High Resolution Electromagnetic Imaging of Lossy Objects in the Presence of Noise

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Abstract — A microwave imaging algorithm based on the stochastic inverse of matrices is proposed. The prior knowledge required in the algorithm is obtained by the application of the Tikhonov regularization which enables us to find a good first approximation. As a consequence, the number of iterations in the proposed algorithm is considerably reduced. Example tests for two-dimensional structures show that this algorithm provides an efficient means for reconstructing accurately lossy or lossless dielectric bodies of various contrasts even in the presence of noise in the scattered field data.

I. INTRODUCTION

Efficient numerical techniques for the reconstruction of the complex permittivity of lossy dielectric bodies from scattered near-field measurements are essential for many applications, such as nondestructive testing, geophysical exploration, remote sensing, medical imaging, and robotic vision. The reconstruction methods proposed so far fall into two main categories, namely the spectral domain methods [1], [2] and the spatial domain methods [3]–[5] (although some other techniques, such as maximum entropy [6] and simulated annealing [7] have also been attempted). In recent years, more research activity based on the spatial domain methods has been reported, since the spectral domain techniques are only valid for low contrast dielectric bodies and weak scatterers. In this paper, we present a new method for reconstructing dielectric bodies, based upon a stochastic inversion transformation. The stochastic treatment of ill-posed problems [8] has been successfully used in image processing and recognition techniques [9], seismology studies [10], and synthetic aperture radar imaging [11]. Here, we apply the stochastic inversion of matrices to the area of microwave imaging of dielectric bodies and illustrate the efficiency of this new approach. A difficulty in this case is that appropriate initial guesses necessary in the associated iterative process are practically impossible to be made and inappropriate guesses can cause the algorithm to be slowly convergent or even divergent. We obtain the required prior knowledge by applying the Tikhonov regularization procedure [12, which we have found to give very good first approximations. The proposed algorithm consists of three main steps. The nonlinear integral equation used for the reconstruction of the dielectric body is first linearized by introducing an equivalent current density. Secondly, the Tikhonov regularization is employed to obtain a good approximation to the a priori data required in the algorithm. Finally, the stochastic inverse is applied to compute the equivalent current distribution within the body. From the reconstructed current density one can simply derive the permittivity distribution which is used directly to develop the object images.

II. FORMULATION

Consider a two-dimensional region \( D \) of investigation with a complex permittivity

\[
\epsilon(r) = \epsilon'(r) - j\epsilon''(r) .
\]

The electromagnetic scattering of an incident transverse magnetic wave is described by

\[
\nabla^2 E^s(r) + k_0^2 E^s(r) = -[k^2(r) - k_0^2]E^e(r) \tag{2}
\]

where \( E^s(r) \) is the scattered electric field intensity, \( k(r) \) and \( k_0 \) are the wave numbers inside and outside \( D \), respectively, with the background of the investigation domain and the outside region considered to be a free space, for instance, and \( E^e(r) \) is the total electric field intensity. By introducing an equivalent current density defined as

\[
J_e(r) = j(\omega \mu_0)^{-1} [k^2(r) - k_0^2]E^e(r) , \tag{3}
\]

with \( \omega \) and \( \mu_0 \) being the angular frequency and the permeability of free space, respectively, and by using the Green function \( G(r, r') \) for an unbounded free space, (2) can be replaced by the equivalent integral equation

\[
E^s(r) = \int_D J_e(r')G(r, r')dr' . \tag{4}
\]

The equivalent current distribution within the investigation domain is to be determined by solving (4) in terms of the scattered field \( E^s \) detected outside the investigation region. Once this is done, the permittivity distribution is derived.
from (3), with the total electric field inside the object computed by integrating the current density,

\[ E'(r) = E_i(r) + \int_{D} J_i(r')G(r, r')dr' \]  

(5)

where \( E_i(r) \) is the electric field intensity in the incident wave. If the background of the investigation domain or the outside region, or both, are not a free space, then the formulation must take into account the actual wave numbers.

Equation (4) can, in principle, be solved numerically by discretizing the investigation region and by using the corresponding matrix equation

\[ [E^f] = [G][J_e] \]  

(6)

where \( [E^f] \) and \( [J_e] \) are column vectors, and \( [G] \) is a rectangular matrix representing the Green function. Unfortunately, in most cases this problem is ill-posed and the solution may not be unique. However, in practical situations, for the range of parameters considered, approximate solutions are sufficient for developing satisfactory images. Taking into account the errors and the noise in the scattered field, (6) is replaced by

\[ [E^f_N] = [G][J_e] + [N] \]  

(7)

with the column vector \( [N] \) indicating the errors and the noise in the scattered field. In this paper, we use a stochastic inversion method to solve this linear equation. The stochastic inversion algorithm is constructed based upon statistical considerations, with \( [J_e] \) and \( [N] \) in (7) simulated by stochastic or random vectors. The estimates \( [J'_e] \) of \( [J_e] \) are derived from the measured data \( [E^f_N] \) to minimize the expected Euclidean norm of the reconstruction error, \( \| [J'_e] - [J_e] \| \). Under the assumption that the random variables \( [J_e] \) and \( [N] \) are uncorrelated, the expression which gives the best estimate \( [J'_e] \) to be employed in the reconstruction algorithm is [8]

\[ [J'_e] = [R_J][G]^H([G][R_J][G]^H + [R_N])^{-1}[E^f_N] \]  

(8)

where \( [R_J] \) is the correlation matrix of \( [J_e] \), \( [R_N] \) is the correlation matrix of \( [N] \), and \( H \) denotes the conjugate transpose of a matrix. In electromagnetic imaging problems, the additive noise can be assumed to be uncorrelated and isotropic, and thus \( [R_N] \) is a diagonal matrix

\[ [R_N] = \sigma_n^2 [I] \]  

(9)

where \( \sigma_n^2 \) is the noise variance, which is available from prior knowledge, and \( [I] \) is a unit matrix. Since \( [J_e] \) is assumed to be uncorrelated, its correlation matrix \( [R_J] \) is approximated as consisting of the elements [11]

\[ R_{ij} = \frac{\rho}{\rho} \delta_{ij} \]  

(10)

with \( \delta_{ij} \) denoting the Kronecker delta symbol. For the problems analyzed so far by using this technique [9]-[11], the initial estimate of \( [J_e] \) is provided by available \textit{a priori} knowledge of the system investigated. Since in the case of our problem the unknown \( [J_e] \) is the equivalent current density, with no initial information about its distribution in most cases, in this paper we propose the usage of the Tikhonov regularization technique [12] in order to obtain a first approximation of \( [J_e] \) in the form

\[ [J'_e] = [G]^H([G][G]^H + g[I])^{-1}[E^f_N] \]  

(11)

which corresponds to letting \( [R_J] \) in (8) be a unit matrix and \( \sigma_n^2 \) be the regularization parameter \( g \). Although this is not the best selection of \( g \), it allows us to obtain a good first estimate in a very simple way. In addition, we have found that the proposed algorithm is practically sensitive only to the order of magnitude of \( g \). Thus, we only use a regularization parameter equal to the order of magnitude of \( \sigma_n^2 \). With the classical definition of the signal-to-noise ratio, \( S/N = 20 \log(\| E^f \| / \| N \|) \), we set \( g = 10^{-2} \) when \( S/N = 20 \text{dB} \), \( g = 10^{-4} \) when \( S/N = 40 \text{dB} \), and so on, for an \( \| E^f \| \) of the order of magnitude of unity. This simplify the implementation of the algorithm since in practical cases the signal-to-noise ratio is readily available.

In the next section, the following iterative algorithm is used for numerical computations based upon the discussion above. An initial estimate of \( [J'_e] \) is obtained by applying the Tikhonov regularization technique, with the regularization parameter in (11) chosen to be equal to the order of magnitude of the elements in the correlation matrix of the noise. The diagonal matrix \( [R_J] \) in (10) is then calculated from this \( [J'_e] \). Subsequent estimates of \( [J'_e] \) are obtained from (8), with \( [R_N] \) from (9) and with the latest evaluation of \( [R_J] \). The iteration process continues until a stable solution is obtained.

III. NUMERICAL RESULTS

As a first illustrative example, consider an investigation domain of a square cross section of \( S/3\lambda_0 \) a side, where \( \lambda_0 \)
is the wavelength of the illuminating incident wave taken to be a plane wave propagating normally to the left hand side of the domain. 25 cells are used to discretize the investigation region and 16 equally spaced detectors are located on a concentric circular loop of a diameter of $3\lambda_0$. The electric field values measured by the detectors are provided by a direct scattering computation using the moment method for the scatterer shown in Fig. 1 and the presence of the noise in the scattered field is simulated by adding to the real and imaginary parts of the field values two independent sequences of Gaussian random variables of zero mean value. Fig. 2 shows the results after four iterations for the reconstructed average permittivity versus signal-to-noise ratio for a relative permittivity $\varepsilon_r = 5.5 - j1.2$ of the scatterer. In order to compare the overall accuracy of the reconstructed permittivities, we use the relative mean square error defined as

$$\alpha = \left[ \frac{\sum_k |\varepsilon_k - \varepsilon_k|^2}{\sum_k |\varepsilon_k|^2} \right]^{1/2}$$

(12)

Fig. 3. Error of reconstruction versus S/N.

Fig. 4. Error of reconstruction versus the number of iterations.
A new numerical method for solving inverse problems regarding electromagnetic imaging has been proposed. It is based on the stochastic inversion of a linear system of equations. The prior knowledge required in the method is determined by employing the Tikhonov regularization to obtain a first approximation to the reconstructed values. Numerical simulations by the proposed method show that the dielectric permittivity distribution can be reconstructed accurately even in the presence of noise. In the examples considered for illustration in this paper only one illumination was used. When multiple illuminations are employed a substantially better accuracy is achieved. Compared with other iterative methods presented so far for dielectric body microwave imaging problems, the stochastic technique requires less computation time since the number of iterations is reduced by the application of the Tikhonov regularization for determining the initial guess; moreover, for each iteration it only needs one matrix inversion, while other iterative techniques require two matrix inversion operations per iteration. These features recommend the method presented for some special applications when a high resolution is required in the presence of noise, especially in defect detection and remote sensing.

**REFERENCES**


