Performance analysis of multiply connected thin shields

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Abstract. By performing holes in a thin electromagnetic shield, its weight and price can be reduced, without practically changing the shielding efficiency for certain geometries and frequencies. A new method for solving the integral equation of the surface current density in thin shields with holes is applied, where the current density is represented by a linear combination of specialized surface vector functions associated with the interior nodes of the discretization mesh and with the sets of nodes on each hole contour. A Galerkin technique is applied to determine the scalar coefficients of these functions. The magnetic flux density in the shielding zone is obtained accurately, in terms of exact analytical formulas, from these vector functions.

Keywords: Shielding efficiency, current sheet integral equations, thin shields with holes

1. Introduction

In numerous applications, the cost and the weight of the electromagnetic shields could be reduced by performing holes in the shield plates. As well, an important role of the holes is to allow for an adequate installation of the measurement and control devices. If the main path of the current induced in the shield is not perturbed substantially in the presence of the holes, then, the shielding efficiency only has a slight decrease. The solution methods for the quasistationary steady-state fields associated with the shields include various finite difference and finite element techniques and, also, surface integral equations specifically formulated for the case of thin shields. When the shield thickness is much smaller than the rest of its linear dimensions, application of the usual finite element method is not adequate. Modelling the shields using the surface impedance concept avoids the discretization of the region inside the conducting material \cite{1-3}. A hybrid finite element - boundary element technique can be employed \cite{4} to avoid discretizing the region outside the shield but, the fine discretization mesh inside the shield yields large system matrices that are not so well-conditioned. If the shield thickness is much smaller than the depth of penetration of the electromagnetic field, the density of the induced electric current can be considered to be constant across the shield sides and distributed in the form of a current sheet whose density vector is tangentially oriented. This density can be expressed in terms of a surface scalar function such that

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the current continuity conditions are fulfilled, as shown in [5], where a Lagrangian equation formulation using as unknowns the nodal values of this scalar function has been presented for shields with no holes.

In a recent work [6], the current sheet density induced in thin shields is determined as the solution of a surface integral equation. A polyhedral mesh with triangular surface elements is defined on the shield and the current density is represented as a linear combination of specialized surface vector functions associated with the interior nodes of the discretization mesh and with the sets of nodes on each hole contour. The important features of this method consist in a small number of unknowns, namely, equal to the sum of the number of interior nodes and of the number of holes, and in an efficient modeling in the case of multiply connected shields.

In this paper, the surface current density integral equation is applied to analyze the efficiency of the thin shields with holes. The magnetic flux density is easily obtained in terms of the specialized surface vector functions used and the shielding efficiency is evaluated by comparing the flux density at specified locations in the presence and in the absence of the shields.

2. Current sheet computations

The time-harmonic integral equation satisfied by the surface density $J_s$ of the current induced over the surface $S$ of a nonmagnetic thin shield is [6]

$$\rho_s J_s(r) + j\lambda \int_S \frac{J_s(r')}{R} dS' = -j2\pi f A_0(r) - \nabla V(r)$$

(1)

where $\rho_s = \rho/\Delta$ is the surface resistivity, $\Delta$ the shield thickness, $j = \sqrt{-1}$, $\lambda = f\mu_0/2$ with $f$ the frequency and $\mu_0$ the permeability of free space, $R = |r - r'|$ with $r$ and $r'$, respectively, the position vectors of the observation point and of the source point, $A_0$ is the magnetic vector potential due to external sources, and $-\nabla V$ is the electric scalar potential component of the electric field intensity. The boundary condition on $S$ is $n \cdot J_s = 0$, where $n$ is the unit vector of the local normal direction. The presence of $-\nabla V$ in Eq. (1) can be ignored [6].

The sheet surface $S$ is approximated to be a polyhedral surface with triangular surface elements. To each node $i$ we associate a surface vector function $U_i$, such that on each element $k$ containing the node $i$ (see Fig. 1),

$$U_i^{(k)} = \frac{1}{2S_k} l_i^{(k)}$$

(2)

where $l_i^{(k)}$ is the length vector along the edge of the element $k$ that is opposed to the node $i$, oriented as shown in Fig. 1, and $S_k$ is the area of the element $k$. The function $U_i$ is null over all the surface elements that do not contain the node $i$.

$J_s$ over $S$ is written in the form

$$J_s(r) \approx \sum_{i=1}^{N} \alpha_i U_i(r) + \sum_{m=1}^{N_h} \beta_m W_m(r)$$

(3)

where the functions $U_i$ are associated with the $N$ interior nodes of $S$ and the functions $W_m$ are associated with the contours of the $N_h$ holes of the shield,

$$W_m(r) = \sum_{i \in \{m\}} U_i(r)$$

(4)
with \( \{m\} \) representing the set of nodes \( i \) on the contour of the hole \( m \). By expressing the surface current density as a linear combination of the functions \( U_i \) and \( W_m \) defined by Eqs (2) and (4), the zero divergence condition for the current density is satisfied everywhere over the discretized \( S \).

The unknown coefficients \( \alpha_i \) and \( \beta_m \) in Eq. (3) are determined by taking the inner products of the two sides of Eq. (1) with \( U_n, n = 1, 2, \ldots, N \), and with \( W_{n'}, n' = 1, 2, \ldots, N_h \). Integrating both sides over the discretized \( S \) yields the following system of \( N + N_h \) algebraic equations in \( \alpha_i \) and \( \beta_m \):

\[
\sum_{i=1}^{N} A_{ni} \alpha_i + \sum_{m=1}^{N_h} B_{nm} \beta_m = C_n, \quad n = 1, 2, \ldots, N + N_h
\]  

(5)

with

\[
A_{ni} = a_{ni} + j\lambda a'_{ni}, \quad B_{nm} = b_{nm} + j\lambda b'_{nm}
\]  

(6)

where

\[
a_{ni} = \int_S \rho_s(r) U_n(r) \cdot U_i(r) dS
\]  

(7)

\[
b_{nm} = \int_S \rho_s(r) U_n(r) \cdot W_m(r) dS
\]  

(8)

\[
a'_{ni} = \int_S \int_{S'} \frac{1}{R} U_n(r) \cdot U_i(r') dS dS'
\]  

(9)

\[
b'_{nm} = \int_S \int_{S'} \frac{1}{R} U_n(r) \cdot W_m(r') dS dS'
\]  

(10)

\[
C_n = -j2\pi f \int_S U_n(r) \cdot A_0(r) dS
\]  

(11)

for \( n = 1, 2, \ldots, N \) and

\[
a_{ni} = \int_S \rho_s(r) W_{n'}(r) \cdot U_i(r) dS
\]  

(12)
\[ b_{nm} = \int_{S} \rho_{s}(r) W_{n'}(r) \cdot W_{m}(r) dS \]  \hspace{1cm} (13)

\[ a'_{ni} = \int_{S} \int_{S'} \frac{1}{R} W_{n'}(r) \cdot U_{i}(r') dS dS' \]  \hspace{1cm} (14)

\[ b'_{nm} = \int_{S} \int_{S'} \frac{1}{R} W_{n'}(r) \cdot W_{m}(r') dS dS' \]  \hspace{1cm} (15)

\[ C_{n} = -j2\pi \int_{S} W_{n'}(r) \cdot A_{0}(r) dS \]  \hspace{1cm} (16)

for \( n = N + 1, N + 2, \ldots, N + N_{h} \), with \( r' \equiv n - N \). The system in Eq. (5) has complex coefficients and only \( N + N_{h} \) unknowns, i.e. \( \alpha_{i} \) and \( \beta_{m} \).

3. Flux density and shielding efficiency

The magnetic flux density can be calculated by applying the Biot-Savart formula. Taking into account Eqs (3) and (2), (4), gives the flux density due to the induced current sheet:

\[ B(r) = \frac{\mu_{0}}{8\pi} \sum_{k=1}^{F} \left( \frac{1}{S_{k}} \sum_{i \in \{k\}} \gamma_{i} t_{i}^{(k)} \right) \times \int_{S_{k}} \frac{R}{R^{3}} dS' \]  \hspace{1cm} (17)

where \( F \) is the number of surface elements, \( \{k\} \) is the set of the nodes \( i \) belonging to the element \( k \), \( R = r - r' \), and \( \gamma_{i} \) has the value \( \alpha_{i} \) for an interior node \( i \) and \( \beta_{m} \) for the nodes \( i \) belonging to the contour of the hole \( m \).

The contribution of a surface element \( k \) can be expressed using three terms, corresponding to the element edges. For the edge \( l_{i}^{(k)} \) we have

\[ B_{i}^{(k)}(r) = \frac{\mu_{0} \gamma_{i}}{8S_{k}} \left[ \int_{S_{k}} \left( n_{k} \cdot \frac{l_{i}^{(k)} \times R}{R^{3}} \right) n_{k} dS' + \int_{S_{k}} \left( \frac{h_{i}^{(k)}}{h_{i}^{(k)}} \cdot \frac{l_{i}^{(k)} \times R}{R^{3}} \right) h_{i}^{(k)} dS' \right] \]  \hspace{1cm} (18)

where \( h_{i}^{(k)} \) is the altitude vector corresponding to the node \( i \) of the element \( k \) (see Fig. 1). The first term in the brackets of Eq. (18) can be written as

\[ \int_{S_{k}} \left( n_{k} \cdot \frac{l_{i}^{(k)} \times R}{R^{3}} \right) n_{k} dS' = - \left( l_{i}^{(k)} \cdot \oint_{\partial S_{k}} \frac{dR}{R} \right) n_{k} \equiv - \left( l_{i}^{(k)} \cdot Y_{k}(r) \right) n_{k} \]  \hspace{1cm} (19)

and the second one as

\[ \int_{S_{k}} \left( \frac{h_{i}^{(k)}}{h_{i}^{(k)}} \cdot \frac{l_{i}^{(k)} \times R}{R^{3}} \right) h_{i}^{(k)} dS' = - \int_{S_{k}} l_{i}^{(k)} \left( \frac{n_{k} \cdot R}{R^{3}} \right) h_{i}^{(k)} dS' = -n_{k} \times l_{i}^{(k)} \Omega_{k}(r) \]  \hspace{1cm} (20)
where $\Omega_k$ is the solid angle under which the facet $S_k$ is seen from the observation point $P$ (see Fig. 1). Therefore, the contribution of the surface element $k$ to the flux density is finally written in the form

$$B^{(k)}(r) = - \left( L^{(k)} \cdot Y_k(r) \right) n_k + L^{(k)} \times n_k \Omega_k(r)$$

(21)

where

$$L^{(k)} \equiv \sum_{i \in (k)} \gamma_i I_i^{(k)}, \quad Y_k(r) \equiv \oint_{\partial S_k} \frac{dR}{R}$$

(22)

Since $Y_k$ and $\Omega_k$ can be determined from exact analytical expressions, the magnetic flux density can be efficiently and accurately calculated.

The shielding efficiency at any point is defined as

$$\eta \equiv \frac{B}{B_0}$$

(23)

where $B$ and $B_0$ are the flux densities in the presence and in the absence of the shield, respectively. For a global evaluation of the shielding we determine

$$\eta_g = \frac{\sum_{p=1}^{M} B_p}{\sum_{p=1}^{M} B_{0p}}$$

(24)

where the flux density is computed at $M$ points chosen within the region of interest.
Fig. 3. Current density lines at a phase of $2\pi f t \approx 51$ degrees for a shield with $\Delta = 5$ mm in a uniform field at $f = 50$ Hz.

Fig. 4. Current lines at a phase of $2\pi f t \approx 88$ degrees for a shield with $\Delta = 1$ mm in the field of a coil of 50 mm radius at $f = 5$ kHz.

4. Illustrative results

Consider two pairs of copper shields of resistivity $\rho = 2 \times 10^{-8} \Omega \text{m}$ occupying a portion of a parabolic surface of equation $x^2 + y^2 = z/5$, namely the portion with $\sqrt{x^2 + y^2} \in [0, 0.1 \text{ m}]$. For the first pair the thickness is $\Delta = 5$ mm and for the second one $\Delta = 1$ mm. In each of the pairs, one shield is without holes, while the other one has seven holes, as shown in Figs 2–5. The shields are placed in the vicinity of coaxial circular coils of radii of 50 mm and 100 mm, placed on the $z = 0$ plane and carrying a sinusoidal current with frequencies of 5 kHz and 50 Hz, respectively, and, also, in a practically uniform field produced by a 1 m radius coil.

The mesh for the shields with holes is plotted in Fig. 2 and the current lines when the shield with $\Delta = 5$ mm is introduced in a uniform magnetic field of frequency $f = 50$ Hz are drawn in Fig. 3. For a shield with holes and $\Delta = 1$ mm in the presence of coils of radii of 50 mm and 100 mm, with $f = 5$ kHz, the
Fig. 5. Current lines at a phase of $2\pi f \approx 88$ degrees for a shield with $\Delta = 1$ mm in the field of a coil of 100 mm radius at $f = 5$ kHz.

Fig. 6. Shielding efficiency: (a) shield with $\Delta = 1$ mm in a uniform magnetic field at $f = 5$ kHz; (b) shield with $\Delta = 1$ mm in the field of a coil of a diameter of 100 mm at $f = 5$ kHz.

Current density lines are shown in Figs 4 and 5, respectively.

The shielding efficiency is computed for points on the line segment $x \in (-50 \text{ mm}, 50 \text{ mm})$, $y= 0$, $z= 20 \text{ mm}$ and is given in Fig. 6.

The Table 1 gives the global shielding efficiency (see Eq. (24)) for shields without and with holes, for different frequencies ($f$) of the external field, thicknesses ($\Delta$), and field coil diameters ($D$).

5. Conclusion

The method employed for the analysis of the quasistationary steady-state fields is based on the surface integral equation satisfied by the current sheet density induced in thin metallic shields. This is solved numerically by using a Galerkin method with a novel choice of the set of vector basis functions such that the current continuity is ensured everywhere. The total number of unknowns is equal to the sum
of the number of interior nodes of the triangular mesh used and the number of holes in the shield. The specialized vector functions proposed to represent the induced current sheet density allow for a calculation of the magnetic flux density in terms of exact analytical formulas.

The application of the above computational procedure proves to be especially useful for the analysis of thin shields with holes which are lighter than the shields without holes and allow a convenient connection of the measurement and control devices, while the global shielding efficiency is almost the same, as shown in Fig. 6 and Table 1.

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References