Field Analysis for Thin Shields in the Presence of Ferromagnetic Bodies

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Application of the current sheet integral equation for an efficient analysis of the electromagnetic field in the presence of thin shields is limited to the case of nonferromagnetic and homogeneous media. An extension of this method to thin shields in the vicinity of ferromagnetic bodies is proposed in this paper for time-harmonic fields. The fixed-point polarization technique is used, where the nonlinear material is replaced by a free space and a distribution of fictitious polarization whose magnitude is corrected iteratively in terms of the magnetic induction. A Fourier decomposition of the polarization is employed and the integral equation is solved for each harmonic. The procedure has a guaranteed convergence and is more and more rapid as the number of harmonics retained decreases. In the beginning only the fundamental harmonic is considered and, afterward, higher order harmonics are added to increase the computation accuracy.

Index Terms—Current sheet integral equations, nonlinear media, thin shields.

I. INTRODUCTION

To overcome the decrease in efficiency of the finite volume solution techniques when the shield thickness decreases, a few dedicated methods have been proposed for linear media, which are especially appropriate for thicknesses smaller than the field depth of penetration. A surface impedance concept was used in [1], while in [2] the surface density of the current induced is expressed in terms of unknown scalar quantities associated with the nodes of the shield surface discretization mesh. In [3], the induced current density is represented by simple vector surface functions, such that the current continuity is preserved everywhere, the unknowns to be determined being scalar coefficients associated with the interior nodes of the mesh, with only one unknown for each hole contour in the case of shields with holes. These unknowns are computed by employing the current sheet integral equation.

Field analysis is drastically complicated when shields are close to ferromagnetic bodies. The nonlinearity can be treated by the Newton–Raphson method or by using a static magnetic permeability, with different techniques for correcting this permeability [4]. Since, usually, in electromagnetic shielding problems only a reduced number of harmonics is required, the harmonic balance method [5] could be applied for solving the periodic regim, but the Newton–Raphson method applied to the associated nonlinear system of equations does not insure the solution convergence. The size of this system becomes huge when more harmonics are considered due to the harmonic coupling introduced by the nonlinearity of the \( B-H \) relationship. The main disadvantage of these methods consists in the fact that they do not allow the construction of an integral equation for the surface current induced in the shield. A method based on the iterative polarization fixed-point method [6] has been presented in [7] for the solution of 2-D field problems of steady-state periodic regim in the presence of ferromagnetic media.

In this paper, a technique employing the current sheet integral equation is developed for the computation of the time-periodic electromagnetic field in 3-D structures containing thin shields and nonlinear ferromagnetic bodies. The nonlinear material is replaced by a linear one, having the permeability of free space, with a distribution of magnetic polarization nonlinearly depending on the magnetic induction. The current sheet integral equation is solved for each harmonic and, at each iteration, the polarization is corrected in time domain in terms of the corresponding magnetic induction. The computation process is started by only considering the fundamental harmonic, with the results being afterward improved by including higher harmonics. At most, three harmonics are sufficient for a reasonable accuracy of the field in practical structures.

II. NONLINEARITY TREATMENT BY THE FIXED-POINT METHOD AND ITERATIVE ALGORITHM

The nonlinear relationship \( \mathbf{B} = \mathbf{F}(\mathbf{B}) \) is replaced by

\[ \mathbf{B} = \mu_0 \mathbf{H} + \mathbf{I} \] (1)

where \( \mu_0 \) is a constant chosen to be the permeability of free space and the magnetic polarization \( \mathbf{I} \) has a nonlinear dependence on \( \mathbf{B} \) [6]

\[ \mathbf{I} = \mathbf{B} - \mu_0 \mathbf{F}(\mathbf{B}) \equiv \mathbf{G}(\mathbf{B}) \] (2)

such that the function \( \mathbf{G} \) is a contraction, i.e.,

\[ \|\mathbf{G}(\mathbf{B}_1) - \mathbf{G}(\mathbf{B}_2)\| \leq \lambda \|\mathbf{B}_1 - \mathbf{B}_2\| \] (3)

for any \( \mathbf{B}_1, \mathbf{B}_2 \), with \( \lambda < 1 \). The norms used are defined as

\[ \|\mathbf{U}\| = \left( \frac{1}{T} \int_0^T \int_{\Omega_{\text{fe}}} \mathbf{U} \cdot d\mathbf{v} dt \right)^{1/2} \] (4)

where \( T \) is the time period and \( \Omega_{\text{fe}} \) the region occupied by ferromagnetic bodies. For any distribution of given current, the magnetic induction is uniquely determined by the corresponding distribution of polarization, \( \mathbf{B} = \mathbf{L}(\mathbf{I}) \), the function \( \mathbf{L} \) being nonexpansive [i.e., \( \lambda = 1 \) in (3)] [6].

Manuscript received December 08, 2009; revised February 17, 2010; accepted February 22, 2010. Current version published July 21, 2010. Corresponding author: F. I. Hantila (e-mail: hantila@elth.pub.ro).
Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.
Digital Object Identifier 10.1109/TMAG.2010.2044641

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The iterative algorithm proposed for the solution of the time-periodic electromagnetic field starts with an arbitrary \( I \), for instance a zero value everywhere, which is iteratively updated with (1) and (2). We approximate the magnetic polarization in the form of a finite Fourier expansion as

\[
I_a(t) = \sum_{n=1}^{N_a} \left( I_n^t \sin(n \omega t) + I_n^t \cos(n \omega t) \right) \tag{5}
\]

with only \( N_a \) harmonics retained, \( I \cong I_a \equiv Y(I) \). The approximation \( Y \) is also a nonexpansive function, i.e., \(|Y(I_1) - Y(I_2)| \leq |I_1 - I_2|\), the inequality being more pronounced when the number of harmonics is smaller. Thus, the iterative algorithm coupled with the series expansion (5) is always convergent, the computation time being smaller when a smaller number of harmonics is considered,

\[
\cdots I^{k-1}_a \xrightarrow{L} \mathbf{B}^k \xrightarrow{G} I^k \mathbf{Y} \xrightarrow{I_a} \mathbf{B}^{k+1} \ldots, \quad k = 1, 2, \ldots \quad (6)
\]

For each harmonic \( n \) of polarization, written in phasor form as

\[
I_n = I_n^t + jI_n^q, \quad j = \sqrt{-1} \tag{7}
\]
we solve the field problem using the surface integral equation of the current induced in the shield and determine the \( n \)th harmonic of magnetic induction

\[
B_n = B_n^t + jB_n^q. \tag{8}
\]

The time-domain expression of \( B \) is derived in the form

\[
B(t) = \sum_{n=1}^{N_a} \left( B_n^t \sin(n \omega t) + B_n^q \cos(n \omega t) \right). \tag{9}
\]

Then, at each time step, (9) and (2) are used to correct the magnetic polarization. The computational effort is substantially reduced if one starts with only the fundamental and, then, the solution accuracy is increased by adding a few more harmonics [7].

### III. FORMULATION OF SHIELD SURFACE INTEGRAL EQUATION

In the model with a free space permeability everywhere and known distributions of source currents and polarization, for each harmonic \( n \), the surface current density \( J_{s,n} \), induced over the thin shield surface \( S \) satisfies the integral equation

\[
\rho_s J_{s,n}(\mathbf{r}) + j \eta n \omega A_{0n}(\mathbf{r}) = -j n \omega A_{n}(\mathbf{r}) - \nabla V(\mathbf{r}) \tag{10}
\]

where \( \rho_s = \rho/\Delta \) is the surface resistivity, with \( \Delta \) being the shield thickness and \( \rho \) its resistivity, \( R = |\mathbf{r} - \mathbf{r}'| \) with \( \mathbf{r} \) and \( \mathbf{r}' \), respectively, the position vectors of the observation point and of the source point, \( A_{0n} \) is the magnetic vector potential due to the \( n \)th harmonic of the external sources, for instance the given current density \( J_{0n} \) over the region \( \Omega_0 \), and \( A_{n} \), the vector potential due to the \( n \)th harmonic of polarization in \( \Omega_k \),

\[
A_{0n}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{\Omega_0} \frac{J_{0n}(\mathbf{r}')}{R} dS', \tag{11}
\]

\[
A_{n}(\mathbf{r}) = \frac{1}{4\pi} \int_{\Omega_k} \frac{(\nabla' \times I_n(\mathbf{r}'))}{R} dS'. \tag{12}
\]

The scalar potential component of the electric field intensity, \(-\nabla V\), can be ignored [3].

### IV. NUMERICAL SOLUTION

The surface \( S \) is approximated by a polyhedral surface with triangular surface elements, as shown in Fig. 1. To each node \( i \) we associate a surface vector function \( U_i \), of constant value \( U_i(p) = 1/(2S_p) \mathbf{I}_i^p \) over each surface element \( (p) \), of area \( S_p \), containing the node \( i \) and of zero value for all the surface elements which do not contain the node \( i \). Each of the current density harmonics is written as a linear combination of the functions \( U_i \) associated with all of the \( N \) nodes of \( S \),

\[
J_{s,n}(\mathbf{r}) \approx \sum_{i=1}^{N} \alpha_{ni} U_i(\mathbf{r}) = \sum_{i=1}^{N} \alpha_{ni} \sum_{p \in \{i\}} \frac{1}{2S_p} \mathbf{I}_i^p(\mathbf{r}) \tag{13}
\]

with unknown complex coefficients \( \alpha_{ni} \), where \( \{i\} \) is the set of surface elements having in common the node \( i \). The region \( \Omega_k \) is discretized in \( N_k \) polyhedral volume elements \( \Omega_k \), within each of them the magnetic polarization \( I_{n,m} \) being assumed to be constant. Equation (12) becomes

\[
\mathbf{A}_{m} = -\frac{1}{4\pi} \sum_{i=1}^{N_k} \sum_{m=1}^{N} \left( \int_{\partial \Omega_m} \frac{\mathbf{n}'}{R} dS' \right) \times I_{n,m} \tag{14}
\]

where \( \partial \Omega_m \) is the boundary of \( \Omega_m \) and \( \mathbf{n}' \) the outward unit vector normal to \( \partial \Omega_m \). With (13) substituted in (10) and, then, scalar multiplying both sides of (10) with the vector functions \( U_j \), associated with the nodes \( j \), and integrating over the shield surface yields for each harmonic \( n \) the following system of algebraic equations:

\[
\sum_{i=1}^{N} (d_{ji} + j n \omega a_{ji}^q) \alpha_{ni} = -j n \omega a_{n,i} - j n \omega a_{n,j}, \quad j = 1, 2, \ldots, N \tag{15}
\]

where

\[
d_{ji} = \int_S \rho_s U_j \cdot U_i dS = \sum_{p \in \{j\} \cap \{i\}} \frac{\mathbf{I}_j^p}{4S_p} \cdot \mathbf{I}_i^p \tag{16}
\]

\[
d_{ji}^q = \frac{\mu_0}{4\pi} \int_S \int_{S'} U_j \cdot U_i \frac{1}{R} dSdS' \tag{17}
\]

with

\[
Y(S_p, S_q) = \int_{S_p} \int_{S_q} \frac{1}{R} dSdS' \tag{18}
\]
\[ b_{m,j} = \int_{S} U_j \cdot A_{0n} dS \quad b_{n,j} = \sum_{m=1}^{N_{\Omega}} w_{jm} \cdot I_{nm} \]  

(19)

with

\[ w_{jm} = \frac{1}{4\pi} \int_{S} \oint_{\Omega_{mn}} \nabla' \times \frac{U_j}{R} dS dS' \]

\[ = \frac{1}{4\pi} \sum_{\sigma \in \{J\}} \sum_{p \in \{m\}} Y(\sigma_p, S_q) \sum_{q \in \{j\}} \frac{\nabla' \times (T^q)}{2S_q} \]  

(20)

where \( \{m\} \) is the set of facets of the volume element \( \Omega_{mn} \) in \( \Omega_{\text{fe}}, \sigma_p \) is a facet of \( \Omega_m \) and \( \mathbf{n}_p \) the outward unit vector normal to \( \sigma_p \).

V. CALCULATION OF MAGNETIC INDUCTION

Magnetic induction is assumed to be constant within each volume element \( \Omega_{m} \) of volume \( V_m \), with a value equal to the average value over that element. Its \( n \)th harmonic is calculated as

\[ \dot{B}_{nm} = \dot{B}_{0nm} + \dot{B}_{Jnm} + \dot{B}_{rmn} \]  

(21)

where \( \dot{B}_{0nm} \) is due to the given source currents and \( \dot{B}_{Jnm} \) is produced by the currents induced in the shield which, with (13) and (20), can be expressed in the form

\[ \dot{B}_{Jnm} = \frac{j \omega}{4\pi V_m} \int_{S_m} \int_{\Omega_{ne}} J_{s_n}(r') \times \frac{R}{R^2} dS' dV 
\]

\[ = \frac{j \omega}{4\pi V_m} \sum_{N} w_{mj} \mathbf{G}_{m,j} \]  

(22)

\[ \dot{B}_{rmn} \] due to the magnetic polarization

\[ \dot{B}_{Jnm} = \frac{1}{4\pi V_m} \int_{\Omega_m} \nabla' \times \frac{I_n(r')}{R^2} dV 
\]

\[ = \frac{1}{4\pi V_m} \sum_{i=1}^{N_{\Omega}} \bar{t}_{mi} I_{ni} \]  

(23)

with

\[ \bar{t}_{mi} = \frac{1}{4\pi V_m} \int_{\Omega_m} \int_{\Omega_{ne}} \left( \mathbf{n}_i \cdot \mathbf{n}_m \right) \frac{\nabla' - \mathbf{n}_m}{R} dS_m dS_i \]

\[ = \frac{1}{4\pi V_m} \sum_{p \in \{m\}, q \in \{i\}} Y(\sigma_p, \sigma_q) (\mathbf{n}_p \cdot \mathbf{n}_q) \bar{t} - (\mathbf{n}_p \cdot \mathbf{n}_q) \]  

(24)

where \( \bar{t} \) is the identity dyadic and \( (\mathbf{uv}) \) is the dyad of \( \mathbf{u} \) and \( \mathbf{v} \).

Once the average value of the magnetic induction phasor is determined for all the harmonics considered, one obtains the magnetic induction in time domain and, then, the magnetic polarization is updated at each time step.

Computation Remarks:

1) One of the two integrals in (18) can always be performed analytically; moreover, when \( \sigma_p \) and \( \sigma_q \) are located in the same plane, the entire double integral in (18) can be expressed analytically.

2) The integrals \( Y(\sigma_p, S_q) \) in (18) and \( Y'(\sigma_p, \sigma_q) \) in (24) are only calculated for \( q \geq p \).

3) Most of the integrals \( Y'(\sigma_p, S_q) \) appear twice in (20) and \( Y(\sigma_p, \sigma_q) \) four times in (24) since they are evaluated for the two sides of the surface \( \sigma_p \).

4) The tensor \( \bar{t}_{mi} \) has the following properties:

\[ \bar{t}_{mi} = \bar{t}_{im}, \quad \bar{t}_{mi} = (\bar{t}_{mi})^T 
\]

\( (T \) denotes the transpose \)

\[ t_{mizi} = t_{miyy} + t_{mizz} = \begin{cases} 0 & \text{for } m \neq i \\ V_m & \text{for } m = i \end{cases} \]  

(25)

(26)

which allow the computation of a total of only \( 2.5(N_{\Omega}^2 + N_{\text{fe}}) \) tensor components out of a total of \( 9N_{\Omega}^2 \).

VI. COMPUTATION EXAMPLES

The median surface of a 1-mm-thick shield of resistivity \( 2 \times 10^{-6} \Omega \cdot \text{m} \), described by \( x^2 + y^2 + (z - 0.15)^2 = 0.09 \) with \( x, y \in [-0.21, 0.21], \) is shown in Fig. 2. On the concave side of the shield, a nonconducting ferromagnetic cube of side 0.2 m is centered at the origin of the coordinate system, with its edges parallel to the coordinate axes. Its \( B-H \) characteristic is given in Fig. 3. Beneath the shield, a circular coil of negligible cross-section and of a diameter of 0.4 m, is placed coaxially with the z-axis and centered at \( z = -0.2 \) m. The coil carries a sinusoidal current of frequency \( f \) and intensity \( \mathcal{S} \) in rms value.

A mesh with 841 nodes and 1600 triangular surface elements has been utilized for the shield, while the cube was discretized in 192 tetrahedrons with 71 nodes. Current lines on the shield surface are sketched in Fig. 2. The average magnetic induction in rms value over the entire cube is calculated as

\[ B_{av} = \left( \frac{1}{2V_{\Omega}} \sum_{m=1}^{N_{\Omega}} \left( \sum_{n=1,3,5,2N_\Omega-1} R_{mn}^2 \right) V_m \right)^{1/2} \]  

(27)

where \( V_{\Omega} \) is the cube volume. The iterative process is ended when a relative error \( \|F - F^{-1}\|/\|F\| < 2 \times 10^{-5} \) is obtained.
In Table I, the magnitude of the average magnetic induction of the first three harmonics separately \((n = 1, 3, 5)\) and of its total value [see (27)], as well as the magnitude of the average magnetic induction in the absence of the shield, \(B_{av}^{(0)}\), are given for two different frequencies and two different values of the inducing current. For smaller values of current, when the magnetic induction is small, it is sufficient to only retain the fundamental, the average values of the third and fifth harmonics being much smaller. For greater values of the coil current, the weight of the higher harmonics increases. The number of iterations required for the examples considered and the computation time, when employing a notebook with a 2.5-GHz processor, are given in Table II.

VII. REMARKS AND CONCLUSION

In the method presented, based on the shield current sheet integral equation, the number of unknowns in the matrix equation is reduced to only the number \(N\) of nodes in the shield mesh. On the other hand, using the fixed-point polarization technique for the treatment of the material nonlinearity allows the solution of the problem to be obtained by dealing with each harmonic separately. First, we only consider the fundamental and, then, the third harmonic is added. If the result is only slightly modified, the computational process is ended. Otherwise, we add the fifth harmonic as well. The number of necessary harmonics to be included is very small when the shield thickness is smaller than the field depth of penetration. The case when a higher harmonic is needed, such that the depth of penetration corresponding to its frequency is less than the shield thickness, could be treated by using an equivalent system containing thinner shields. Their associated current sheets are evaluated for each harmonic separately, with the coupling of various harmonics caused by the medium nonlinearity taking place only in the ferromagnetic material and being independent of the shield thickness. This approach is especially efficient for the analysis, evaluation and design of the electromagnetic shielding related to large power transformers, electric machines and transmission lines.

The method developed in this paper, benefitting from the fixed-point iterative technique used, is always convergent. Its rate of convergence can be improved by employing a dynamic overrelaxation technique [6]. For the periodic regim, one can start with an overrelaxation factor of 1.99, for instance, and whenever increases in the error \(\|f^k - f^{k-1}\|\) occur this factor is reduced in steps, for a unity value of it the convergence being guaranteed.

ACKNOWLEDGMENT

This work was supported in part by the Romanian National Council of Scientific Research under Grants ID_2010 and ID_2197.

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