Electromagnetic levitation of rotating cylinders


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Abstract: Stable electromagnetic levitation of spinning conducting cylindrical rotors has been demonstrated theoretically for the first time. General expressions of the quasi-stationary magnetic vector potential, for a rotating, conducting, cylindrical shell in the proximity of a distribution of parallel straight wires carrying currents, have been obtained by assuming a two-dimensional field. For a practical range of geometric and electric parameters of a thin cylindrical shell rotor, the static stability conditions, losses and torque/speed characteristics are quantitatively derived. Speeds of rotation of up to twenty thousand revolutions per second have been analysed. The results of this study can be applied to the design of electromagnetically levitated and driven rotors.

List of principal symbols

- \( A \) = magnetic vector potential
- \( a_0 \) = mean radius of cylindrical shell
- \( c \) = half the distance between the two wires
- \( d \) = thickness of cylindrical shell
- \( F_x, F_y \) = \( x \) and \( y \) components of force per unit length of rotor
- \( f, \omega \) = frequency and angular frequency of \( i \)
- \( i, I \) = instantaneous and RMS values of wire currents
- \( I_m \) = minimum current for levitation
- \( j \) = \( \sqrt{-1} \)
- \( k_x, k_y \) = complex symbols (see eqns. 33 and 24)
- \( P \) = time-average losses per unit length of rotor
- \( r, \theta \) = circular cylindrical co-ordinates relative to rotor axis
- \( r_k, \theta_k \) = \( r, \theta \) for filamentary conductors (see Figs. 1 and 2)
- \( T \) = torque per unit length of rotor
- \( t \) = time
- \( x, y \) = cartesian co-ordinates of rotor axis
- \( x_m, y_m \) = \( x, y \) for minimum current of levitation
- \( \gamma \) = complex symbols (see eqn. 18)
- \( \delta(u) \) = Dirac delta function
- \( \mu, \mu_0 \) = permeability of rotor and outside medium
- \( v, \omega \) = speed of rotation in rev/s and rad/s
- \( \sigma \) = conductivity of rotor
- \( \text{Re}, \text{Im} \) = real and imaginary parts of

1 Introduction

The electromagnetic levitation forces acting on a solid conductor occur as a result of the interaction between the eddy currents induced within the conductor and the inducing external magnetic field. In an electromagnetic levitation system, the electromagnetic forces on a solid conducting body compensate fully for the gravitational forces, thereby putting the body into a state of suspension: 'floating on the external magnetic field'. To design such a system it is imperative to analyse the necessary conditions of stable equilibrium. The static stability region of an electromagnetically levitated conductor is that space region within which the conductor, when placed, would move to an equilibrium position under the action of the resultant of the levitation and gravitational forces. The problem of dynamic stability is much more difficult than that of static stability because it involves both mechanical and electric transients.

Experimental and theoretical studies on electromagnetic levitation systems thus far have mainly been related to two problems: first, heating and melting of electroconducting materials by avoiding their contact with the crucible (in order to produce very high-purity metals and alloys), and secondly, the design of electromagnetically levitated vehicles [1]. Rigorous analytical solutions, based on the quasi-stationary electromagnetic field equations, were obtained for a few simple geometric configurations with immobile levitated solid conductors [2–5].

This paper relates to the electromagnetic levitation of a long, conducting, cylindrical shell rotating in the presence of a distribution of parallel wires carrying alternating current.

An exact analytical solution for the electromagnetic field associated with this system can be obtained by adopting a two-dimensional model consisting of a solid circular cylindrical shell of radii \( a \) and \( b \), \( a > b \), which is linear, homogeneous and isotropic, of permeability \( \mu \) and conductivity \( \sigma \); the cylinder rotates about its axis with constant angular velocity \( \omega \), in the magnetic field produced by a distribution of parallel filamentary conductors carrying the current

\[
i = \sqrt{2}I \sin \omega t
\]

The medium outside the rotating cylindrical shell is non-conducting and of permeability \( \mu_0 \). The peripheral velocities considered are negligible with respect to the speed of light.

In Section 2.1 the exact vector potential solution corresponding to a single straight filamentary current in the presence of the rotating cylindrical shell is derived, from which the field due to a given two-dimensional current distribution can be determined by superposition.

In Section 2.2 the exact field solution is simplified correspondingly for the special case of a thin, conducting, cylindrical shell rotating in the presence of two filamentary currents, for which the analysis of forces, losses and torque is developed in Section 3. The complete treatment of static stability and numerical results for a wide range of parameters for the two-dimensional system considered are presented in Section 4. The problem of dynamic stability is not treated analytically in this paper. Practical additions to the theoretical two-dimensional system are suggested, based on preliminary experiments performed on levitated finite-length aluminium shells.
2 Electromagnetic field analysis

The magnetic-flux density and the induced electric-field intensity (with the field quantities defined in the local system of reference, attached to the moving media) can be expressed in terms of the magnetic vector potential as:

\[ B = \text{curl} \ A \]

\[ E = -\frac{\partial A}{\partial t} + v \times \text{curl} \ A \]

where \( v \) is the local velocity, \( v = r \omega_r \).

Owing to the parallel-plane structure of the magnetic field, the vector potential is chosen as having only an axial component and depending only on time and cylindrical co-ordinates \( r \) and \( \theta \), \( A(r, \theta, t) \).

2.1 Exact vector potential solution

The vector potential may be determined by using either the co-ordinate system attached to the rotor, in which the wire, located at \( r_0, \theta_0 \) (see Fig. 1), has a tangential velocity \(-r_0 \omega_r \), or the co-ordinate system fixed with respect to the wire.

In rotor-fixed co-ordinates, for instance, the vector potential equations are:

\[ \nabla^2 A_r = 0 \quad r < b \]

\[ \nabla^2 A = -\mu_0 \frac{\partial A}{\partial t} = 0 \quad b < r < a \]

\[ \nabla^2 A_e = -\mu_0 \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial A_e}{\partial r} \right) \quad r > a \]

The boundary conditions can be written as follows:

\[ A_r = A_e \quad \text{and} \quad \frac{\partial A_r}{\partial r} = \frac{\mu_0}{\mu} \frac{\partial A_e}{\partial r} \quad \text{at} \quad r = b \]

\[ A = A_e \quad \text{and} \quad \frac{\partial A}{\partial r} = \frac{\mu_0}{\mu} \frac{\partial A_e}{\partial r} \quad \text{at} \quad r = a \]

The general solutions of eqns. 4-6 can be obtained by using the Bessel and Fourier series representation of the Dirac delta functions in eqn. 6 and the separation of variables for each time-harmonic component of the potentials, taking into account the regularity conditions at \( r = 0 \) and \( r \to \infty \). Applying the superposition of these components and the boundary conditions, and assuming that the return path of the wire current is at infinity, finally yields

\[ A_i = \frac{\mu I}{2\pi} \text{Im} \sum_{n=1}^{\infty} \left( \frac{a r}{b r_0} \right) \left( e^{i \omega t} - e^{-i \omega t} \right) \times e^{i(n\theta - \theta_0 + \omega t)} \quad r \leq b \]

\[ A = -\frac{\mu I}{2\pi} \text{Im} \sum_{n=1}^{\infty} \left( \frac{a r}{b r_0} \right) \left[ R_d(\gamma_r^+ r) e^{i \omega t} - R_d(\gamma_r^- r) e^{-i \omega t} \right] \times e^{i(n\theta - \theta_0 + \omega t)} \quad b < r \leq a \]

\[ A_e = \frac{\mu_0 I \sqrt{2}}{4\pi} \text{Im} \sum_{n=1}^{\infty} \left( \frac{1}{r} \right) \left[ G_n^+ \left( \frac{a}{r} \right) e^{i \omega t} - G_n^- \left( \frac{a}{r} \right) e^{-i \omega t} \right] e^{i(n\theta - \theta_0 + \omega t)} \quad r > a \]

where

\[ r_0 \equiv \min (r, r_0) \]

\[ r_\infty \equiv \max (r, r_0) \]

and

\[ G_n^+ \left( \frac{a}{r} \right) \equiv 1 - \left( \frac{a}{r} \right)^{2n} \left[ 1 + 2 \frac{\mu}{\mu_0} n R_d(\gamma_r^+ a) \right] \]

\[ R_d(\gamma_r^+ r) \equiv \frac{1}{D_n^+} \left[ T_n^+ J_n(\gamma_r^+ r) - S_n^+ Y_n(\gamma_r^+ r) \right] \]

\[ D_n^+ \equiv S_n^+ \left[ \left( \frac{\mu}{\mu_0} + 1 \right) n J_n(\gamma_r^+ a) + \gamma_r^+ a Y_n(\gamma_r^+ a) \right. \]

\[- T_n^+ \left[ \left( \frac{\mu}{\mu_0} + 1 \right) n J_n(\gamma_r^- a) + \gamma_r^- a Y_n(\gamma_r^- a) \right] \]

with

\[ S_n^+ \equiv \left( \frac{\mu}{\mu_0} + 1 \right) n J_n(\gamma_r^+ a) \quad \gamma_r^+ b J_n(\gamma_r^- a) \quad \gamma_r^- b Y_n(\gamma_r^- a) \]

\[ J_n \] and \( Y_n \) being the Bessel functions of the first and second kind, respectively, and integral order \( n \) [6].

In the special case when \( \omega_r = 0 \), the general eqns. 9-11 become the known expressions for a wire carrying AC in the presence of an immobile cylindrical shell [7].

For \( b = 0 \), eqns. 9-11 yield those corresponding to a wire carrying current in the proximity of a rotating solid circular cylinder [8].

2.2 Approximation for thin cylindrical shells

From a practical viewpoint, the interesting case of electromagnetic levitation is that of a very thin cylindrical shell rotor, of permeability \( \mu = \mu_0 \). When the rotor thickness \( d = a - b \) (see Fig. 2) is very small compared with its mean radius \( a_o \) and with the distance between the shell wall and the carrying-current wire, the general expressions in eqns. 9-11 can be considerably simplified.

A good approximation for \( d/a_0 \leq 0.1 \) consists in assuming an infinitely thin cylindrical shell of finite surface conductivity \( \sigma_s = ad \).

In this case the rotor will constitute a rotating sheet of induced currents, of surface density \( J_s \). For this simplified case
model, eqns. 4 and 6 remain the same, but eqns. 7 and 8 are replaced by the following boundary conditions at

\[ \frac{\partial A_i}{\partial r} - \frac{\partial A_2}{\partial r} = -\mu_0 \sigma d \frac{\partial A_{1/2}}{\partial t} \]

Applying the same procedure as in the previous Section now yields

\[ A_i = \frac{\mu_0 I}{\sqrt{2\pi a_0 d}} \sum_{n=1}^{\infty} \frac{r}{(r_0)^n} \left[ k_n e^{in\theta} - k_n e^{-in\theta} \right] e^{-j\theta} \]

\[ A_{1/2} = \frac{\mu_0 I}{4\pi} \sum_{n=1}^{\infty} \frac{r}{(r_0)^n} \left[ 1 + k_n^* \left( \frac{a_0}{r_0} \right)^{2n} e^{-j\theta} \right] e^{in\theta} \]

\[ J_i = -\sigma d \frac{\partial A_{1/2}}{\partial t} \]

where \( r < a_0 \) and \( r > a_0 \) are the symbols in eqn. 12, and

\[ k_n^* = \frac{(y_n^2)^d a_d/2n}{1 - (y_n^2)^2 a_d/2n} \]

These results can also be derived from the general expressions in eqns. 9-11 by expanding correspondingly the Bessel functions in series and retaining their first terms.

### 3 Levitation of thin shell rotors

The rigorous study of electromagnetic levitation in the general case of cylindrical rotors should be elaborated on the basis of eqns. 9-11. The fact that extremely large values of current are necessary for the levitation of solid cylindrical conductors, as previously shown for immobile conductors [3], indicates that, from a practical standpoint, thin metallic cylindrical tubes are appropriate for the construction of devices with levitated rotors. In this paper we analyze the case of thin cylindrical shells and apply the results presented in Section 2.2.

Let us consider a thin cylindrical shell rotor above two parallel wires carrying the currents \( i \) and \(-i\) (eqn. 1), as shown in Fig. 2. The relationship between the cartesian co-ordinates of the rotor axis and the circular cylindrical co-ordinates of the two wires with respect to the rotor axis is given by

\[ r_1 e^{i\theta_1} = -(c + x) - jy \]

\[ r_2 e^{i\theta_2} = c - x - jy \]

### 3.1 Force on the rotor

The elementary force on a wire carrying the current \( i \) can be calculated with the aid of the expression

\[ dF = idl \times B_{ext} \]

where \( dl \) is the vector length element of the wire and \( B_{ext} \) is the magnetic-flux density produced at the points on the wire by the currents induced within the rotor and by the other wire current. The time-average force on unit length of wire can be written in the form

\[ f_i = I \text{Re} \{ \text{grad} \ A_{ex} \} \]

with \( A_{ex} \) being the complex representation of the vector potential corresponding to \( B_{ext} \), in wire-fixed co-ordinates. For the wire 1, carrying the current \( i \), eqn. 28 becomes

\[ f_1 = I \text{Re} \{ \text{grad} \ A_1 \} \]

and for the wire 2, carrying the current \(-i\),

\[ f_2 = -I \text{Re} \{ \text{grad} \ A_2 \} \]

in which \( a_0 \) is the unit vector of the positive x-axis and \( A_j \) is the complex representation of the vector potential for \( r > a_0 \), in wire-fixed co-ordinates, produced only by the currents within the rotor induced by both wire currents (see eqn. 22 and Fig. 2).

\[ A_j = \mu_0 I \sum_{n=1}^{\infty} \frac{1}{(r_0)^n} \left[ k_n \left( \frac{a_0}{r_0} \right)^{2n} e^{in\theta} - k_n^* \left( \frac{a_0}{r_0} \right)^{2n} e^{-in\theta} \right] \]

\[ + \mu_0 I^2 \frac{a_0}{4\pi} \]

with the asterisk denoting the complex conjugate.

The x and y components of the time-average force on unit length of the rotor can be derived as the real and imaginary parts, respectively, of the following complex expression:

\[ F_x + jF_y = \frac{\mu_0 I^2}{4\pi} \sum_{n=1}^{\infty} \frac{1}{(r_1)^n} \left[ k_n \left( \frac{a_0}{r_1} \right)^{2n} e^{in\theta_1} - k_n^* \left( \frac{a_0}{r_1} \right)^{2n} e^{-in\theta_1} \right] \]

\[ \times \frac{e^{i\theta_1}}{r_1} \sum_{n=1}^{\infty} k_n \left( \frac{a_0}{r_2} \right)^{2n} e^{in\theta_2} \]

\[ \times \frac{e^{i\theta_2}}{r_2} \sum_{n=1}^{\infty} k_n \left( \frac{a_0}{r_2} \right)^{2n} e^{in\theta_2} \]

where

\[ k_n \equiv k_n + k_n^* \]

For the special case when \( \omega_0 = 0 \), eqn. 32 becomes the known expression corresponding to an immobile thin cylindrical shell [4].

For \( \omega = 0 \), i.e. the case of the thin cylindrical shell rotating in the field of the two wires carrying DC, \( I \) and \(-I\), eqn. 32 can be written as
in which

\[
R_1^2 = r_1^2 + \frac{\alpha_0^2}{r_2^2} - 2r_1 \frac{\alpha_0^2}{r_2} \cos (\theta_1 - \theta_2) \\
R_2^2 = r_2^2 + \frac{\alpha_0^2}{r_1^2} - 2r_2 \frac{\alpha_0^2}{r_1} \cos (\theta_1 - \theta_2)
\]

(35)

(36)

\[
k = -2j \frac{d\alpha_0/\delta^2}{1 + jd\alpha_0/\delta^2}, \quad \delta_\omega = \sqrt{\frac{2}{\omega_\omega \omega_0}} \sqrt{2}
\]

(37)

The static stability conditions for the electromagnetic levitation of the rotor, the position for maximum force of levitation and minimum levitation current can be determined from eqn. 32, taking into account the rotor weight, as illustrated in Section 4. It should be noted that for \( \omega_\omega \neq 0 \), in general, the horizontal force \( F_x \neq 0 \) at \( x = 0 \), i.e. the stationary position of levitation is not symmetrical with respect to the two wires, as it is in the case of an immobile cylinder.

### 3.2 Power loss

The time-average electromagnetic power received per unit length of rotor can be evaluated for a thin cylindrical shell as

\[
P = \frac{\alpha_0}{2\pi} \int_0^{2\pi} \frac{1}{\delta^2} \mathbf{J} \cdot \mathbf{J}^* d\theta
\]

(38)

where \( \delta^2 \) is given by eqn. 19 and \( \mathbf{J} \) is the complex representation of the surface current density in wire-fixed coordinates, induced in the rotor by both wire currents (see eqn. 23)

\[
\mathbf{J} = \frac{1}{2\pi \alpha_0} \sum_{n=1}^{\infty} \left\{ \sum_{\pm} \frac{a_0}{r_1} e^{-jn\theta_1} - \frac{a_0}{r_2} e^{-jn\theta_2} \right\} e^{jn\theta}
\]

\[+ \frac{1}{2 \pi \alpha_0} \sum_{\pm} \frac{a_0}{r_1} e^{jn\theta_1} - \frac{a_0}{r_2} e^{jn\theta_2} e^{-jn\theta} \]

(39)

Substituting this expression in eqn. 38 gives

\[
P = \frac{1}{2\pi \alpha_0} \sum_{n=1}^{\infty} (-p_n) \left\{ \frac{a_0}{r_1} e^{-jn\theta_1} \frac{a_0}{r_2} e^{-jn\theta_2} \right\} + \frac{1}{2 \pi \alpha_0} \sum_{\pm} \frac{a_0}{r_1} e^{jn\theta_1} - \frac{a_0}{r_2} e^{jn\theta_2} e^{-jn\theta} - 2 \left( \frac{a_0^2}{r_1 r_2} \right)^n \cos n(\theta_1 - \theta_2)
\]

(40)

where \( p_n \) is the real part of \( k_n \):

\[
k_n = p_n + jq_n
\]

(41)

In the case of an immobile cylinder, eqn. 40 yields the result obtained in Reference 4.

For \( \omega = 0 \), when the conducting cylindrical shell rotates in the field of the two wires carrying DC, \( I \) and \( -I \), eqn. 40 becomes (see eqns. 35 and 37)

\[
P = 4\pi \frac{\mu_0 I^2 a_0^2}{4\pi} q \left\{ \frac{1}{r_2 - a_0} + \frac{1}{r_2 - a_0^2} \right\}
\]

\[-2 \frac{r_2}{R_2^2} \cos (\theta_1 - \theta_2) - \frac{a_0}{R_2^2} \frac{r_2}{r_1} \cos (\theta_1 - \theta_2) - \frac{a_0^2}{R_2^2} \frac{r_2}{r_1}
\]

(42)
In the case of an immobile cylinder, the curve represented by eqn. 47 is symmetrical with respect to the y-axis, e.g. the curve for \( v = 0 \) in Fig. 3a. For a cylinder in rotation about its axis, there are, in general, two distinct branches of this curve for a given speed (see Fig. 3). A first branch rises from the co-ordinate origin, reaches a maximum and is located either in the first quadrant (for speeds \( v < 900 \) rev/s in the case illustrated in Fig. 3a) or mainly in the second quadrant (for speeds \( v \geq 900 \) rev/s in Fig. 3b). The second branch is located on the other side of the y-axis. Theoretically, the upper boundary of the static stability region is defined by these two branches, up to the level corresponding to the maximum of the first branch.

For each conducting cylindrical shell there is a critical frequency \( f_{cr} \) (\( f_{cr} \approx 1700 \) Hz for \( a_0 = 1 \) cm, \( d = 0.5 \) mm, \( c = 2 \) mm, \( f = 2000 \) Hz) below which all the first branches are located in the second quadrant; for each frequency \( f > f_{cr} \) there is a critical speed \( v_{cr} \) (\( v_{cr} \approx 900 \) rev/s for the case in Fig. 3) below which the first branches are located in the first quadrant and above which the first branches are located in the second quadrant. The values of \( f_{cr} \) decrease with \( d \) and \( c \), and those of \( v_{cr} \) increase with \( f \) for a given geometry, e.g. \( v_{cr} \approx 9700 \) rev/s for \( f = 10000 \) Hz and the geometry used in Fig. 3.

Let us denote by \( y_m \) the height \( y \) on the rising section of the first branch for which the levitation current \( I \) is minimum, corresponding to a maximum levitation force \( F_y \). The height \( y_m \) increases with the distance \( c \), as shown in Fig. 4. The variation of \( y_m \) in terms of \( f \) is small for immobile cylinders, the values of \( y_m \) being close to those obtained for equivalent solid cylinders \([3]\) and even for perfectly conducting cylinders \([4]\). Within the range \( 1.5 < c/a_0 < 2.5 \), the dependence of \( y_m \) upon the speed is also small; for instance, the value of \( y_m \) increases by 4% when \( v \) increases from 0 to 7500 rev/s, for \( a_0 = 1 \) cm, \( d = 0.5 \) mm, \( c = 2 \) cm, \( f = 10000 \) Hz.

The minimum current for levitation \( I_m \) is given by the equation

\[
F_y(x_m, y_m) - W = 0
\]

where \( x_m, y_m \) are the co-ordinates of the rotor axis for maximum levitation force, on the rising section of the first branch of the curve \( F_y(x, y) = 0 \). Some of the numerical results are graphically represented in Fig. 5. The current \( I_m \) depends on the thickness \( d \), i.e. on cylinder weight; its dependence on speed is small, in general, presenting, for example, an increase of 4% when \( v \) increases from 0 to 7500 rev/s, for \( a_0 = 1 \) cm, \( d = 0.5 \) mm, \( c = 2 \) cm, \( f = 10000 \) Hz. For \( d \geq 0.5 \) mm and \( f > 5000 \) Hz, the values of \( I_m \) can be considered approximately equal to those corresponding to equivalent perfectly conducting cylinders \([4]\), taking into account the actual weight.

The lower boundary of the static stability region in the plane \( xy \) is determined by the upper branch of the curve represented by

\[
F_y(x, y) \bigg|_{t = I_m} - W = 0
\]

Up to a certain speed, specific to each geometry and frequency (approximately equal to 1000 rev/s in Fig. 3) the location of the lower boundary of the static stability region is close to that for \( v = 0 \), being practically independent of speed and frequency; in each case its minimum is at the corresponding point \( x_m, y_m \). Above this specific speed the static stability region is bounded laterally by the vertical line \( x = x_m \).

The levitated rotor is in stable equilibrium for those currents \( I > I_m \) for which its axis is located on the rising section of the first branch of the curve \( F_y(x, y) = 0 \), above
the point $x_m$, $y_m$. If the rotor axis is displaced from the position of equilibrium to points within the static stability region (e.g. $M$ or $M'$ for speeds of up to 1000 rev/s in the case illustrated in Fig. 3a), the horizontal force will be oriented towards the vertical line of the point of stable equilibrium, whereas the vertical force will be either downwards or upwards, so that the rotating cylinder will be brought back to the initial position of equilibrium. If the rotating cylinder axis is displaced beyond the static stability region boundaries (e.g. at the points $N$ or $N'$ for the case in Fig. 3a), the horizontal electromagnetic force will move the cylinder away from the vertical line of the initial point of stable operation and equilibrium will be lost.

For each given geometry and frequency, there is a speed limit (of about 1600 rev/s for the example in Fig. 3) above which there is no static stability region at all. In the case of the geometry chosen for illustration in Fig. 7, these speed limits are approximately equal to the maximum speeds indicated on the corresponding torque/speed characteristics. When the suspension wires carry DC, there is no static stability region at any speed.

### 4.2 Losses and torque/speed characteristics

In all the cases considered, the power loss and torque were evaluated numerically for the rotor located with its axis at the point $x_m$, $y_m$, for which the levitation current is minimum, $I = I_m$. Some of the results are shown in Figs. 6 and 7. In the range of parameters considered, the losses depend little on $d$, $f$ and $v$; for example, in the case $a_0 = 1$ cm and $c = 2$ cm, the losses increase by 4.3% when $d$ increases from 0.25 mm to 1 mm, for $f = 2000$ Hz and $v = 0$, they increase by 7.7% when $f$ increases from 50 Hz to 10000 Hz, for $d = 0.5$ mm and $v = 0$, and there is a decrease of 4.3% of the power loss when $v$ increases from 0 to 7500 rev/s, for $d = 0.5$ mm and $f = 10000$ Hz.

Some of the torque/speed characteristics are presented in Fig. 7. These characteristics are closely related to the location of the curves defining the static stability regions (see Fig. 3). If the first branches of the curves $F(L, y) = 0$ are all in the second quadrant for a given frequency, then the corresponding torques are all negative, and this happens when the frequency $f < f_c$ ($f_c \approx 1700$ Hz in the case of the geometry in Fig. 7). For any value of $f > f_c$, the torque is positive in the range $0 < v < v_c$, when the first branches of the curves $F(L, y) = 0$ are located in the first quadrant; at a speed approximately equal to the critical speed $v_c$ (which corresponds to the passage of these first branches to the second quadrant) the torque becomes negative.

It is obvious that there is no starting torque and that the stable operation as a motor is confined to the range of the torque/speed characteristics with positive torques and negative slopes.

![Fig. 7 Torque/speed characteristics for $a_0 = 1$ cm, $d = 0.5$ mm, $c = 2$ cm and various frequencies](image)

### 5 Conclusion

General expressions of the two-dimensional quasistationary electromagnetic field in the presence of a rotating cylindrical shell conductor, as well as simplified formulas for the case of a thin cylindrical shell were developed.

The analysis performed in this paper demonstrates theoretically the necessary conditions for a stable electromagnetic levitation of long cylindrical rotors. In Section 4, general properties relative to a system with levitated rotating cylinders and some of the quantitative results obtained are presented.

The output of this study can be applied to the design of small electromagnetically levitated rotor motors. In terms of the torque/speed relationships and stability regions suitable for a given purpose, the system geometry and current source can be chosen by using such characteristics as those illustrated in Figs. 3, 4, 5 and 7. A wider static stability region corresponds to a larger value of the distance between the two suspension wires, but the necessary minimum current for levitation is larger in this case. For a stable operation at higher values of speed, higher values of frequency must be supplied, as shown in Fig. 7. By using a combined current and frequency regulation, a larger range of speeds and torques can be covered.

Preliminary qualitative experiments were performed by using an aluminium cylindrical shell of 30 cm in length and two main inducing conductors of 50 cm in length. In order to prevent the rotor from moving and tilting longitudinally, the coil ends were bent slightly upward. To improve the static and dynamic stability conditions, secondary conductors in series with the main ones were placed higher and slightly outward, the practical coil producing a 'trough' of magnetic field for the levitated cylinder.

An electromagnetically levitated rotor motor presents interesting prospects for specialised applications. Some
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7 References

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