Abstract—A general formulation for the scattering from multiple waveguide discontinuities is presented. It is based on the modal analysis technique where the corresponding fields are expanded in terms of vector eigenfunctions and the boundary conditions are imposed at each junction. Analytic expressions for the global scattering matrices are derived using a recurrence procedure that requires substantially less computation than the traditional cascading techniques. The matrix equations are truncated in a manner that satisfies the conditions for a good overall convergence, which is illustrated for step-discontinuities in circular waveguides. Numerical results are presented for a thick iris in a circular waveguide and for iris matched dielectric window designs.

I. INTRODUCTION

THE ANALYSIS of waveguide discontinuities is a classic electromagnetic theory and microwave engineering problem with application to the design of microwave components and antennas. Multiple-step discontinuities in waveguides are encountered in filters, transformers, irises, directional couplers [1], mode converters [2], and the modelling of horn antennas [3].

Structures with multiple waveguide discontinuities are typically analyzed by cascading the generalized scattering matrices of individual discontinuities [2], [4]. Cascading can also be performed using the transmission matrix representation, but this method has some disadvantages not encountered with the generalized scattering matrix approach. The method can be numerically unstable when too many higher order modes are included in the waveguide sections between the discontinuities or the total length of the structure is too large [5]. Also, the transmission matrix method requires the field expansions in each waveguide to have the same number of modes which may violate the criterion for avoiding the relative convergence phenomenon [6], [7]. This restriction has been overcome using an improved transmission matrix formulation [8]. Cascading with an admittance matrix formulation has also been proposed [9].

Problems relative to multiple discontinuities in waveguides have been solved by numerous techniques. The variational method has been used to solve for interacting irises and steps [10], [11]. Double discontinuities have been treated by using the moment method with point matching [12] and the Galerkin technique [13], [14]. An earlier mode matching technique was applied to double discontinuities [15]; however, the solution required large matrix operations which were avoided in [16] by reducing the sets of equations and using back substitution. An admittance matrix formulation based on modal analysis has been used to solve for a special class of double discontinuities [9]. All the mode matching based techniques may suffer from the relative convergence phenomenon which is eliminated by using an appropriate ratio of modal terms in the waveguide field expansions [6], [7].

The purpose of this paper is to present a solution for multiple waveguide discontinuities where the scattering parameters are directly calculated by simultaneously solving for the interaction between discontinuities rather than by cascading discontinuities. The electric and magnetic fields in each waveguide section are expressed as an exact expansion of orthonormal modes and the boundary conditions on the tangential field components are enforced over each discontinuity plane. By applying the mode orthogonality, an infinite set of linear algebraic equations is obtained. For numerical computation, this set of equations is appropriately truncated. The scattering matrices are derived by using a recurrence technique.

The formulation, based on the modal analysis technique [17], is compatible with the criterion for avoiding the relative convergence phenomenon and more efficient than the commonly used cascading techniques. Although the formulation is valid for cylindrical waveguides of any cross section, only multiple-step discontinuities in circular waveguides will be considered in this paper for quantitative illustrations. A thick iris in a circular waveguide is analyzed in detail to verify the convergence and accuracy of the formulation. Numerical results are compared with data available in the literature. Also, improved designs for iris matched dielectric windows are presented.

II. FORMULATION

Consider the multiple-step discontinuities shown in Fig. 1. There are N transverse discontinuities with N + 1 waveguide regions along a common axis. An arbitrary multimode incident field is assumed from waveguide 1.

The total transverse electric and magnetic fields can be written in modal form as follows:
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Fig. 1. Section with multiple waveguide discontinuities.

in the 1st region

\[ E_{t1} = \sum_m (A_{1m} e^{-\gamma_{1m}(z+z_1)} + B_{1m} e^{\gamma_{1m}(z+z_1)}) e_{1m} \]
\[ H_{t1} = \sum_m (A_{1m} e^{-\gamma_{1m}(z+z_1)} - B_{1m} e^{\gamma_{1m}(z+z_1)}) h_{1m}, \] (1)

in the jth region \((j = 2, \ldots, N)\)

\[ E_{tj} = \sum_m (A_{jm} e^{-\gamma_{jm}(z+z_j)} + B_{jm} e^{\gamma_{jm}(z+z_j)}) e_{jm} \]
\[ H_{tj} = \sum_m (A_{jm} e^{-\gamma_{jm}(z+z_j)} - B_{jm} e^{\gamma_{jm}(z+z_j)}) h_{jm}, \] (2)

and in the \(N+1\)th region

\[ E_{t,N+1} = \sum_m A_{N+1,m} e^{-\gamma_{N+1,m}z} e_{N+1,m} \]
\[ H_{t,N+1} = \sum_m A_{N+1,m} e^{-\gamma_{N+1,m}z} h_{N+1,m}, \] (3)

\(A_{im}\) and \(B_{im}\) are the forward and the backward complex coefficients, respectively, of the \(m\)th mode in the \(i\)th region, \(\gamma_{im}\) is the propagation constant, and \(e_{im}\) and \(h_{im}\) are the corresponding transverse electric and magnetic field functions of the \(m\)th mode. The modal field functions form an orthonormal set, i.e.,

\[ \langle e_{im}, h_{in} \rangle_{S_i} = \int_{S_i} (e_{im} \times h_{in}) \cdot d\mathbf{s} = \delta_{mn} \] (4)

where \(S_i\) is the \(i\)th waveguide cross section, and \(\delta_{mn}\) is the Kronecker delta.

At each discontinuity plane, the transverse fields must be continuous over each aperture and the tangential electric field must be zero on the walls. The continuity of the transverse electric and magnetic field intensities over each aperture cross section are expressed as follows:

at \(z = -z_1\)

\[ \sum_m (A_{1m} + B_{1m}) e_{1m} = \sum_m (A_{2m} e^{\gamma_{2m}L_2} + B_{2m} e^{-\gamma_{2m}L_2}) e_{2m} \]
\[ \sum_m (A_{1m} - B_{1m}) h_{1m} = \sum_m (A_{2m} e^{\gamma_{2m}L_2} - B_{2m} e^{-\gamma_{2m}L_2}) h_{2m}, \] (5)

at \(z = -z_j\) \((j = 2, \ldots, N - 1)\)

\[ \sum_m (A_{jm} + B_{jm}) e_{jm} = \sum_m (A_{j+1,m} e^{\gamma_{j+1,m}L_{j+1}} + B_{j+1,m} e^{-\gamma_{j+1,m}L_{j+1}}) e_{j+1,m} \]
\[ \sum_m (A_{jm} - B_{jm}) h_{jm} = \sum_m (A_{j+1,m} e^{\gamma_{j+1,m}L_{j+1}} - B_{j+1,m} e^{-\gamma_{j+1,m}L_{j+1}}) h_{j+1,m}, \] (6)

and at \(z = 0\)

\[ \sum_m (A_{Nm} + B_{Nm}) e_{Nm} = \sum_m A_{N+1,m} e_{N+1,m} \]
\[ \sum_m (A_{Nm} - B_{Nm}) h_{Nm} = \sum_m A_{N+1,m} h_{N+1,m}, \] (7)

where

\[ L_i \equiv -z_i + z_{i-1} \quad (i = 2, \ldots, N). \] (8)

In order to properly include the boundary condition on the transverse walls, the boundary enlargement and reduction discontinuity cases are handled separately [17]. For the boundary enlargement case, at each discontinuity we take the vector product of the terms in the electric field continuity equation with a magnetic mode function from the following waveguide section, and the vector product of the terms in the magnetic field continuity equation with an electric mode function from the preceding waveguide section. Applying the orthogonality of the modes, (5)–(7) become:

at \(z = -z_1\)

\[ \sum_m (A_{1m} + B_{1m}) (e_{1m}, h_{2n})_{S_1} = F_{2n} \cosh (\gamma_{2n}L_2) + D_{2n} \sinh (\gamma_{2n}L_2) \quad (n = 1, 2, 3, \cdots) \]
\[ A_{1n} - B_{1n} = \sum_m (D_{2m} \cosh (\gamma_{2m}L_2) + F_{2m} \sinh (\gamma_{2m}L_2)) \cdot (e_{1m}, h_{2m})_{S_1} \quad (n = 1, 2, 3 \cdots), \] (9)
at $z = -z_j (j \neq 1, N)$

\[ \sum_m F_{jm} (e_{jm}, h_{j+1,n}) S_j = F_{j+1,n} \cosh (\gamma_{j+1,n} l_{j+1}) + D_{j+1,n} \sinh (\gamma_{j+1,n} l_{j+1}) \quad (n = 1, 2, 3, \cdots) \]

\[ D_{jn} = \sum_m (D_{j+1,m} \cosh (\gamma_{j+1,m} l_{j+1}) + F_{j+1,m} \sinh (\gamma_{j+1,m} l_{j+1})) \cdot (e_{jn}, h_{j+1,m}) S_j \quad (n = 1, 2, 3, \cdots), \] (10)

and at $z = 0$

\[ \sum_m F_{Nm} (e_{Nm}, h_{N+1,n}) S_N = A_{N+1,n} \quad (n = 1, 2, 3, \cdots) \]

\[ D_{Nn} = \sum_m A_{N+1,m} \cdot (e_{Nm}, h_{N+1,m}) S_N \quad (n = 1, 2, 3, \cdots) \] (11)

where

\[ D_{ir} \equiv A_{ir} - B_{ir} \]

\[ F_{ir} \equiv A_{ir} + B_{ir}. \] (12)

Truncating the infinite series to $M_i$ modes in each $i$th region, (9)–(11) can be written in matrix form:

at $z = -z_1$

\[ G_1 (A_1 + B_1) = C_2 E_2 + S_2 D_2 \]

\[ A_1 - B_1 = C_1^T (C_2 D_2 + S_2 E_2), \] (14)

at $z = -z_j (j \neq 1, N)$

\[ G_j E_j = C_{j+1} E_{j+1} + S_{j+1} D_{j+1} \]

\[ D_j = G_j^T (C_{j+1} D_{j+1} + S_{j+1} E_{j+1}), \] (15)

and at $z = 0$

\[ G_N E_N = A_{N+1} \]

\[ D_N = G_N^T A_{N+1} \] (16)

where the $T$ superscript indicates matrix transpose, $A_1$ is the known incident modal coefficient column matrix of $M_1$ elements, $B_1$ and $A_{N+1}$ are the column matrices of $M_1$ and $M_{N+1}$ elements of the unknown modal coefficients for the fields reflected in region 1 and transmitted in region $N + 1$, respectively, and

\[ D_i = [A_{im} - B_{im}] M_i \times M_1 \] (17)

\[ E_i = [A_{im} + B_{im}] M_i \times M_1 \] (18)

\[ C_i = [\delta_{mn} \cosh (\gamma_{im} L_i)] M_i \times M_i \] (19)

\[ S_i = [\delta_{mn} \sinh (\gamma_{im} L_i)] M_i \times M_i \] (20)

\[ G_i = [(e_{im}, h_{i+1,n}) S_i] M_{i+1} \times M_i. \] (21)

For the boundary reduction case, the sets of matrix equations would be similar in form to (14)–(16) with the quantities described in (17)–(20), except we let

\[ G_i = [(e_{i+1,n}, h_{im}) S_{i+1}] M_{i+1} \times M_i. \] (22)

As a result of the truncations, the solution to the scattering problem will be an approximate solution where the accuracy is dependent on the number of modes selected in each region. Choosing the ratio of modes approximately equal to the ratio of the waveguide cross section dimensions prevents the relative convergence phenomenon where the solution may converge to an incorrect result or may not converge at all [6], [7].

In scattering matrix formulation, $B_1$ and $A_{N+1}$ can be expressed as

\[ B_1 = S_{11} A_1 \]

\[ A_{N+1} = S_{21} A_1 \] (23)

where $S_{11}$ and $S_{21}$ are the complex reflection and transmission matrices for the entire structure, respectively. Solving the sets of linear matrix equations for an arbitrary series of boundary enlargement and reduction discontinuities, the scattering matrices are obtained as

\[ S_{i} = \begin{cases} \frac{I - 2G_i^T G_j U^{-1} G_j}{G_i}, & \text{for a 1st discontinuity boundary enlargement (B.E.)} \\ \frac{-I + 2G_i^T G_j U^{-1} G_j}{G_i}, & \text{for a 1st discontinuity boundary reduction (B.R.)} \end{cases} \] (24)

\[ S_{i} \in [2(G_i R_i) (G_i R_i)] \cdots \] (25)

where $G_i (i = 1, \cdots, N)$ is determined by (21) or (22) for the $i$th discontinuity B.E. or B.R. case, respectively, $I$ is the unit matrix and

\[ \begin{pmatrix} U \\ U \end{pmatrix} = \begin{pmatrix} (C_2 + G_1 G_1^T S_2) + (S_2 + G_1 G_1^T C_2) G_2 \tilde{R}_1^{-1} G_2, & \text{for the 1st and 2nd discontinuities both B.E. or B.R.} \\ (S_2 + G_1 G_1^T C_2) + (C_2 + G_1 G_1^T S_2) G_2 \tilde{R}_1^{-1} G_2, & \text{otherwise.} \end{pmatrix} \] (26)

$Q_i$ and $R_i$ are $M_i \times M_i$ matrices calculated from the following recurrence formulas:

\[ Q_i = \begin{pmatrix} S_{i} + C_i G_i^T Q_i R_i^{-1} G_i, & \text{for the $i$th and $i + 1$th discontinuities both B.E. or B.R.} \\ C_i + S_i G_i^T Q_i R_i^{-1} G_i, & \text{otherwise} \end{pmatrix} \] (27)

\[ R_i = \begin{pmatrix} S_{i} + C_i G_i^T Q_i R_i^{-1} G_i, & \text{for the $i$th and $i + 1$th discontinuities both B.E. or B.R.} \\ C_i + S_i G_i^T Q_i R_i^{-1} G_i, & \text{otherwise} \end{pmatrix} \] (28)

with

\[ Q_{N+1} = R_{N+1} = I. \] (29)
III. CONVERGENCE AND ACCURACY

Consider a thick circular iris of radius \( b \) and length \( L \) in a circular waveguide of radius \( a \), as shown in Fig. 2. The iris forms a double discontinuity comprised of a boundary reduction followed by a boundary enlargement at a distance \( L \).

The optimum ratio of the number of modes should be \( M/N \approx a/b \), with \( M = P \), where \( M, N, \) and \( P \) are the number of modes in regions 1, 2, and 3, respectively [6], [7]. Choosing the number of modes according to this ratio should ensure fast and accurate results. This is illustrated here by studying the numerical results for an incident TE_{11} mode.

In Fig. 3, the magnitude of the reflection coefficient is shown as a function of \( M \) for a thick iris with \( a = 2b, L/a = 0.01 \), and \( k \alpha = 3.2 \). The recurrence formulas, (27) and (28), require that the calculation of the scattering matrices begins at the \( N \)th discontinuity and move successively to the 1st discontinuity. The proposed formulation is valid for cylindrical waveguides of any cross section; however, only multiple step-discontinuities in circular waveguides are considered for numerical computations in this paper.

The convergence of the magnitude of the reflection coefficient with different ratios of \( M/N \) (\( P = M \) from now on) is shown in Fig. 4 for two values of \( k \alpha \), where \( k \) is the wave number inside the structure. It is clear that the solution converges for all ratios of \( M/N \), but the convergence is always fastest when \( M/N = a/b \). Also, the rate of convergence is observed to be slower as the frequency increases.

The reflection and transmission coefficients for decreasing values of \( L/a \) are shown in Table I. The ratio \( M/N \) has a noticeable effect on the solution as \( L/a \) decreases. In the case of an infinitely thin iris, the formulation mathematically collapses for \( M/N = 1 \) in the sense that (24) and (25) becomes zero and unit matrices, respectively. The numerical results always converge to the correct solution when \( M/N = a/b \) as a result of the linear system being well conditioned [7].

To verify the accuracy of the formulation, comparison with numerical results calculated by the moment method [14] are shown in Table II and Table III. For \( L \) less than 0.2 inches, excellent agreement of results are achieved with \( M = N(a/b) = 40 \). However, as \( L \) increases the number of modes must be reduced to avoid numerical instabilities as \( U \) becomes too ill-conditioned for inversion. Significant disagreement, particularly in the phase, is observed for large \( L \) as the frequency increases and the ratio \( a/b \) decreases. In Table II, the magnitude of the transmission coefficient decreases to
the reduction in modes may compromise the accuracy of the region. As a result, the corresponding matrix for the waveguide between discontinuities is very large, the analysis can be problem by omitting the troublesome higher order modes, but in inversion. Reducing the number of modes used alleviates the highest order modes, which are below cutoff, cause the hyperbolic functions. In cases where the separation distance more stable than solution as already demonstrated.

As the separation distance between a discontinuity increases, the inversion of one matrix inversions are needed in [14]. Proposed Formulation requires zero as from the effect of the hyperbolic functions in (19) and (20). 

TABLE I COMPARISON OF THE REFLECTION AND TRANSMISSION COEFFICIENTS AS FUNCTIONS OF L FOR A THICK IRIS WITH a = 2b and \(\frac{a}{b} = 3.2\)

<table>
<thead>
<tr>
<th>L/b</th>
<th>Reflection Coefficient</th>
<th>Transmission Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^{-1}</td>
<td>(-0.19497 + j0.44710)</td>
<td>(-0.18518 + j0.44955)</td>
</tr>
<tr>
<td>10^{-2}</td>
<td>(-0.05996 + j0.31072)</td>
<td>(-0.05996 + j0.39773)</td>
</tr>
<tr>
<td>10^{-3}</td>
<td>(-0.00996 + j0.29597)</td>
<td>(-0.00417 + j0.25437)</td>
</tr>
<tr>
<td>10^{-4}</td>
<td>(-0.00483 + j0.29323)</td>
<td>(-0.00204 + j0.17789)</td>
</tr>
<tr>
<td>10^{-5}</td>
<td>(-0.00483 + j0.29173)</td>
<td>(-0.00122 + j0.17589)</td>
</tr>
<tr>
<td>10^{-6}</td>
<td>(-0.00483 + j0.29015)</td>
<td>(-0.00095 + j0.16878)</td>
</tr>
<tr>
<td>0.0</td>
<td>(-0.00483 + j0.29015)</td>
<td>(-0.00095 + j0.16878)</td>
</tr>
</tbody>
</table>

TABLE II COMPARISON OF THE REFLECTION AND TRANSMISSION COEFFICIENTS AS FUNCTIONS OF L FOR A THICK IRIS WITH a = 0.50175 in, b = 0.25 in, and f = 9 GHz

<table>
<thead>
<tr>
<th>L (in)</th>
<th>Reflection Coefficient</th>
<th>Transmission Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>(-0.067 + 149.8^\circ)</td>
<td>(-0.067 + 150.1^\circ)</td>
</tr>
<tr>
<td>0.008</td>
<td>(-0.074 + 153.7^\circ)</td>
<td>(-0.074 + 153.4^\circ)</td>
</tr>
<tr>
<td>0.010</td>
<td>(-0.086 + 156.6^\circ)</td>
<td>(-0.086 + 156.8^\circ)</td>
</tr>
<tr>
<td>0.200</td>
<td>(-0.099 + 161.0^\circ)</td>
<td>(-0.099 + 160.9^\circ)</td>
</tr>
<tr>
<td>0.500</td>
<td>(-0.100 + 162.0^\circ)</td>
<td>(-0.100 + 162.0^\circ)</td>
</tr>
<tr>
<td>1.000</td>
<td>(-0.100 + 162.0^\circ)</td>
<td>(-0.100 + 162.0^\circ)</td>
</tr>
</tbody>
</table>

TABLE III COMPARISON OF THE REFLECTION AND TRANSMISSION COEFFICIENTS AS FUNCTIONS OF L FOR A THICK IRIS WITH a = 0.50175 in, b = 0.375 in, and f = 12 GHz

<table>
<thead>
<tr>
<th>L (in)</th>
<th>Reflection Coefficient</th>
<th>Transmission Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>(-0.099 + 89.3^\circ)</td>
<td>(-0.097 + 89.4^\circ)</td>
</tr>
<tr>
<td>0.008</td>
<td>(-0.082 + 88.8^\circ)</td>
<td>(-0.082 + 88.7^\circ)</td>
</tr>
<tr>
<td>0.010</td>
<td>(-0.063 + 88.6^\circ)</td>
<td>(-0.063 + 88.4^\circ)</td>
</tr>
<tr>
<td>0.200</td>
<td>(-0.056 + 185.6^\circ)</td>
<td>(-0.056 + 185.6^\circ)</td>
</tr>
<tr>
<td>0.500</td>
<td>(-0.040 + 190.8^\circ)</td>
<td>(-0.040 + 190.9^\circ)</td>
</tr>
<tr>
<td>1.000</td>
<td>(-0.007 + 194.3^\circ)</td>
<td>(-0.007 + 194.3^\circ)</td>
</tr>
<tr>
<td>3.000</td>
<td>(-0.000 + 194.3^\circ)</td>
<td>(-0.000 + 194.3^\circ)</td>
</tr>
</tbody>
</table>

Design curves presented in [18] show the iris radius b that minimizes the reflection coefficient of an incident TE_{11} mode for a given a, \(\varepsilon_r\), L, I, and operating frequency f. The reflection coefficient of the dielectric window was calculated in [18] by cascading the generalized scattering matrices of the four waveguide discontinuities. Ten modes were used for the field expansions in each waveguide region, but Fig. 4 in the present paper shows that this may be an inadequate number of modes to account for the higher mode interaction of closely separated discontinuities. A better choice would be to use a higher number of modes in the ratio \(MN = a/b\) where M is the number of modes in the waveguide and dielectric regions, and N is the number of modes in the iris regions. An equal number of modes are used in the waveguide and dielectric regions since studies performed by the authors for single air-dielectric discontinuities, within the same range of the ratio a/b, have shown that this is a suitable choice.

To demonstrate the improvement in accuracy of the results by using higher order modes, the results calculated by the proposed formulation, using \(M = N(a/b) = 30\), are compared with those in [18], as shown in Fig. 6. All dimensions are normalized to the waveguide radius a, and the frequency is normalized to the cutoff frequency \(f_c\) of the TE_{11} mode. The optimization was performed using the ZXGZN minimization subroutine of the International Mathematical and Statistical Library (IMSL). It is observed that the results for \(L = 0.04a\) are within 1.2 percent, but for \(L = 0.01a\), the maximum difference between the results obtained in this paper and those in [18] is 4.2 percent. The frequency response of some of zero as L increases due to the iris region being below cutoff at the operating frequency. The proposed formulation requires the inversion of one \(N \times N\) matrix, with N shown in the tables, while two 40 \(\times\) 40 matrix inversions are needed in [14].
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Fig. 6. Optimum iris radius as a function of the normalized frequency for an iris matched dielectric window with $\varepsilon_r = 2.8$ and $L_i = L$, and comparison with results in [18].

![Graph showing Optimum Iris Radius](image)

The choice of ten modes in each waveguide region was made in [18] partially on the basis of maintaining a reasonable computational time. Using a larger number of modes, as done here, would have dramatically increased the computation time for the optimization procedure. Table IV shows the size and number of full complex matrix inversions required by the proposed method compared with cascading as in [18]. It is assumed that the dielectric region is bisected with an electric and magnetic wall to use the even and odd mode excitation analysis about the symmetry plane. The proposed formulation proves to be a more efficient method by requiring only two relatively small matrix inversions, compared to six matrix inversions in [18]. This allows more modes to be used than in [18] without a large increase in computation time.

In Table V, the minimum number of full complex matrix multiplications and inversions required to calculate the scattering parameters for $N$ discontinuities is compared with the commonly used cascading techniques. The proposed formulation has a definite advantage over the two cascading techniques by requiring fewer matrix multiplications and inversions. Compared to the proposed formulation, cascading generalized scattering matrices (S-matrix) [8] requires almost twice as many matrix multiplications and three times as many matrix inversions, and the transmission matrix method (T-matrix) [5] requires almost 60 percent more matrix multiplications and two extra matrix inversions. The proposed formulation has the advantage over the transmission matrix method in being compatible with the criterion for avoiding the relative convergence phenomenon, which also gives the fastest convergence, and is numerically stable when the total length of the waveguide sections, $L_1 + L_2 + \cdots + L_N$, becomes large. A transmission matrix formulation compatible with the criterion for avoiding the relative convergence phenomenon has been presented in [8], but requires slightly more computation than in [5].

![Graph showing Return Loss](image)

![Graph showing Return Loss](image)

**Fig. 7.** Return loss as a function of the normalized frequency for an iris matched dielectric window optimized for $f_s/f_c = 1.1, 1.5, 1.5$ with $\varepsilon_r = 2.8, L_i = L$, and: (a) $L/a = 0.01$; (b) $L/a = 0.04$.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td>Mult</td>
<td>Inv</td>
<td>Mult</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
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<td>2</td>
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<td>4</td>
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<tr>
<td>5</td>
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</table>
V. CONCLUSION

A general solution for the scattering from multiple discontinuities in waveguides based on the modal analysis technique has been presented. The proposed formulation uses an efficient recurrence procedure that requires substantially fewer full matrix multiplications and inversions than the commonly used cascading techniques. For circular waveguide step-discontinuities, choosing the ratio of the number of modes approximately equal to the ratio of the waveguide cross section dimensions ensures a very rapid convergence of results, and is critical for achieving accurate results with closely separated discontinuities. The accuracy of the proposed formulation has been confirmed by comparing results for a thick iris in a circular waveguide available in the literature. The computation for the thick iris required only one relatively small matrix inversion while two \( 40 \times 40 \) matrix inversions are required for the moment method [14]. As an application, designs for iris matched dielectric windows in circular waveguides have been improved by as much as 4.2 percent using more modes in the optimum ratio for the calculations to account for the higher order mode interaction and increase accuracy. Although only step-discontinuities in circular waveguides have been discussed, the formulation can be applied to transverse discontinuities in cylindrical waveguides of any cross section.

REFERENCES


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