] Time-harmonic electromagnetic fields in the presence of heterogeneous media can efficiently be analyzed using surface integral equations. Within each homogeneous subregion the field can be represented in terms of unknown electric and magnetic currents defined over the boundary of that subregion. A direct imposition of the interface conditions yields systems of coupled integral equations in the densities of these electric and magnetic currents. There are two main disadvantages of the formulations based on such equations. First, there are two unknown vector functions to be determined over all the interfaces and, second, all these unknown functions are to be determined simultaneously, which makes the size of the resultant matrix equation in the numerical computation very large. A more efficient alternative is to use surface integral equations satisfied by only one unknown function per interface.

[3] A formulation using a single unknown surface function for the scattering of a plane wave by a penetrable homogeneous long cylinder of arbitrary cross section was first presented by Maystre and Vincent [1972]. Later, a recursive procedure making possible the analysis of plane-wave scattering from a two-dimensional layered structure using an integral equation involving a single unknown function only over the outer surface was developed [Maystre, 1978] for applications to periodical structures in optical gratings. Marx [1982] extended the construction of a single surface integral equation to the three-dimensional electromagnetic scattering of time-harmonic and, also, general time-varying fields from homogeneous dielectric bodies. Glisson [1984] showed how to derive such a single integral equation for time-harmonic fields based on equivalence theorems. Recently, general recursive procedures for two-dimensional systems of heterogeneous bodies and complex nested structures, incorporating the properties of invariance to translation and rotation of the reduction operators, have been presented and numerically implemented [Swatek and Ciric, 1998, 2000a, 2000b]. As well, reduced surface integral equations have been derived for the analysis of two-dimensional eddy-current fields in solid conductors [Ciric and Curiae, 2005] and of Laplacian fields in the presence of layered dielectric structures [Ciric, 2006].

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[5] The aim of this paper is to present the formulation of reduced vector integral equations for the three-dimensional electromagnetic wave scattering from arbitrarily shaped layered bodies. Expressions in a matrix form of the integral operators involved, adequate for computer implementation, and a detailed analysis of the high efficiency of the numerical solution of field problems using the proposed formulation will be presented in subsequent papers.

2. Electromagnetic Field Modeling

[5] A time-harmonic electromagnetic field \((E, H)\) in a sourceless, homogeneous region \(V\) of permittivity \(\varepsilon\) and permeability \(\mu\), bounded by a smooth surface \(S\) can be represented as [Stratton, 1941]

\[
E(r) = \oint_{S} \left\{- \nabla G \times J^m(r') \right\} dS' - \frac{j}{\omega \varepsilon} \nabla \times \left[ \nabla G \times J(r') \right] dS' \tag{1}
\]

\[
H(r) = \oint_{S} \left\{ \nabla G \times J(r') \right\} dS' - \frac{j}{\omega \mu} \nabla \times \left[ \nabla G \times J^m(r') \right] dS' \tag{2}
\]

where a time dependence \(\exp(j \omega t), j \equiv \sqrt{-1}\), has been assumed and suppressed, \(J\) and \(J^m\) have dimensions of surface densities of electric current and magnetic current, respectively,

\[
J^e = -\hat{n} \times H, \quad J^m = \hat{n} \times E \tag{3}
\]

with \(\hat{n}\) the unit vector normal to \(S\) and outwardly oriented, and \(G\) is a Green function for an unbounded homogeneous space,

\[
G = \frac{e^{-j \beta R}}{4 \pi R}, \quad \beta \equiv \omega \sqrt{\varepsilon \mu}, \quad R = |r - r'|. \tag{4}
\]

Consider a layered body as shown in Figure 1, each subregion \(V_i\) consisting of a homogeneous dielectric of material constants \(\varepsilon_i, \mu_i\), and a given incident field \((E_{inc}, H_{inc})\) in the region \(V_0\) outside the outermost surface \(S_1\). The electric and magnetic field intensities \(E_i\) and \(H_i\) in \(V_i\), \(i = 0, 1, 2, \ldots, n\), satisfy the interface conditions

\[
\hat{n}(r) \times E_i(r) = \hat{n}(r) \times E_{i+1}(r), \tag{5}
\]

\[
\hat{n}(r) \times H_i(r) = \hat{n}(r) \times H_{i+1}(r), \tag{6}
\]

\[r \in S_{i+1}, \quad i = 0, 1, 2, \ldots, n - 1.\]

Using the representation (1)–(2) for each subregion of the body, the application of the conditions in (5), (6) yields the classical system of coupled integral equations with two unknown vector functions \(J^e_{i+1}\) and \(J^m_{i+1}\) on each interface \(S_{i+1}\).

[6] Assume that instead of the representation (1)–(2), \(E_i\) and \(H_i\) in \(V_i\) can be represented in terms of \(J^e_{i+1}\) and \(J^m_{i+1}\) only over \(S_{i+1}\) (as in (1), (2)), and of only a single unknown function \(J_i\) defined over \(S_i\). Below, it is shown that both \(J^e_{i+1}\) and \(J^m_{i+1}\) on each interface can be expressed in terms of \(J_i\) on the same interface, and a recursive relationship between the \(J_i\) 's on successive interfaces can be derived, from \(J_n\) to \(J_1\). This will allow us to determine \(J_1\) on \(S_1\) from a single integral equation relative to \(S_1\), which involves the given incident field in \(V_0\). We use the term reduced surface integral equation [Ciric, 2006] for such an equation since it refers to only one interface instead of all the interfaces. The scattered field in \(V_0\) is calculated from only \(J_1\). To illustrate the derivation of a reduced surface integral equation, take \(J_i\) to be the surface density of an electric current. Then, \(E_i\) and \(H_i\), \(i = 1, 2, \ldots, n - 1\), due to \(J_i\), \(J^e_{i+1}, J^m_{i+1}\), are expressed as

\[
E_i(r) = \int_{S_i} \left\{ - \frac{j}{\omega \varepsilon_i} \nabla \times \left[ \nabla G_i \times J_i(r') \right] \right\} dS' \tag{7}
\]

\[
H_i(r) = \int_{S_i} \nabla G_i \times J_i(r') dS' + \int_{S_{i+1}} \left\{ - \frac{j}{\omega \mu_i} \nabla \times \left[ \nabla G_i \times J^m_{i+1}(r') \right] \right\} dS', \tag{8}
\]

where \(G_i\) is \(G\) in (4) with \(\beta\) replaced by \(\beta_i = \omega \sqrt{\varepsilon_i \mu_i}\), and \(J^e_{i+1}\) and \(J^m_{i+1}\) are defined with respect to the direction of the normal in Figure 1, i.e. (see (1)–(3)),

\[
J^e_{i+1} = \hat{n} \times H_i, \quad J^m_{i+1} = -\hat{n} \times E_i, \tag{9}
\]

\[\hat{n}, E_i, H_i\] on \(S_{i+1}\).
In $V_n$, whose only boundary is $S_n$, $E_n$ and $H_n$ are only given by the first integral in (7) and (8) (with $i = n$), respectively,

$$E_n(r) = \int_{S_n} \left\{-\frac{j}{\omega \varepsilon_n} \nabla \times [\nabla G_n \times J_n(r')] \right\} dS', \quad r \in V_n$$

$$H_n(r) = \int_{S_n} \nabla G_n \times J_n(r') dS', \quad r \in V_n$$

while the scattered fields $E_{sc}$ and $H_{sc}$ in $V_0$ are only given by the second integral in (7) and (8) (with $i = 0$)

$$E_{sc}(r) = \int_{S_0} \left\{ \left\{-\nabla G_0 \times J_0'(r') \right\} - \frac{j}{\omega \varepsilon_0} \nabla \times [\nabla G_0 \times J_0'(r')] \right\} dS', \quad r \in V_0$$

$$H_{sc}(r) = \int_{S_0} \left\{ \nabla G_0 \times J_0'(r') \right. \left. - \frac{j}{\omega \mu_0} \nabla \times [\nabla G_0 \times J_0''(r')] \right\} dS', \quad r \in V_0.$$
where \( u \) is the surface density of electric or magnetic current, and the sign of the first term in the right-hand side is \(-\) when \( S_q = S_{i+1} \) and \(+\) when \( S_q = S_i \). Thus, from (7) and (8) we obtain on \( S_i \) and \( S_{i+1} \), just inside \( V_i \), \( i = 1, 2, \ldots, n - 1 \),

\[
\hat{n} \times E_i = -j \frac{1}{\omega \varepsilon_i} [Q_i J_i - \frac{1}{\omega \mu_i} [i + i + 1] Q_i J_{i+1}^m, \quad r \in S_i
\]

\[
\hat{n} \times H_i = \left( \frac{1}{2} I + i + 1 \right) L_i J_{i+1}^m - \frac{1}{\omega \mu_i} i + 1 Q_i J_i^m, \quad r \in S_{i+1}
\]

(17)

with \([\cdot]^{-1}\) denoting the inverse of an operator. From (21) and (17), with (22) and (23), we obtain the recursion

\[
T_i^e = \left( 1 + \frac{j}{\omega \varepsilon_i} L_i T_{i+1}^e - j \frac{i + 1}{\omega \varepsilon_i} Q_i T_{i+1}^m, r \in S_{i+1}, \right)
\]

(25)

Similarly, from (21) and (19), with (22) and (23), another recursion is obtained, i.e.

\[
T_i^m = \left( \frac{j}{\omega \mu_i} + j \frac{i + 1}{\omega \varepsilon_i} Q_i T_{i+1}^e + i + 1 L_i T_{i+1}^m, r \in S_{i+1}, \right)
\]

(26)

The following theorem summarizes the above results: Let the electromagnetic field in each layer of a layered dielectric body be represented as in (7)–(9). Then, the recursions (23)–(26) exist for the function \( J_i \) in (7), (8) and for the surface operators \( T_i^e \) and \( T_i^m \) in (21).

The determination of the operators \( T_i^e \) and \( T_i^m \) is performed recursively from \( S_n \) to \( S_1 \), while the functions \( J_i \) are computed recursively from \( J_1 \) to \( J_m \) with \( J_1 \) obtained as a solution of a reduced surface integral equation relative to \( S_1 \), which involves the given incident field in \( V_0 \). Since the fields in (7), (8) and (10)–(13) are Maxwellian and, with (22), all the boundary conditions are satisfied, according to the uniqueness theorem the equations (7), (8) and (10)–(13) give the electromagnetic field everywhere if the unknown function \( J_1 \) on \( S_1 \) is uniquely determined in terms of the given \((E_{inc}, H_{inc})\) in \( V_0 \).

### 3. Reduced Boundary Integral Equations

The total fields in the homogeneous unbounded region \( V_0 \) are

\[
E_0 = E_{inc} + E_{sc}, \quad H_0 = H_{inc} + H_{sc}, \quad r \in V_0
\]

(27)

where the scattered electromagnetic field \((E_{sc}, H_{sc})\) is given in (12), (13) and satisfies the usual far-field radiation condition. Imposing the boundary conditions (5), (6) for \( S_1 \) (i.e., for \( i = 0 \) and using (21) we have

\[
\hat{n} \times (E_{inc} + E_{sc}) = T_i^e J_1
\]

(28)

\[
\hat{n} \times (H_{inc} + H_{sc}) = T_i^m J_1, \quad r \in S_1.
\]

(29)
Substituting $J^e_1$ and $J^m_1$ from (22) (with $i = 0$) in (12), (13) and using the operators in (14), (15) yields

$$\hat{n} \times E_{sc} = \left[ \left( \frac{1}{2} I + \frac{1}{\omega_0^2} \right) T^e_1 - \frac{j}{\omega_0} \frac{1}{Q_0} T^m_1 \right] J_1$$  \hspace{2cm} (30)

$$\hat{n} \times H_{sc} = \left[ \left( \frac{1}{2} I + \frac{1}{\omega_0^2} \right) T^m_1 + \frac{j}{\omega_0} \frac{1}{Q_0} T^e_1 \right] J_1.$$  \hspace{2cm} (31)

Two reduced integral equations in $J_1$ are obtained, namely, from (28) and (30) an electric field integral equation in the form

$$\left[ \left( \frac{1}{2} I + \frac{1}{\omega_0^2} \right) T^e_1 + \frac{j}{\omega_0} \frac{1}{Q_0} T^m_1 \right] J_1 = \hat{n} \times E_{inc},$$  \hspace{2cm} (32)

and from (29) and (31) a magnetic field integral equation in the form

$$\left[ \left( \frac{1}{2} I + \frac{1}{\omega_0^2} \right) T^m_1 - \frac{j}{\omega_0} \frac{1}{Q_0} T^e_1 \right] J_1 = \hat{n} \times H_{inc},$$  \hspace{2cm} (33)

$[11]$ $T^e_1$ and $T^m_1$ in these reduced equations are determined starting with the innermost region $V_n$ and, then, performing the recursions (25), (26) outwardly from $S_n$ to $S_1$. Equations (10) and (11) give
defined over the outer surface $S_1$ of the body. The field scattered in $V_0$ is determined from (12), (13), with (22) for $i = 0$. If needed, the field within a layer $\ell$, $1 \leq \ell \leq n - 1$, inside the body is derived by calculating $J_2$, $J_3$, ..., $J_{\ell+1}$ with (23) and, then, $J^e_{\ell+1}$ and $J^m_{\ell+1}$ with (22), applying the operators $R_2$, $R_3$, ..., $R_{\ell+1}$ and $T^e_{\ell+1}$, $T^m_{\ell+1}$ already determined; $E_1$ and $H_1$ are obtained from (7), (8), with $i = \ell$. $E_0$ and $H_0$ in $V_0$ are determined by (10), (11).  

[13] One can easily see that for the special case of a single homogeneous dielectric body of permittivity $\varepsilon_1$ and permeability $\mu_1$, bounded by $S_1$, one has (see (36))

$$T^e_1 = -\frac{j}{\omega_0^2} \frac{1}{Q_1}, \hspace{0.5cm} T^m_1 = -\frac{1}{2} I + \frac{1}{\omega_0^2} \frac{1}{Q_1} C_1.$$  \hspace{2cm} (37)

Equations (32) and (33) become, respectively,

$$\left[ \frac{j}{\omega_0} \frac{1}{Q_1} + \frac{j}{\omega_0} \frac{1}{Q_0} \left( \frac{1}{2} I - \frac{1}{\omega_0^2} \frac{1}{Q_1} C_1 \right) \right] J_1 = \hat{n} \times E_{inc},$$  \hspace{2cm} (38)

$$\left[ \frac{j}{\omega_0^2} \frac{1}{Q_0} \left( \frac{1}{2} I - \frac{1}{\omega_0^2} \frac{1}{Q_0} \left( \frac{1}{2} I - \frac{1}{\omega_0^2} \frac{1}{Q_1} C_1 \right) \right) \right] J_1 = \hat{n} \times H_{inc},$$  \hspace{2cm} (39)

which were previously presented [Martin and Ola, 1993; Yeung, 1999; Ciric and Jayasekera, 2007].

[14] The reduced integral equations for a perfect conductor occupying the region $V_n$ coated with $n - 1$ dielectric layers, $V_{n-1}$, $V_{n-2}$, ..., $V_1$ are obtained from (32), (33) taking into account that the surface current density $J_n$ is just the density of the actual current induced on its surface $S_n$ and

$$T^e_n = -\frac{j}{\omega} \frac{1}{Q_n}, \hspace{0.5cm} T^m_n = -\frac{1}{2} I + \frac{1}{\omega} \frac{1}{Q_n} n C_n.$$  \hspace{2cm} (40)

and, thus (see (21))

$$T^e_n = -\frac{j}{\omega} \frac{1}{Q_n}, \hspace{0.5cm} T^m_n = -\frac{1}{2} I + \frac{1}{\omega} \frac{1}{Q_n} n C_n.$$  \hspace{2cm} (36)

Using these expressions and (24)–(26) we obtain $R_n$ and $T^e_{n-1}$, $T^m_{n-1}$ for $S_{n-1}$ and then, recursively, all $R_n$, $T^e_n$, $T^m_n$, $i = n - 2$, $n - 3$, ..., 1. It is important to remark that the operators $R_{n+1}$, $T^e_{n+1}$, $T^m_{n+1}$ associated with $S_{n+1}$ take into account only the geometric and material structure of the region inside $S_n$ being independent of the geometry and material characteristics outside $S_n$ and of the incident field.

[12] In the model presented, the problem of electromagnetic wave scattering from a layered dielectric body is solved by solving a surface integral equation only in $J_1$ and using the operators in (14), (15) yields

$$\hat{n} \times E_{inc} = \left[ \left( \frac{1}{2} I + \frac{1}{\omega_0^2} \right) T^e_1 - \frac{1}{\omega_0} \frac{1}{Q_0} T^m_1 \right] J_1$$  \hspace{2cm} (30)

and from (29) and (31) a magnetic field integral equation in the form

$$\left[ \left( \frac{1}{2} I + \frac{1}{\omega_0^2} \right) T^m_1 + \frac{j}{\omega_0} \frac{1}{Q_0} T^e_1 \right] J_1 = \hat{n} \times H_{inc},$$  \hspace{2cm} (33)

4. Remarks

[16] The reduced surface integral equations presented involve only one unknown vector function defined over the outermost surface of a layered body, with the reduction procedure performed using operators relative
to individual interfaces. In the numerical computation, these operators are converted into matrices whose size is determined only by the number of unknowns on the respective individual interfaces. Multiplication and inversion of various operators become multiplication and inversion of the corresponding matrices. The amount of numerical computation needed in the proposed procedure increases practically proportionally with the number of layers, whereas an increase with the square of the number of layers is encountered when solving the sparse systems of algebraic equations resulting from the simultaneous solution of the systems of classical coupled integral equations. Reduced surface integral equations are most advantageous for the analysis of the wave scattering from a system of identical multilayered or multiply nested dielectric bodies when the reduction procedure is performed only once, for one of the bodies, since the reduction operators depend only on the geometry and material of the body, being independent of the structure of the outside region and of the incident field. Once these operators are constructed, they can be reused for various relative positions of the bodies and different incident fields, thus reducing substantially the computational effort required in design and optimization studies. It should be pointed out that a reduction procedure is possible even for coupled surface integral equations, such that the scattered field can be determined from two unknown vector functions defined over the outermost surface of the bodies. But, the field analysis based on such reduced coupled integral equations would require, for same accuracy, about ten times more numerical computation than when applying the reduced single integral equations presented in this paper.

[17] Three other kinds of reduced single surface integral equations can be derived for the electromagnetic scattering by a layered dielectric body. Namely, another equation is obtained if instead of representing the fields in \( V \) with \( J_\text{o} \) on \( S_\text{i} \) and \( J_{\text{r},1}, J'_{\text{r},1} \) on \( S_{\text{r},1} \) we use \( J'_\text{o}, J''_\text{o} \) on \( S_\text{i} \) and \( J_{\text{r},1}, J'_{\text{r},1} \) on \( S_{\text{r},1} \); two more kinds of integral equations are obtained by employing instead of a single electric current a single magnetic current on each interface.

[18] In order to derive correct results at “irregular” frequencies corresponding to internal resonances, one can use instead of the unknown surface current density \( J_\text{i} \) on \( S_\text{i} \), a combination of electric and magnetic surface currents expressed simply in terms of a single unknown vector function, as in the case of a homogeneous body [Mautz, 1989; Martin and Ola, 1993], or a combined electric field-magnetic field integral equation ((32)–(33)), with the optimum combination from point of view of computational efficiency determined from numerical experiments, as shown for a homogeneous dielectric sphere by Yeung [1999]. Properties of the integral operators used in the construction of the reduced surface integral equations, potentially useful for numerical computations, should be investigated as in the case of classical operators involved in the coupled surface integral equations [Hsiao and Kleinman, 1997].

[19] The conversion into matrices of the surface operators introduced in this paper, necessary for a numerical implementation of the reduced integral equations presented, is shown in a next paper, where the computational complexity and the overall efficiency of the proposed method are also dealt with. The computational effort required in the case of a system of a few identical layered bodies is greatly reduced. For example, for a system of three identical three-layer dielectric bodies in arbitrary configuration, the number of arithmetic operations involved is more than an order of magnitude smaller than that when applying existing methods based on coupled surface integral equations.

References


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