Scattering of Electromagnetic Waves by a System of Two Dielectric Spheroids of Arbitrary Orientation

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Abstract—By means of modal series expansions of the incident, scattered, and transmitted electric and magnetic fields in terms of appropriate vector spheroidal eigenfunctions, an exact solution is obtained to the problem of electromagnetic scattering by two dielectric spheroids of arbitrary orientation. The incident wave is considered to be a monochromatic, uniform plane electromagnetic wave of arbitrary polarization and angle of incidence. To impose the boundary conditions at the surface of one spheroid, the electromagnetic field scattered by the other spheroid is expressed as an incoming field to the first one, in terms of the spheroidal coordinates associated with it, using rotational–translational addition theorems for vector spheroidal wave functions. The column matrix of the total transmitted and scattered field expansion coefficients is equal to the product of a matrix which is independent of the direction and polarization of the incident wave, and the column matrix of the known incident field expansion coefficients. The solution of the associated set of algebraic equations gives the unknown expansion coefficients. Numerical results are presented in the form of plots for the bistatic and backscattering cross sections of two lossless prolate spheroids having various axial ratios, center-to-center separations, and orientations.

I. INTRODUCTION

Even though analytic solutions to scattering of electromagnetic waves by a single spheroid [1], two spheroids with parallel major axes [2], [3], and two spheroids of arbitrary orientation [4], [5] have been available in the literature for the perfectly conducting case, the only such solution available so far in the dielectric case is that for the single spheroid in a plane wave incident field [6]. Recently, an exact solution has been presented to the problem of electromagnetic scattering by a system of two dielectric spheroids with parallel major axes [7], using translational addition theorems for vector spheroidal wave functions derived in [8]. In this communication, we obtain an analytic solution for the general case of scattering by two dielectric spheroids of arbitrary orientation, from which the formulation corresponding to conducting spheroids can be derived by particularization. As in the case of two perfectly conducting spheroids of arbitrary orientation, the solution given here has also been obtained on the basis of rotational-translational addition theorems for vector spheroidal wave functions derived by the authors [9], [10], and independently in [11]. As mentioned in [12], the importance of obtaining such a solution lies in the fact that it can be used along with the exact solution for two perfectly conducting spheroids of arbitrary orientation, as benchmarks, to be considered in development of a database for validating numerical codes and also in cross validating numerical and experimental results.

II. FORMULATION AND ANALYSIS

Let $A$ and $B$ be two prolate spheroids as shown in Fig. 1, with the unprimed coordinate system attached to the spheroid $A$, and the primed coordinate system attached to the spheroid $B$. The major axes of $A$ and $B$ are along the $z$ and $z'$ axes of the Cartesian systems $Oxyz$ and $O'x'y'z'$, respectively. The system $Ox_1y_1z_1$ is parallel to $O'x'y'z'$, and is rotated with respect to $Oxyz$ through the Euler angles $\alpha, \beta, \gamma$, as defined in [13]. The center $O'$ of $B$ has spherical coordinates $d, \theta_d, \phi_d$ relative to $Ox_1y_1z_1$ and $d, \theta_0, \phi_0$ relative to $Oxyz$. The prolate spheroidal coordinates associated with the unprimed and primed systems are denoted by $\xi, \eta, \phi$ and $\xi', \eta', \phi'$, respectively. Consider a linearly polarized, monochromatic plane electromagnetic wave, with an electric field of unit amplitude, being incident at an angle $\theta_i$ with respect to the major axis of $A$, the plane of incidence being chosen as the $x$-$z$ plane ($\phi_i = 0$), as shown in Fig. 1. The polarization angle $\gamma_k$ is the angle between the direction of the incident electric field intensity vector and the direction of the normal to the plane of incidence. For an oblique incidence, the polarized incident wave can be resolved into two modes: the transverse electric (TE) mode for which $\gamma_k$ is zero and the transverse magnetic (TM) mode for which it is $\pi/2$. It is assumed that the dielectric media inside and outside the spheroids are linear, isotropic, and homogenous.

The incident electric field in the unprimed coordinate system $E_{iA}$ can be expanded in a series of prolate spheroidal vector wave functions in the matrix form

$$E_{iA} = \overline{M}_{iA}^{(1)} \overline{J}_A$$

(1)

with the overbar denoting a column matrix and $T$ the transpose of a matrix. The elements of $\overline{M}_{iA}^{(1)}$ which are spheroidal vector wave functions of the first kind expressed in terms of unprimed spheroidal coordinates, and those of $\overline{J}_A$ which are the corresponding known incident field expansion coefficients, are all defined in [2, appendix I].

The electromagnetic field scattered by the spheroid $B$ corresponds to a nonplane wave whose electric field intensity $E_{sB}$ can be expanded in terms of a set of prolate spheroidal vector wave functions as [2]

$$E_{sB} = \overline{M}_{sB}^{(2)} \overline{\beta}$$

(2)

in which $\overline{M}_{sB}^{(2)}$ and $\overline{\beta}$ are column matrices whose elements are prolate spheroidal vector wave functions of the fourth kind, expressed in terms of the primed spheroidal coordinates and the corresponding unknown expansion coefficients, respectively. Using the rotational–translational addition theorems for vector spheroidal

Fig. 1. Scattering system geometry.
wave functions [10], it is possible to express this outgoing electric field \( \mathbf{E}_{sB} \) from spheroid \( B \) as an incoming field \( \mathbf{E}_{sBA} \) to the spheroid \( A \) in the form [4]
\[
\mathbf{E}_{sBA} = \mathbf{M}^{(1)T}_{sB} [\Gamma] ^T \tilde{\mathbf{\beta}}.
\]

The electric fields \( \mathbf{E}_{sA} \) and \( \mathbf{E}_{sBA} \) incident on the spheroid \( A \), determine an electric field \( \mathbf{E}_{sA'} \), scattered by \( A \), which can be expanded in a matrix form similar to \( \mathbf{E}_{sB} \) as
\[
\mathbf{E}_{sA'} = \mathbf{M}^{(4)T}_{sA} \tilde{\mathbf{\alpha}}.
\]

The electric field intensity \( E_{sA} \) of the electromagnetic field transmitted inside the spheroid \( A \) can also be expanded in terms of a set of prolate spheroidal vector wave functions in the form [7]
\[
E_{sA} = \mathbf{M}^{(3)T}_{sA} \tilde{\mathbf{\gamma}}.
\]

The elements of \( \mathbf{M}^{(1)}_{sA} \) are prolate spheroidal vector wave functions of the first kind, expressed in terms of unprimed spheroidal coordinates, taking into account the permittivity of the material inside the spheroid. The elements of \( \tilde{\mathbf{\gamma}} \) are the corresponding unknown coefficients in the series expansion.

Using Maxwell's equation
\[
\mathbf{H} = jk^{-1}(\varepsilon/\mu)^{1/2} \nabla \times \mathbf{E}
\]
where \( \varepsilon \) and \( \mu \) are the permittivity and the permeability, respectively, it is possible to obtain the expansions of the different magnetic \( (\mathbf{H}) \) fields in terms of appropriate vector spheroidal wave functions from those of the corresponding electric \( (\mathbf{E}) \) fields. This is done by replacing \( \mathbf{M} \) by \( \mathbf{N} \), where \( \mathbf{N} = k^{-1}(\mathbf{V} \times \mathbf{M}) \), and multiplying each expansion by the appropriate value of \( j(\varepsilon/\mu)^{1/2} \).

III. IMPOSING THE BOUNDARY CONDITIONS

On the surface of each dielectric spheroid, the tangential components of both electric and magnetic fields should be continuous across the boundary. Thus on the surface of the spheroid \( A(\xi = \xi_A) \) we have
\[
(\mathbf{M}^{(1)T}_{sA} I_A + \mathbf{M}^{(1)T}_{sB} [\Gamma] ^T \tilde{\mathbf{\beta}} + \mathbf{M}^{(2)T}_{sA} \tilde{\mathbf{\alpha}}) \times \hat{\mathbf{\xi}} \big|_{\xi = \xi_A} = (\mathbf{M}^{(1)T}_{sA} \tilde{\mathbf{\gamma}}) \times \hat{\mathbf{\xi}} \big|_{\xi = \xi_A}
\]
\[
(\mathbf{N}^{(1)T}_{sA} I_A + \mathbf{N}^{(1)T}_{sB} [\Gamma] ^T \tilde{\mathbf{\beta}} + \mathbf{N}^{(2)T}_{sA} \tilde{\mathbf{\alpha}}) \times \hat{\mathbf{\xi}} \big|_{\xi = \xi_A} = \left( \epsilon_A / \epsilon_1 \right)^{1/2} (\mathbf{N}^{(1)T}_{sA} \tilde{\mathbf{\gamma}}) \times \hat{\mathbf{\xi}} \big|_{\xi = \xi_A}
\]
which \( \epsilon_1 \) and \( \epsilon_A \) are the permittivities of the media outside and inside the spheroid \( A \), respectively. Similarly, imposing the boundary conditions at the surface of the spheroid \( B(\xi' = \xi_B) \) yields
\[
(\mathbf{M}^{(1)T}_{sB} I_B + \mathbf{M}^{(1)T}_{sB} [\Gamma] ^T \tilde{\mathbf{\beta}} + \mathbf{M}^{(2)T}_{sB} \tilde{\mathbf{\alpha}}) \times \hat{\mathbf{\xi}}' \big|_{\xi' = \xi_B} = \left( \epsilon_B / \epsilon_2 \right)^{1/2} (\mathbf{M}^{(1)T}_{sB} \tilde{\mathbf{\gamma}}) \times \hat{\mathbf{\xi}}' \big|_{\xi' = \xi_B}
\]
\[
(\mathbf{N}^{(1)T}_{sB} I_B + \mathbf{N}^{(1)T}_{sB} [\Gamma] ^T \tilde{\mathbf{\beta}} + \mathbf{N}^{(2)T}_{sB} \tilde{\mathbf{\alpha}}) \times \hat{\mathbf{\xi}}' \big|_{\xi' = \xi_B} = \left( \epsilon_B / \epsilon_2 \right)^{1/2} (\mathbf{N}^{(1)T}_{sB} \tilde{\mathbf{\gamma}}) \times \hat{\mathbf{\xi}}' \big|_{\xi' = \xi_B}
\]

The elements of the column matrix \( \tilde{\mathbf{\delta}} \) are the unknown expansion coefficients in the series expansion of the electromagnetic field transmitted inside the spheroid \( B \), in terms of vector spheroidal wave functions of the first kind in primed coordinates. \( \mathbf{M}^{(1)}_{sB} \), \( I_B \), and \( \mathbf{M}^{(2)}_{sB} \) are all defined in [5], and \( \mathbf{M}^{(1)}_{sB} \) is defined in [7]. The elements of \( \mathbf{N}^{(1)}_{sB} \), \( \mathbf{N}^{(2)}_{sB} \), \( \mathbf{N}^{(3)}_{sB} \), and \( \mathbf{N}^{(4)}_{sB} \) are obtained from the corresponding elements of \( \mathbf{M}^{(1)}_{sB} \), \( \mathbf{M}^{(2)}_{sB} \), \( \mathbf{M}^{(3)}_{sB} \), and \( \mathbf{M}^{(4)}_{sB} \), respectively, with \( \mathbf{M} \) replaced by \( \mathbf{N} \), \( \varepsilon_B \) is the permittivity of the medium inside the spheroid \( B \). The structure of the matrix \( [\Gamma'] \) is similar to that of \( [\Gamma] \), with its elements consisting of rotational–translational coefficients, which appear in the rotational–translational addition theorems that are used to express vector wave functions of the fourth kind in unprimed spheroidal coordinates in terms of those of the first kind in primed coordinates for \( r' \leq \alpha \) (see Fig. 1), and are given by
\[
Q_{m\gamma}(\alpha, \beta, \gamma; \mathbf{d}) = \sum q_{m\gamma}^{\pm}(h) \cdot \sum_{q=0,1} \cdot \sum_{v=-(|m|+q)} R_{m\gamma}(\alpha, \beta, \gamma) \cdot \sum_{r=0,1} \cdot d_{m\gamma}^{\pm}(\alpha, \beta, \gamma; \mathbf{d})
\]
where \( d_{m\gamma}^{\pm}(h) \) and \( d_{m\gamma}^{\pm}(h') \) are the spherical expansion coefficients, and \( N_{m\gamma}(h') \) is the normalization constant [14]. The rest of the notation used is as follows:
\[
N_{m\gamma} = \frac{2}{(2l + 1)} \left( \frac{l + \mu}{l - \mu} \right)
\]
\[
R_{m\gamma}(\alpha, \beta, \gamma) = (-1)^{l-m} \frac{N_{m\gamma}}{N_{sg}} \left( \frac{\sin \beta}{\beta} \right)^{1/2} \sum_{l=s} j^{l-m} (2l + 1)
\]
\[
Q_{m\gamma}^{\pm}(\alpha, \beta, \gamma; \mathbf{d}) = \left( \frac{s+1}{s+m} \right) \cdot a(v, s) - \mu, k \cdot \psi^{[\pm]}_{m\gamma}(\mathbf{d})
\]
where \( a(v, s) = \mu, l \cdot p \cdot \psi^{[\pm]}_{m\gamma}(\mathbf{d}) \) is the linearization expansion coefficient, and \( \psi^{[\pm]}_{m\gamma}(\mathbf{d}) \) is the associated Legendre function of the first kind.

Taking the scalar product of both sides of (7) and (8) by
\[
\left\{ I_{s} \theta \right\} S_{m, \mid m \mid + \epsilon(h, \eta)} e^{\pm j m \epsilon (u v)} \text{ and of both sides of (9) and (10) by}
\left\{ I_{s} \phi \right\} S_{m, \mid m \mid + \epsilon(h, \eta)} e^{\pm j m \epsilon (u v)}
\]
in which \( \psi^{[\pm]}_{m\gamma}(\mathbf{d}) \) is the spherical Hankel function of the second kind and \( P_{l}^{\pm m} \) is the associated Legendre function of the first kind.
spondingly over the surfaces of the two spheroids, yields [7]

\[
\begin{bmatrix}
P_{MA} & 0 & Q_{MA} & R_{MBA} \Gamma^T \\
0 & Q_{NA} & R_{NBA} \Gamma^T \\
0 & P_{MB} & R_{MBA} \Gamma^T & Q_{MB} \\
0 & 0 & R_{NAB} \Gamma^T & Q_{NB}
\end{bmatrix}
\]

\[= \begin{bmatrix}
R_{MA} & 0 \\
0 & R_{NA} \\
0 & R_{MB} \\
0 & R_{NB}
\end{bmatrix}
\begin{bmatrix}
\vec{i}_A \\
\vec{i}_B
\end{bmatrix}
\]  

(17)

\[\begin{bmatrix}
P_{MA} & 0 & Q_{MA} & R_{MBA} \Gamma^T \\
0 & Q_{NA} & R_{NBA} \Gamma^T \\
0 & P_{MB} & R_{MBA} \Gamma^T & Q_{MB} \\
0 & 0 & R_{NAB} \Gamma^T & Q_{NB}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
R_{MA} & 0 \\
0 & R_{NA} \\
0 & R_{MB} \\
0 & R_{NB}
\end{bmatrix}
\begin{bmatrix}
\vec{i}_A \\
\vec{i}_B
\end{bmatrix}
\]

\[\begin{bmatrix}
P_{MA} & 0 & Q_{MA} & R_{MBA} \Gamma^T \\
0 & Q_{NA} & R_{NBA} \Gamma^T \\
0 & P_{MB} & R_{MBA} \Gamma^T & Q_{MB} \\
0 & 0 & R_{NAB} \Gamma^T & Q_{NB}
\end{bmatrix}
\]

\[= \begin{bmatrix}
R_{MA} & 0 \\
0 & R_{NA} \\
0 & R_{MB} \\
0 & R_{NB}
\end{bmatrix}
\begin{bmatrix}
\vec{i}_A \\
\vec{i}_B
\end{bmatrix}
\]

(18)

\[\begin{bmatrix}
P_{MA} & 0 & Q_{MA} & R_{MBA} \Gamma^T \\
0 & Q_{NA} & R_{NBA} \Gamma^T \\
0 & P_{MB} & R_{MBA} \Gamma^T & Q_{MB} \\
0 & 0 & R_{NAB} \Gamma^T & Q_{NB}
\end{bmatrix}
\]

\[= \begin{bmatrix}
R_{MA} & 0 \\
0 & R_{NA} \\
0 & R_{MB} \\
0 & R_{NB}
\end{bmatrix}
\begin{bmatrix}
\vec{i}_A \\
\vec{i}_B
\end{bmatrix}
\]

(19)

Equation (17) can be rewritten in the form

\[\begin{bmatrix}
P_{MA} & 0 & Q_{MA} & R_{MBA} \Gamma^T \\
0 & Q_{NA} & R_{NBA} \Gamma^T \\
0 & P_{MB} & R_{MBA} \Gamma^T & Q_{MB} \\
0 & 0 & R_{NAB} \Gamma^T & Q_{NB}
\end{bmatrix}
\]

\[= \begin{bmatrix}
R_{MA} & 0 \\
0 & R_{NA} \\
0 & R_{MB} \\
0 & R_{NB}
\end{bmatrix}
\begin{bmatrix}
\vec{i}_A \\
\vec{i}_B
\end{bmatrix}
\]

(20)

\[
(G) \text{ is the system matrix, which is independent of the direction and polarization of the incident wave.}
\]

\[
\begin{align*}
\text{IV. SCATTERING CROSS SECTIONS IN THE FAR FIELD} & \\
\text{Let us consider a point of observation which has spherical coordinates r, } \theta, \phi \text{ and } r', \theta', \phi' \text{ relative to the two systems } Oxyz & \\
\text{and } O'x'y'z', \text{ respectively. If each of the scattered electric fields is } & \\
\text{expanded in terms of appropriate vector spheroidal wave functions, then as } r \to \alpha, r' \to \alpha, \text{ using the asymptotic expressions of these } & \\
\text{vector wave functions it is possible to write an expression for the scattered electric field intensity in the far zone as} & \\
E_s = E_{sA} + E_{sB} & \\
\frac{e^{-jkr}}{kr} \left[ F_\theta(\theta, \phi) \theta + F_\phi(\theta, \phi) \phi \right] & \\
\text{where} & \\
F_\theta(\theta, \phi) = F_{sA}(\theta, \phi) + F_{sB}(\theta, \phi) & \\
F_\phi(\theta, \phi) = F_{sA}(\theta, \phi) + F_{sB}(\theta, \phi)
\end{align*}
\]

(21)

(22)

\[\text{with } F_{sA}(\theta, \phi), F_{sB}(\theta, \phi) \text{ given in [2], and } F_{sA}(\theta, \phi), F_{sB}(\theta, \phi) \text{ given in [4].}
\]

The normalized bistatic cross section is defined as

\[\frac{\pi \sigma(\theta, \phi)}{\lambda^2} = |F_\theta(\theta, \phi)|^2 + |F_\phi(\theta, \phi)|^2.
\]

(23)

The normalized bistatic cross sections in the E- and H-planes are obtained by substituting \(\phi = \pi/2\) and \(\phi = 0\), respectively, in (23).

When \(\theta = \theta_1\) and \(\phi = \phi_1 = 0\), the normalized backscattering cross section is obtained from (23)

\[\frac{\pi \sigma(\theta_1)}{\lambda^2} = |F_\theta(\theta_1, 0)|^2 + |F_\phi(\theta_1, 0)|^2.
\]

(24)

\[
\begin{align*}
\text{V. NUMERICAL RESULTS} & \\
\text{Numerical results are given for the normalized bistatic and backscattering cross sections in the far field for dielectric prolate spheroids of axial ratios 2 and 5, having various displacements of their centers, and for the relative orientation corresponding to the Euler angles } \alpha = 30^\circ, \beta = 45^\circ, \gamma = 60^\circ. \text{ The rotation } \alpha \text{ is about the } z \text{ axis, the rotation } \beta \text{ is about the new } y \text{ axis, and the final rotation } \gamma \text{ is about the new } z \text{ axis. The medium outside the spheroids is considered to be free space, so that the permittivity of it defined as } \varepsilon_0 \text{ in (8) and (10) is equal to } \varepsilon_0, \text{ the permittivity of free space. Since the series expansions of all the electromagnetic fields in terms of appropriate vector spheroidal wave functions are infinite in extent, when these expansions are written in matrix form as given in Sections II and III, all the corresponding matrices also have infinite dimensions. Therefore, to obtain numerical results with a prescribed accuracy, one has to truncate these series and matrices appropriately. To obtain a two significant digit accuracy in the computed scattering cross sections one has to consider only the } \phi \text{ harmonics } e^{j\phi}, e^{j\phi} \text{ and } e^{2j\phi}. \text{ All the results given in this communication have thus been obtained by considering the values of } m \text{ corresponding to the above } \phi \text{ harmonics, } n = |m|, |m| + 1, \cdots, |m| + 5 \text{ and } k = 0, 1, \cdots, 5 \text{ in truncating all the series and matrices.}
\end{align*}
\]

Fig. 2 shows the normalized bistatic cross section for TE polarization of the incident wave as a function of the scattering angle for two identical prolate spheroids \(A\) and \(B\) each of axial ratio 2, semimajor axes \(\lambda/4\), and dielectric constant 3.0, with the spheroid centers displaced along the \(z\) axis by a distance \(d\), and Euler angles \(30^\circ, 45^\circ, 60^\circ\).

![Normalized bistatic cross section for TE polarization of the incident wave](image-url)
Fig. 3. Normalized backscattering cross section as a function of $\theta_i$ for two identical spheroids having an axial ratio 2, $\varepsilon_r = 3.0$, with their centers displaced along the $z$ axis by a distance $d$, and Euler angles $30^\circ$, $45^\circ$, $60^\circ$.

Fig. 4. Normalized backscattering cross section as a function of $\theta_i$ for two spheroids of axial ratios 2 and 5, having different dielectric constants, with Euler angles $30^\circ$, $45^\circ$, $60^\circ$, and their centers displaced by a distance $\lambda/2$ along the direction specified by $\theta_0 = 60^\circ$, $\phi_0 = 20^\circ$.

spheroids changes from $\lambda/2$ to $\lambda$, both $E$- and $H$-plane patterns tend to show more oscillations with deeper and sharper minima, due to the effect of multiple scattering becoming more pronounced.

In Fig. 3 we present the plots of normalized backscattering cross section versus angle of incidence for two spheroids having the same configurations, orientation, and dielectric constant as in Fig. 2. When compared with the corresponding plots for two parallel dielectric spheroids we observe that the amount of oscillations in the curves has increased. This is due to the fact that more area is now available in general for the interaction between the two spheroids, than when they are in the parallel configuration.

In Fig. 4 the plots of normalized backscattering cross section versus angle of incidence are given for two nonidentical spheroids $A$ and $B$ of axial ratios 2 and 5, respectively, with each spheroid having a semimajor axis $\lambda/4$ and separated center-to-center by a distance $\lambda/2$ in the direction specified by the spherical coordinates $\theta_0 = 60^\circ$, $\phi_0 = 20^\circ$. The orientation of $B$ with respect to $A$ is specified by the Euler angles $\alpha = 30^\circ$, $\beta = 45^\circ$, $\gamma = 60^\circ$. In the first plot the dielectric constants of the spheroids $A(\varepsilon_{rA})$ and $B(\varepsilon_{rB})$ are 3.0 and 4.0, respectively, and in the second one they are 4.0 and 3.0, respectively. In both cases the behavior of the backscattering cross sections for both TE and TM polarizations is almost the same. However, in the first plot the minimum and the second maximum are sharper than the corresponding ones in the second plot, and occur around $\theta_i = 120^\circ$ and $150^\circ$, respectively.

The results for a system of perfectly conducting spheroids of arbitrary orientation can be obtained as a special case from the results presented here, for $\varepsilon \to \infty$. The normalized backscattering cross sections calculated for two spheroids $A$ and $B$, each of an axial ratio 2, semimajor axis $\lambda/4$, having an orientation $\alpha = 0^\circ$, $\beta = 90^\circ$, $\gamma = 0^\circ$, with centers displaced along the $z$ axis of the spheroid $A$ by $\lambda/2$ and a relative permittivity of $10^4$ was compared with the corresponding results for the perfectly conducting case, and were found to be in good agreement. The same computer algorithm was used for all the permittivity range in order to check its efficiency. The number of terms that have to be used in truncating the series, with increasing relative permittivity is practically the same for a given accuracy.

The formulation presented in this communication and the software used for calculating numerical results is general. However, it is interesting to note that for a particular system of only two spheroids, the relative position of one with respect to the other can always be obtained by choosing the $x$ and $y$ axes appropriately, and then by performing only one rotation through the Euler angle $\beta$, i.e., with $\alpha = 0^\circ$, $\gamma = 0^\circ$, followed by the corresponding translation. In order to demonstrate the generality of the theory presented and the validity of the software being used, all the other results presented in this communication have been obtained by choosing $\alpha$ and $\gamma$ to be different from zero. The reduction in the total amount of computation time required, however, when using $\alpha = 0^\circ$, $\gamma = 0^\circ$ is only about $5\%$ with respect to the case when they are different from zero.

VI. CONCLUSION

On the basis of rotational-translational addition theorems for vector spheroidal wave functions, an exact analytic solution to the problem of electromagnetic scattering by a system of two dielectric spheroids of arbitrary orientation has been obtained for the first time. These exact solutions for systems of complex objects are important for validating large numerical codes and experimental results. Numerical results are given for the normalized bistatic and backscattering cross sections in the resonance region, where the major axis lengths of the spheroids are comparable to the wavelength of the exciting wave. From the results obtained it is clearly demonstrated that the magnitudes of the backscattering cross sections are much less than those of the corresponding perfectly conducting case. The results for electromagnetic scattering by systems of perfectly conducting spheroids of arbitrary orientation, by systems of parallel dielectric spheroids, parallel perfectly conducting spheroids, and by spheres are obtained as special cases from those presented here.

REFERENCES


