Scattering of Electromagnetic Waves by a Coated Dielectric Spheroid

M. F. R. Cooray and I. R. Ciric*

Department of Electrical and Computer Engineering
University of Manitoba
Winnipeg, Manitoba, R3T 2N2, Canada

Abstract—The method of separation of variables is used to derive an analytic solution to the problem of electromagnetic scattering by a homogeneous dielectric spheroid with a confocal lossy dielectric coating of arbitrary thickness. The electric and magnetic fields in each different region are expressed in terms of a set of vector spheroidal eigenfunctions, and the solution is obtained by imposing the appropriate boundary conditions at each spheroidal surface. Computed results illustrate the dependence of the scattered field in the far zone on the coating material and its thickness, the size and the material of the spheroid, and on the angle of incidence.

INTRODUCTION

Recently, there has been an increasing interest in the study of reflective and absorptive properties of objects that can be modeled using cylinders, spheres, and spheroids with a dielectric coating [1–6]. Analytic solutions are available for circular and elliptic cylinders, and for spheres coated with dielectric materials. The purpose of this paper is to obtain an exact analytic solution to the problem of electromagnetic scattering from a dielectric spheroid with a confocal dielectric coating. This type of a model is useful in radar engineering, to analyze the change in the scattering patterns, and in biomedical engineering for studying the amount of electromagnetic radiation being absorbed by a human body. It can also be used as a benchmark in determining the accuracy of various numerical and approximate methods applied to three-dimensional systems [7]. Extending the formulation for scattering by a spheroid with a single layer of coating to the case of scattering by a spheroid with two or more layers of coatings is straightforward.

FORMULATION OF THE PROBLEM

Consider a linearly polarized monochromatic plane electromagnetic wave, with an electric field intensity of unit amplitude, incident on a spheroid coated with a confocal layer, as shown in Fig. 1. The media of the spheroid and of the coating layer are both assumed to be linear, homogeneous, and isotropic with permittivities \( \varepsilon_2 \) and \( \varepsilon_1 \), in general complex quantities, and permeabilities \( \mu_2 \) and \( \mu_1 \), respectively. The medium outside the spheroidal structure is free space with per-

* To whom correspondence should be sent.
mittivity \( \varepsilon_0 \) and permeability \( \mu_0 \). The semi axial lengths of the spheroidal core are \( a_2 \) and \( b_2 \), and those of the spheroid formed by the confocal outer layer are \( a_1 \) and \( b_1 \). The inner and outer spheroidal surfaces are defined by \( \xi = \xi_2 \) and \( \xi = \xi_1 \), respectively, in which \( \xi \) is the radial coordinate of the spheroidal coordinate system. The major axis of the spheroid is along the \( z \)-axis of the Cartesian system \( Oxyz \) whose origin \( O \) is at the spheroid center.

Figure 1. Geometry of the scattering system.
Without any loss of generality the incident plane can be taken to be the $x$-$z$ plane ($\phi_i = 0$), with the incident propagation vector $k_0$ making an angle $\theta_i$ with the $z$-axis. The polarization angle $\gamma_k$ is the angle between the direction of the incident electric field intensity and the direction of the normal to the plane of incidence. A linearly polarized incident wave can be resolved in general into transverse electric (TE) and transverse magnetic (TM) components. For TE polarization $\gamma_k = 0$ and for TM polarization $\gamma_k = \pi/2$. A time dependence of $e^{j\omega t}$ is assumed and suppressed throughout. In this paper we present results only for prolate spheroids. The corresponding results for oblate spheroids can be obtained by using the transformation $\xi \rightarrow j\xi$ and $F \rightarrow -jF$, with $F$ being the semi-interfocal distance of the spheroid.

The incident electric and magnetic fields $\bar{E}_i$ and $\bar{H}_i$ can be expanded in terms of a set of vector spheroidal wave functions in the form

$$\bar{E}_i = \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} (p_{mn}^+ M_{mn}^{(1)} + p_{mn}^- M_{mn}^{(-1)})$$

$$\bar{H}_i = j\sqrt{\frac{\mu_0}{\varepsilon_0}} \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} (p_{mn}^+ N_{mn}^{(1)} + p_{mn}^- N_{mn}^{(-1)})$$

where $[8-11]

$$p_{mn}^{\pm} = \frac{2j^{n-1}}{k_0 N_{mn}} S_{mn}(h_0, \cos \theta_i) \left( \frac{\cos \gamma_k}{\cos \theta_i} \mp j \sin \gamma_k \right)$$

with $N_{mn}$ being the normalization constant of the spheroidal angle function $S_{mn}(h_0, \cos \theta_i)$ [12], $h_0 = k_0 F$, with $k_0$ the wavenumber in free space. The spheroidal vector wave functions $M_{mn}$ are those given in [8], and $N_{mn} = k^{-1} \nabla \times M_{mn}$. Equation (1) can now be written in a matrix form as

$$\bar{E}_i = \bar{M}_i^{(1)T} \bar{I}$$

with the double overbar denoting a column matrix, and $T$ denoting the transpose of a matrix,

$$\bar{M}_i^{(1)} = \begin{pmatrix} \bar{M}_{i0} \\ \bar{M}_{i1} \\ \bar{M}_{i2} \\ \vdots \end{pmatrix}, \quad \bar{I} = \begin{pmatrix} \bar{p}_0 \\ \bar{p}_1 \\ \bar{p}_2 \\ \vdots \end{pmatrix}$$

where

$$\bar{M}_i^{(1)T} = \begin{bmatrix} \bar{M}_{i0}^{+(1)T} & \bar{M}_{i1}^{-(1)T} \\ \bar{M}_{i1}^{+(1)T} & \bar{M}_{i0}^{-(1)T} \end{bmatrix}$$

$$\bar{M}_i^{(1)T} = \begin{bmatrix} \bar{M}_{\sigma-1}^{+(1)T} & \bar{M}_{\sigma+1}^{-(1)T} & \bar{M}_{\sigma+1}^{+(1)T} & \bar{M}_{\sigma-1}^{-(1)T} \\ \bar{M}_{\sigma+1}^{+(1)T} & \bar{M}_{\sigma-1}^{-(1)T} & \bar{M}_{\sigma-1}^{+(1)T} & \bar{M}_{\sigma+1}^{-(1)T} \end{bmatrix}, \quad \sigma \geq 1$$

with $\sigma$ being a positive integer.
with
\[
\overline{M}^{\pm(1)}_\tau = \begin{bmatrix} \overline{M}^{\pm(1)}_{\tau,|\tau|} (h_0; \tau) & \overline{M}^{\pm(1)}_{\tau,|\tau|+1} (h_0; \tau) & \overline{M}^{\pm(1)}_{\tau,|\tau|+2} (h_0; \tau) & \cdots \end{bmatrix}
\]
(7)
\(\overline{M}\) denoting the spheroidal coordinate triad \((\xi, \eta, \phi)\) and
\[
\overline{P}_0^T = \begin{bmatrix} \overline{P}_0^T & \overline{P}_1^T \end{bmatrix}
\]
(8)
with
\[
\overline{P}_\sigma^T = \begin{bmatrix} \overline{P}_{\sigma-1}^T & \overline{P}_{\sigma+1}^T & \overline{P}_{\sigma+1}^T & \overline{P}_{\sigma-1}^T \end{bmatrix}, \quad \sigma \geq 1
\]
(9)
Similarly, (2) can be written in matrix form as [13–14]
\[
\overline{H}_i = j \sqrt{\frac{\epsilon_0}{\mu_0}} \overline{N}_i^{(1)T} \overline{\bar{y}}
\]
(10)
The elements of \(\overline{N}_i^{(1)T}\) are obtained from the corresponding elements of \(\overline{M}_i^{(1)T}\) by replacing \(\overline{M}\) by \(\overline{N}\). The scattered field \(\overline{E}_s\) for \(\xi > \xi_1\) can be expanded in terms of vector spheroidal wave functions [9] as
\[
\overline{E}_s = \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \left( \alpha_{mn}^+ \overline{M}_{mn}^{+(4)} + \alpha_{m+1,n+1}^z \overline{M}_{m+1,n+1}^{z(4)} \right)
\]
(11)
This takes the matrix form
\[
\overline{E}_s = \overline{M}_s^{(4)} \overline{\alpha}
\]
(12)
where
\[
\overline{M}_s^{(4)} = \begin{bmatrix} \overline{M}_{s0} \\ \overline{M}_{s1} \\ \overline{M}_{s2} \\ \vdots \end{bmatrix}, \quad \overline{\alpha} = \begin{bmatrix} \overline{\alpha}_0 \\ \overline{\alpha}_1 \\ \overline{\alpha}_2 \\ \vdots \end{bmatrix}
\]
(13)
in which
\[
\overline{M}_s^{(4)} = \begin{bmatrix} \overline{M}_{s0}^{+(4)T} & \overline{M}_{s0}^{z(4)T} \\ \overline{M}_{s1}^{+(4)T} & \overline{M}_{s1}^{z(4)T} \\ \overline{M}_{s2}^{+(4)T} & \overline{M}_{s2}^{z(4)T} \\ \vdots & \vdots \end{bmatrix}, \quad \sigma \geq 1
\]
(14)
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with

\[
\overline{M}_{+}^{(4)T} = \left[ M_{r_{1}r_{1}}^{(4)}(h_{0};\bar{r}) M_{r_{1}r_{1}+1}^{(4)}(h_{0};\bar{r}) M_{r_{1}r_{1}+2}^{(4)}(h_{0};\bar{r}) \ldots \right]
\]

\[
\overline{M}_{z}^{(4)T} = \left[ M_{r_{1}r_{1}}^{(4)}(h_{0};\bar{r}) M_{r_{1}r_{1}+1}^{(4)}(h_{0};\bar{r}) M_{r_{1}r_{1}+2}^{(4)}(h_{0};\bar{r}) \ldots \right]
\]

and

\[
\overline{\alpha}_{0}^{T} = \left[ \overline{\alpha}_{0}^{+T} \overline{\alpha}_{0}^{zT} \right]
\]

\[
\overline{\alpha}_{\sigma}^{T} = \left[ \overline{\alpha}_{\sigma-1}^{+T} \overline{\alpha}_{\sigma}^{zT} \overline{\alpha}_{-(\sigma-1)}^{T} \overline{\alpha}_{-\sigma}^{T} \right], \quad \sigma \geq 1
\]

with

\[
\overline{\alpha}_{+}^{T} = \left[ \alpha_{r_{1}r_{1}}^{+} \alpha_{r_{1}r_{1}+1}^{+} \alpha_{r_{1}r_{1}+2}^{+} \ldots \right]
\]

\[
\overline{\alpha}_{z}^{T} = \left[ \alpha_{r_{1}r_{1}}^{z} \alpha_{r_{1}r_{1}+1}^{z} \alpha_{r_{1}r_{1}+2}^{z} \ldots \right]
\]

The scattered magnetic field \( \overline{H}_{s} \) for \( \xi > \xi_{1} \) has the form

\[
\overline{H}_{s} = j\sqrt{\frac{\mu_{0}}{\xi}} \overline{N}_{s}^{(4)T} \overline{\alpha}
\]

with the elements of \( \overline{N}_{s}^{(4)T} \) obtained from those of \( \overline{M}_{s}^{(4)T} \) by replacing \( \overline{M} \) by \( \overline{N} \). The electric field transmitted in the region \( \xi_{2} < \xi < \xi_{1} \) can be expressed as

\[
1\overline{E}_{t} = 1\overline{M}_{t}^{(1)T} \overline{\beta} + 1\overline{M}_{t}^{(2)T} \overline{\gamma}
\]

where the elements of the matrices \( 1\overline{M}_{t}^{(1)T} \) and \( 1\overline{M}_{t}^{(2)T} \) are obtained from the corresponding elements of the matrix \( \overline{M}_{s}^{(4)T} \) by replacing the spheroidal vector wave functions of the fourth kind by those of the first kind and the second kind appropriately, and \( h_{0} \) by \( h_{1} = k_{1}F \), with \( k_{1} = \sqrt{\mu_{1}/\mu_{0}\sqrt{\varepsilon_{1}/\varepsilon_{0}}} k_{0} \). The matrices \( \overline{\beta} \) and \( \overline{\gamma} \) are obtained from \( \overline{\alpha} \) by replacing \( \alpha \) by \( \beta \) and by \( \gamma \), respectively. For the transmitted electric field in the region \( \xi < \xi_{2} \) we can write

\[
2\overline{E}_{t} = 2\overline{M}_{t}^{(1)T} \overline{\delta}
\]

with the matrix \( 2\overline{M}_{t}^{(1)T} \) obtained from \( 1\overline{M}_{t}^{(1)T} \) by replacing \( h_{1} \) by \( h_{2} = k_{2}F \), where \( k_{2} = \sqrt{\mu_{2}/\mu_{0}\sqrt{\varepsilon_{2}/\varepsilon_{0}}} k_{0} \). \( \overline{\delta} \) is obtained from \( \overline{\alpha} \) by replacing \( \alpha \) by \( \delta \). The expansions of the corresponding magnetic fields are

\[
1\overline{H}_{t} = j\sqrt{\frac{\varepsilon_{1}}{\mu_{1}}} \left[ 1\overline{N}_{t}^{(1)T} \overline{\beta} + 1\overline{N}_{t}^{(2)T} \overline{\gamma} \right]
\]

\[
2\overline{H}_{t} = j\sqrt{\frac{\varepsilon_{2}}{\mu_{2}}} 2\overline{N}_{t}^{(1)T} \overline{\delta}
\]

where the matrices \( 1\overline{N}_{t}^{(1)T} \) and \( 1\overline{N}_{t}^{(2)T} \), \( i = 1, 2 \), are obtained from \( 1\overline{M}_{t}^{(1)T} \) and \( 1\overline{M}_{t}^{(2)T} \), respectively, by replacing \( \overline{M} \) by \( \overline{N} \).
IMPOSING THE BOUNDARY CONDITIONS

The tangential components of the total electric and magnetic fields across each of the spheroidal surfaces \( \xi = \xi_1 \) and \( \xi = \xi_2 \) are continuous, that is on the spheroidal surface \( \xi = \xi_1 \)

\[
(E_i + E_s) \times \hat{\xi} = \frac{1}{2} E_t \times \hat{\xi} \\
(H_i + H_s) \times \hat{\xi} = \frac{1}{2} H_t \times \hat{\xi}
\]  

(22)

and on the spheroidal surface \( \xi = \xi_2 \)

\[
1E_t \times \hat{\xi} = 2E_t \times \hat{\xi} \\
1H_t \times \hat{\xi} = 2H_t \times \hat{\xi}
\]

(23)

In terms of the corresponding expansions in series of vector spheroidal wave functions, (22) and (23) can be rewritten as

\[
\left( \frac{M_i}{\alpha} \right)^{(1)} (T) = \left( \frac{M_i}{\alpha} \right)^{(2)} (T) \times \hat{\xi} |_{\xi = \xi_1} = \left( \frac{1}{\beta} \frac{M_i}{\alpha} \right)^{(1)} T + \frac{1}{\beta} \frac{M_i}{\alpha} \right)^{(2)} \times \hat{\xi} |_{\xi = \xi_1} \\
\]

\[
\left( \frac{N_i}{\alpha} \right)^{(1)} (T) = \left( \frac{N_i}{\alpha} \right)^{(2)} (T) \times \hat{\xi} |_{\xi = \xi_1} = \sqrt{\frac{\mu_0}{\mu_1}} \sqrt{\frac{\varepsilon_1}{\varepsilon_0}} \left( \frac{1}{\beta} \frac{N_i}{\alpha} \right)^{(1)} T + \frac{1}{\beta} \frac{N_i}{\alpha} \right)^{(2)} \times \hat{\xi} |_{\xi = \xi_1} \\
\]

\[
\left( \frac{1}{\beta} \frac{M_i}{\alpha} \right)^{(1)} T + \frac{1}{\beta} \frac{M_i}{\alpha} \right)^{(2)} \times \hat{\xi} |_{\xi = \xi_2} = \frac{1}{\delta} \left( \frac{1}{\beta} \frac{M_i}{\alpha} \right)^{(1)} T + \frac{1}{\beta} \frac{M_i}{\alpha} \right)^{(2)} \times \hat{\xi} |_{\xi = \xi_2} \\
\]

(25)

Applying the orthogonality properties of the trigonometric functions and the spheroidal angle functions, and integrating correspondingly over each spheroidal surface, we finally obtain a matrix equation of the form [13]

\[
\bar{S} = [G] \bar{T}
\]  

(26)

in which

\[
\bar{S} = \begin{pmatrix}
\bar{\alpha} \\
\bar{\beta} \\
\bar{\gamma} \\
\bar{\delta}
\end{pmatrix}
\]

\[
\bar{T} = \begin{pmatrix}
\bar{\alpha} \\
\bar{\beta} \\
\bar{\gamma} \\
\bar{\delta}
\end{pmatrix}
\]

is the column matrix of the unknown coefficients and \([G]\) is the system matrix whose elements are independent of the direction and polarization of the incident wave. The calculation of the elements of \([G]\) is given in [11]. The details regarding the calculation of the integrals that appear as a result of applying the orthogonality of the spheroidal wave functions are given in [15] and [11]. Once the coefficients \(\alpha, \beta, \gamma, \) and \(\delta\) are known, it is possible to calculate the fields outside the spheroidal system, inside the coating, and inside the spheroid by substituting back in the appropriate series expansions of the fields. In this paper we
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present quantitative results for the normalized bistatic and backscattering cross sections.

scattered field in the far zone

When using the asymptotic forms of the vector spheroidal wave functions $\overline{M}$, the scattered electric field $\overline{E_s}$ in the far zone becomes

$$\overline{E_s} = \frac{e^{-jkr}}{kr} \left[ F_\theta(\theta, \phi) \hat{\theta} + F_\phi(\theta, \phi) \hat{\phi} \right]$$  \hspace{1cm} (27)$$

where

$$F_\theta(\theta, \phi) = - \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} j^{n+1} \frac{S_{mn}(h_0, \cos \theta)}{2} \left( (\alpha_{mn}^{+} - \alpha_{mn}^{-}) \cos(m+1)\phi ight. \right.$$  

$$+ j(\alpha_{mn}^{+} + \alpha_{mn}^{-}) \sin(m+1)\phi \left. - \sum_{n=1}^{\infty} j^{n+1} \frac{S_{ln}(h_0, \cos \theta)}{2} \alpha_{ln}^{+} \right) \hspace{1cm} (28)$$

$$F_\phi(\theta, \phi) = \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} j^n \left[ \cos \theta \frac{S_{mn}(h_0, \cos \theta)}{2} \right.$$  

$$\cdot \left\{ (\alpha_{mn}^{+} + \alpha_{mn}^{-}) \cos(m+1)\phi + j(\alpha_{mn}^{+} - \alpha_{mn}^{-}) \sin(m+1)\phi \right\}$$  

$$- j \sin \theta S_{m+1,n+1}(h_0, \cos \theta)$$  

$$\cdot \left\{ (\alpha_{m+1,n+1}^{+} + \alpha_{m+1,n+1}^{-}) \cos(m+1)\phi \right.$$  

$$+ j(\alpha_{m+1,n+1}^{+} - \alpha_{m+1,n+1}^{-}) \sin(m+1)\phi \right\} \right]$$  

$$+ \cos \theta \sum_{n=1}^{\infty} j^n \frac{S_{ln}(h_0, \cos \theta)}{2} \alpha_{ln}^{+} \sin \theta \sum_{n=0}^{\infty} j^n S_{0n}(h_0, \cos \theta) \alpha_{0n}^{+} \right) \hspace{1cm} (29)$$

in which

$$\alpha_{mn}^{+} = k_0 \alpha_{mn}^{+}, \quad \alpha_{mn}^{-} = k_0 \alpha_{mn}^{-}$$

and $r, \theta, \phi$ are the spherical coordinates of the point of observation.

The normalized bistatic cross section is obtained from

$$\frac{\pi \sigma(\theta, \phi)}{\lambda_0^2} = |F_\theta(\theta, \phi)|^2 + |F_\phi(\theta, \phi)|^2$$  \hspace{1cm} (30)$$

with $\lambda_0 = 2\pi/k_0$. Numerical computation has been performed for the normalized bistatic cross sections in the $E$- and $H$- planes, i.e., $\phi = \pi/2$ and $\phi = 0$, respectively, for different values of the scattering angle $\theta$. When $\theta = \theta_i$ and
\( \phi = \phi_i = 0 \) we obtain the normalized backscattering cross section

\[
\frac{\pi \sigma(\theta_i)}{\lambda_0^2} = |F_\theta(\theta_i,0)|^2 + |F_\phi(\theta_i,0)|^2
\]

for different values of the angle of incidence \( \theta_i \).

**NUMERICAL RESULTS AND DISCUSSION**

Computed numerical results are presented in the form of plots of normalized bistatic and backscattering cross sections in the far field, for spheroids with different axial ratios, materials and coatings. Since the series expansions in terms of vector spheroidal wave functions are infinite in extent, all the matrices involved in the field expressions have infinite dimensions. Thus, to obtain numerical results of a required accuracy, these infinite series and matrices should be appropriately truncated. The number of terms in the series expansion of the fields (or the number of matrix elements) required, to obtain a given accuracy in the computed scattering cross sections, depends on the frequency, the size and the material of the spheroid and the coating material. For the sizes and the permittivities of the spheroids and the coatings considered in this paper, we have found it sufficient to consider only the \( \phi \)-harmonics \( e^{j0}, e^{\pm j\phi}, \) and \( e^{\pm 2j\phi} \) in the vector spheroidal wave functions \( \tilde{M}_{mn} \) and \( \tilde{N}_{mn} \), and \( n = |m|, |m| + 1, \ldots, |m| + 5 \) for each value of \( m \), to obtain a two significant digit accuracy in the computed scattering cross sections.

![Figure 2](image.png)

**Figure 2.** Comparison between the normalized backscattering cross section for a dielectric spheroid of axial ratio 2, semi-major axis length 0.2\( \lambda_0 \), and \( \epsilon_{r2} = 2.25 \), “coated” with a layer of \( \epsilon_{r1} = 1.0 \) and thickness 0.02\( \lambda_0 \), and that for an uncoated identical spheroid (shown by crosses).
The efficiency of the computer program used and the accuracy of the results obtained were checked by performing the following experiments. First we considered a homogeneous dielectric spheroid, "coated" with a confocal layer having the same material parameters as those of free space. The spheroid has an axial ratio of 2, with a semi-major axis length of 0.2\(\lambda_0\), and a relative permittivity of 2.25. The thickness of the coating (i.e., \(a_1 - a_2\)) is 0.02\(\lambda_0\). The backscattering cross section computed on the basis of the formulation presented in this paper has been compared with that calculated directly for an identical single dielectric spheroid without any coating on its outer surface. As it can be seen from Fig. 2, the results for the two cases are in very good agreement. Then, in order to verify the computation accuracy in the case of complex values of permittivities, we have calculated the normalized bistatic cross section for a conducting spheroid of an axial ratio of 1.0001 and semi-major axis length of 0.175\(\lambda_0\), coated with a layer of thickness 0.025\(\lambda_0\) and dielectric constant 3.0 - j0.006. The results have been compared with those given in [16] for a conducting sphere of radius 0.175\(\lambda_0\) coated with a layer of the same thickness and dielectric constant, and have been found to be in very good agreement. The special case of a conducting spheroid has been simulated from a dielectric spheroid by using a very high value (10\(^6\)) for the relative permittivity and a very low value (10\(^{-6}\)) for the relative permeability. In addition we have also obtained the results given in [17] for a conducting sphere and a spherical shell coated with a layer of lossy dielectric material, using our program for a spheroid of very low eccentricity. To the best of our knowledge, there are no numerical results available in the resonance region for scattering by a coated dielectric spheroid, for the purpose of comparison.

![Graph](image-url)

**Figure 3.** \(E\)-plane patterns for a dielectric spheroid of axial ratio 2, semi-major axis length 0.25\(\lambda_0\), and \(\varepsilon_r = 2.25\), with confocal coatings of thickness 0.01\(\lambda_0\) and different dielectric constants.
A series of results for a dielectric spheroid with a confocal dielectric coating in the resonance region are presented in Figs. 3 to 9. For all the cases presented in this paper, we have taken $\mu_1$ and $\mu_2$ to be equal to $\mu_0$ and all the normalized bistatic cross sections calculated are for the case of an axial incidence ($\theta_i = 0^\circ$).

Figure 3 shows the variation of the $E$-plane pattern with the type of material being used for coating. The spheroid in this case is of axial ratio 2, having a semi-major axis length of $0.25\lambda_0$, and a dielectric constant $\epsilon_r^2 = 2.25$. The thickness of the lossless coating is $0.01\lambda_0$. There is a similarity in the shape of these $E$-plane patterns. However, as the dielectric constant $\epsilon_r^1$ of the coating material increases we observe that the magnitude of the forward scattering cross section ($\phi = 180^\circ$) increases. The change in the magnitude of the backscattering cross section is much smaller.

In Fig. 4 we present the variation of the normalized bistatic cross section of a spheroidal shell with the scattering angle. The spheroidal shell has a size and shape similar to the spheroid in Fig. 3, and the coating thickness is again $0.01\lambda_0$, but lossy, with $\epsilon_r^1 = 2.13 - j0.055$. In this case we find that the $E$-plane pattern is very similar to those in Fig. 3, but with a much lower magnitude, and with a minimum located at a scattering angle slightly greater than $90^\circ$. The $H$-plane pattern shows a gradual increase in magnitude from $\theta = 0^\circ$ to $180^\circ$.

![Figure 4](image)

**Figure 4.** Normalized bistatic cross section versus scattering angle for a dielectric shell of axial ratio 2, semi-major axis length $0.25\lambda_0$, with a coating of thickness $0.01\lambda_0$ and $\epsilon_r^1 = 2.13 - j0.055$. 
The variation of the $E$-plane pattern with the type of material of the spheroid is shown in Fig. 5. The size and the shape of the spheroid are exactly the same as those in Fig. 3. The thickness of the coating is $0.01\lambda_0$, with $\epsilon_r = 2.13 - j0.055$. As the value of $\epsilon_r$ increases we find that the magnitudes of both the backscattering and the forward scattering cross sections increase. The minimum of each pattern moves towards $\theta = 90^\circ$ with decreasing $\epsilon_r$ and also it becomes more deep and sharp.

Shown in Fig. 6 are the plots of normalized bistatic cross section versus scattering angle for two values of the axial ratio of a single dielectric spheroid with a confocal coating. The spheroid considered in Fig. 6(a) has an axial ratio 2 and that in Fig. 6(b) has an axial ratio 5, but they both have a semi-major axis length $0.25\lambda_0$. The thickness of the coating on each spheroid is $0.01\lambda_0$, with $\epsilon_r = 2.13 - j0.055$, and $\epsilon_r$ is 2.25. Here we observe that the variation of both the $E$- and $H$-plane patterns is almost the same for the two cases. However, the magnitudes of the scattering cross sections in Fig. 6(b) are much less than the corresponding ones in Fig. 6(a). This is because the area available for scattering is less when the axial ratio of the spheroid is 5 than when it is 2.

In Fig. 7 we have a plot that shows the variation of the normalized backscattering cross section with the angle of incidence for the spheroid in Fig. 6(b), for both TE and TM polarizations of the incident wave. For low values of the angle of incidence $\theta_i$, the magnitudes of the backscattering cross sections are almost the same for both polarizations. However, as the value of $\theta_i$ increases, the magnitude for the TM polarization becomes higher than that for the TE polarization, the difference becoming maximum at broadside incidence ($\theta_i = 90^\circ$).

![Figure 5. $E$-plane patterns for a dielectric spheroid of axial ratio 2, semi-major axis length $0.25\lambda_0$, and different core materials, with a confocal coating of thickness $0.01\lambda_0$ and $\epsilon_r = 2.13 - j0.055$.](image)
Figure 6. Normalized bistatic cross section versus scattering angle for a dielectric spheroid of semi-major axis length $0.25\lambda_0$ and $\varepsilon_{r_2} = 2.25$, with a coating of thickness $0.01\lambda_0$ and $\varepsilon_{r_1} = 2.13 - j0.055$ and an axial ratio of: (a) 2.0; (b) 5.0.
Figure 7. Plots of normalized backscattering cross section versus angle of incidence for the coated dielectric spheroid in Fig. 6(b).

The variation of the normalized bistatic cross section for a coated dielectric spheroid with the scattering angle and with the thickness of the layer of coating of dielectric constant $2.13 - j0.055$ is shown in Fig. 8. The spheroid in this case has an axial ratio 2. Its semi-major axis length is defined by $k_0a_2 = 2$ and $\varepsilon_r = 2.25$. In Fig. 8(a) we have the $E$-plane patterns and in Fig. 8(b) the $H$-plane patterns. The magnitudes of both the backscattering and the forward scattering cross sections increase with increasing thickness of the coating. However, the increment in the backscattering cross section is larger than that in the forward scattering cross section. All the minima in Fig. 8(a) occur close to $\theta = 60^\circ$, but become less deep as the thickness of the coating increases. On the other hand the value of $\theta$ at which the minima occur in Fig. 8(b) increases, and the minima become deeper and sharper, with increasing thickness of the coating.

Finally, in Fig. 9 we present plots of the normalized backscattering cross section versus angle of incidence for the spheroid in Fig. 8, for two different thicknesses of the coating with $\varepsilon_r = 2.13 - j0.055$. The minima that occur in the case of TE polarization are deeper than those in the case of TM polarization. It is interesting to note that the thinner coating gives a higher magnitude of the scattering cross section for $\theta_i < 45^\circ$. For all $\theta_i > 0$, the magnitude for TM polarization is higher than that for TE polarization.
Figure 8. Normalized bistatic cross section versus scattering angle for a dielectric spheroid of axial ratio 2 and $\epsilon_r^2 = 2.25$, with coatings of different thicknesses and $\epsilon_{r1} = 2.13 - j0.055$. 
Figure 9. Normalized backscattering cross section versus angle of incidence for the spheroid in Fig. 8, having a coating of $\varepsilon_r = 2.13 - j0.055$ and two different thicknesses.
CONCLUSIONS

An exact analytic solution to the problem of scattering of electromagnetic waves by a dielectric spheroid coated with a confocal lossy dielectric layer of arbitrary thickness has been obtained by using the method of separation of variables, in terms of spheroidal vector wave functions. Numerical results have been presented in the resonance region, to show the variation of the normalized bistatic and backscattering cross sections with the size, shape, the dielectric constant of the material of the spheroid, and the thickness and dielectric constant of the coating. These results indicate that the magnitude of the scattering cross section can be changed by coating the spheroid with an appropriate material of proper thickness. The solution to the problem of scattering by a perfectly conducting spheroid with a lossy/lossless confocal coating can be obtained from the above solution as a special case. These solutions are valuable as benchmarks to be considered in the development of a database for validating numerical codes and also in cross validating numerical and experimental results. Future work on this subject will focus on the problem of scattering by a spheroid with a nonconfocal coating.

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REFERENCES


M. Francis R. Cooray received the B.Sc. degree in electrical engineering (with Honors) from the University of Moratuwa, Sri Lanka in 1981, the M.Eng. degree in electrical engineering from the Memorial University of Newfoundland, St. John’s, NF, Canada in 1987 and the Ph.D. degree in electrical engineering from the University of Manitoba, Winnipeg, MB, Canada in 1990. His current research interests are in the areas of electromagnetic scattering, numerical modeling, and electromagnetic compatibility.

Ioan R. Ciric is a Professor in the Department of Electrical and Computer Engineering at the University of Manitoba, Winnipeg, Canada. His major interests are in the mathematical modeling of stationary and quasistationary fields, electromagnetic fields in the presence of moving solid conductors and levitation, field theory of special electrical machines, dc corona ionized fields, analytical and numerical methods for wave scattering problems, propagation along waveguides with discontinuities, and transients. He has authored and co-authored over 100 technical papers and three electrical engineering textbooks, and has been a contributor to chapters on magnetic field modeling and electromagnetic scattering by complex objects for three books.