REDUCTION OF MULTIPLY-NESTED DIELECTRIC BODIES FOR WAVE SCATTERING ANALYSIS BY SINGLE SOURCE SURFACE INTEGRAL EQUATIONS

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Abstract—A recursive single source surface integral equation formulation for the problem of electromagnetic wave scattering by multiply-nested bodies yields an equivalent outer surface model that is independent of the material and the illumination in the exterior region, and therefore invariant under rotation and translation. The proposed algorithm is applicable to both far-field and near-field problems, and also gives, through a fast backward recursion, the field values at interior points. Such a reduced model may be duplicated and reused in an assortment of complex scattering problems without repeating the reduction calculation. Thus, the high computational efficiencies realized by the recursive formulation, with respect to the previous, direct, single source surface integral equation formulations, are further enhanced for problems involving some degree of repetition, as in geometry and material optimization of scattering structures under various types of illumination. Numerical examples are included in order to demonstrate the features and the efficiency of the recursive multiply-nested algorithm in comparison to direct, simultaneous solution of all unknowns by the electric field integral equation method.

1. INTRODUCTION

As the complexity of the wave scattering structures increases, the numerical solution of the associated field problems can quickly become impractical if the method employed requires the simultaneous solution of all the unknowns. This is, for instance, the case if we attempt to analyze a densely nested structure by way of the well known coupled
surface integral equation methods, such as the electric field integral equation, the magnetic field integral equation, and the combined field integral equation [9, 15]. In single source surface integral equation formulations the coupled pair of unknown electric and magnetic surface current distributions — used in the coupled surface integral equation methods — is replaced with only a single unknown surface current density, thus reducing the total number of unknowns by one half. The nature of this single unknown current density may be either electric [4, 5, 11], magnetic [7], or a combination of both electric and magnetic at a fixed ratio [1, 3, 10, 17]. Rigorous proofs of the uniqueness theorems for scalar and vector wave scattering analysis by single source surface integral equations have been presented in [1, 3] and [10]. The numerical efficiency of such a single source surface integral equation, as compared to that of the coupled electric field integral equation, has been demonstrated for the problem of wave scattering by multiply-connected bodies in [16]. By their nature, single source surface integral equations are particularly well suited to recursive formulation. That is, for equivalence theorem based formulations, the electromagnetic fields generated by only a single surface current can only be constrained on one side of the source-bearing surface. This natural advantage was first recognized by Maystre for the problem of wave scattering by layered, periodic, optical gratings [12, 13], and has only recently been applied to the problem of wave scattering by arbitrary dielectric cylinders [18, 19] and [20].

In this paper, we propose an efficient region-by-region solution algorithm for the electromagnetic scattering by complex bodies that enjoy a general multiply-nested structure based on a recursive formulation of the single source surface integral equation method. A multiply-nested body is defined as being a homogeneous dielectric body that encloses a set of inhomogeneous inclusions, each of these inclusions consisting itself, in general, of a homogeneous dielectric body enclosing its own set of inhomogeneous inclusions (as shown in Fig. 1). In this procedure, the electric field within the homogeneous dielectric medium surrounding a particular set of inclusions is decomposed into the sum of an excitation field generated by a fictitious electric current density distributed on the enveloping surface, and a perturbation field due to the presence of the inclusions in the excitation field. We refer to these two fields as "local incident" and "local scattered" fields, respectively, so as to form an affinity between this recursive formulation and the better-known
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Figure 1. Transverse magnetic illumination of a system of multiply-nested dielectric cylinders; expanded views are shown for the cylinder $V_a$ and its multiply-nested inclusion $V_1$.  

coupled surface integral equation formulations [9, 15]. However, in the latter the decomposition into incident and scattered fields is reserved only for the free space region (in which the incident field is assumed to be known). In the proposed method, the necessary relationship between the local incident and local scattered fields allows to express the actual fields in the enclosed region only in terms of the single electric surface current density on the enveloping surface. This is performed by employing a pair of so-called exclusive operators which fully account for the inner complexity of the multiply-nested body by yielding the actual electric and magnetic field components tangent to the enveloping surface in terms of the single unknown current density distributed on this same surface (i.e., the source of the local incident field). So, having formed the exclusive operators for a particular multiply-nested body, which insure the unique and correct determination of the electromagnetic field within it, this body is now treated as an inclusion within a greater body, and, again, the necessary relationship between
the local incident and the local scattered field (as formulated in this new surrounding medium) allows for the expression of the interior fields in terms of a new single electric current density distributed on the new enveloping surface; and thus, the recursive procedure continues until, finally, all the fields within the multiply-nested body are expressed in terms of a single unknown electric current density distributed only on the outermost surface.

The exclusive operators are constructed in a manner that is independent of the illumination and the material outside the body considered, and therefore are invariant when the body is rotated and translated. Thus, in addition to providing an efficient solution method, the proposed recursive algorithm yields an equivalent surface representation of the scattering body that may be archived and reused in an assortment of scattering problems. This algorithm is equally suited to near- and far-field calculations; as well, the field values at interior points are recovered through a selective backward recursion. By comparison with previous algorithms based on single source surface integral equations (see [6, 8] and [14]), the proposed recursive method does not require the simultaneous solution of all the unknowns, and the necessary computer storage space is substantially smaller; its versatility and efficiency is further increased due to the associated invariance properties. In the special case of a single homogeneous dielectric cylinder, the proposed method reduces to the single source surface integral equation derived in [2], which is the Hermitian adjoint of the original single source surface integral equation derived by Maystre and Vincent [11].

The field modeling procedure used for the derivation of the proposed algorithm has a general validity. In what follows, it is illustrated for the special problem of transverse magnetic (TM) wave scattering by a system of multiply-nested dielectric cylinders.

2. REDUCTION MODELING PROCEDURE

Consider a system of source-free multiply-nested dielectric cylinders \( V_a, V_b, \ldots, V_m \) arranged parallel to the \( z \)-axis and illuminated by a TM incident field as shown in a cross section in Fig. 1. The electric field intensity only has a \( z \)-directed component \( E_z \) which satisfies a homogeneous Helmholtz equation subject to the continuity of the tangential components of the electric and magnetic field intensities across each interface,

\[
(\nabla^2 + k^2) E_z = 0,
\]  

(1)
\[ \Delta E_z(\bar{r}) = 0, \quad \bar{r} \in \text{any interface}, \]  
\[ \Delta H_t(\bar{r}) = 0, \quad \bar{r} \in \text{any interface}, \]  

where \( H_t = (1/j\omega \mu)\partial E_z/\partial n \) is the component of the magnetic field intensity tangent to the interface, \( \partial/\partial n \) indicates the derivative in the direction of the unit vector normal to the surface \( \hat{n} \), \( k = \omega \sqrt{\varepsilon \mu} \) is the wave number that varies from region to region, \( \varepsilon \) and \( \mu \) are the local permittivity and permeability, respectively, \( \omega \) is the angular frequency, \( \Delta \) indicates the step discontinuity in the direction of \( \hat{n} \), and \( j \equiv \sqrt{-1} \). A time dependence \( \exp(j\omega t) \) has been assumed and suppressed. The actual field \( E_z \) in the unbounded, homogeneous region \( V_0 \) is decomposed into the sum of a known incident field \( E_z^{inc} \) and an unknown scattered field \( E_z^{sc} \). This free space scattered field satisfies the radiation condition

\[ \sqrt{r} \left( \frac{\partial}{\partial r} E_z^{sc}(\bar{r}) + jk_0 E_z^{sc}(\bar{r}) \right) \to 0, \quad \text{as} \quad r \equiv |\bar{r}| \to \infty. \]  

We particularize our analysis to the multiply-nested body \( V_a \) whose structure (shown in an expanded view in Fig. 1) consists of an outer surface \( S_a \) enclosing a homogeneous, multiply-connected region \( V'_a \) of permittivity \( \varepsilon_a \) and permeability \( \mu_a \), surrounding a collection of inhomogeneous inclusions \( V_q, q = 1, 2, \ldots, n \).

The field problem within \( V'_a \) is formulated as a local scattering problem as shown in Fig. 2. All the material inside and outside \( S_a \) is replaced with that of \( V'_a \), and the actual electric field \( E_z \) in \( V'_a \) is decomposed into the sum of a local incident field \( E_z^{inc} \) and a local scattered field \( E_z^{sc} \); the local incident field is assumed to be generated by a single layer of electric current \( \hat{z} J_a \) distributed on the closed surface \( S_a \), while the local scattered field is assumed to be generated by coupled layers of electric and magnetic current \( \hat{z} J^e \) and \( \hat{t} J^m \), respectively, distributed over the surfaces \( S_q \) of the inclusions \( V_q, q = 1, 2, \ldots, n \), such that the sum of the local incident field and the local scattered field vanishes within each inclusion \( V_q \). By using the boundary conditions, the coupled surface densities \( J^e \) and \( J^m \) are expressed in terms of the actual magnetic and electric fields tangent to the scattering surfaces,

\[ \hat{z} J^e(\bar{r}) = \hat{n} \times \Delta \bar{H}(\bar{r}) = \hat{z} H_t(\bar{r}), \]
\[ \hat{t} J^m(\bar{r}) = -\hat{n} \times \hat{z} \Delta E_z(\bar{r}) = \hat{t} E_z(\bar{r}), \]
Figure 2. The local scattering problem for the field in the subregion $V_a'$.

where $\bar{r} \in S_q$, $q = 1, 2, \cdots, n$, and $\hat{n}$ and $\hat{t} = \hat{z} \times \hat{n}$ are the unit vectors normal and tangential, respectively, to the surface at $\bar{r}$. The fact that we have placed only one current density (i.e., $J_a$) on $S_a$ implies that the sum of the local incident and local scattered fields is unconstrained outside $S_a$. Thus, the local incident and local scattered fields provide an integral representation of the actual field $E_z$ in the linear, homogeneous, multiply-connected subregion $V_a'$,

$$E_a^{inc}(\bar{r}) + E_a^{sc}(\bar{r}) = E_z, \quad \bar{r} \in V_a', \quad (7)$$

with

$$E_a^{inc}(\bar{r}) + E_a^{sc}(\bar{r}) = \begin{cases} 0, & \bar{r} \in V_q, \quad q = 1, 2, \cdots, n \\ ?, & \bar{r} \notin V_a \end{cases} \quad (8)$$

in which

$$E_a^{inc}(\bar{r}) = -\frac{\omega \mu_a}{4} \int_{S_a} J_a(\bar{r}') H_0^{(2)}(k_a R) d\bar{l}', \quad (9)$$
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\[ E_{a}^{sc}(\vec{r}) = \sum_{q=1}^{n} \int_{S_q} \left( \frac{-\omega \mu_a}{4} \mathcal{J}^e(\vec{r}') H_{0}^{(2)}(k_a R) - j \frac{E_z(\vec{r}')}{4} \frac{\partial}{\partial n'} H_{0}^{(2)}(k_a R) \right) dl', \]  

(10)

where \( H_{0}^{(2)} \) is the Hankel function of the second kind and order zero, \( k_a = \omega \sqrt{\varepsilon_a \mu_a} \) is the wave number in the homogeneous subregion \( V_a' \), \( R = |\vec{r} - \vec{r}'| \), with \( \vec{r} \) and \( \vec{r}' \) being the position vectors of the field point and the source point, respectively, \( \partial / \partial n' \) denotes the derivative in the direction of the unit vector normal to the surface \( S_q \) at \( \vec{r}' \). Substitution of (5) and (6) into (10) yields an expression for the local scattered field in terms of the tangential components of the actual fields on the included surfaces,

\[ E_{a}^{sc}(\vec{r}) = \sum_{q=1}^{n} \int_{S_q} \left( \frac{-\omega \mu_a}{4} H_{t}(\vec{r}') H_{0}^{(2)}(k_a R) - j \frac{E_z(\vec{r}')}{4} \frac{\partial}{\partial n'} H_{0}^{(2)}(k_a R) \right) dl'. \]  

(11)

The reduction algorithm proceeds as follows. Assume that the actual fields \( E_z \) and \( H_t \) tangent to each included surface \( S_q, q = 1, 2, \ldots, n \), can be expressed in terms of a single unknown electric current density \( J_q \) distributed on that same surface, by means of a pair of exclusive operators \( \mathcal{E}^{ex}_q \) and \( \mathcal{H}^{ex}_q \),

\[ E_z(\vec{r}) = \mathcal{E}^{ex}_q J_q, \quad \vec{r} \in S_q, \quad q = 1, 2, \ldots, n, \]  

(12)

\[ H_t(\vec{r}) = \mathcal{H}^{ex}_q J_q, \quad \vec{r} \in S_q, \quad q = 1, 2, \ldots, n. \]  

(13)

Substitution of (12) and (13) into (11) and enforcing the null condition of (8) on the surface of each inclusion lead to a linear relationship between these unknown electric current densities \( J_q, q = 1, 2, \ldots, n \), and the unknown source of the local incident field, i.e., the electric current of density \( J_a \) distributed on \( S_a \),

\[ \sum_{q=1}^{n} \left( \frac{p \mathcal{E}^{ex}_a \mathcal{H}^{ex}_q}{q} + \left( -\delta_{pq} I + \frac{p \mathcal{E}^{em}_a}{q} \right) \mathcal{E}^{ex}_q \right) J_q = -\frac{p \mathcal{E}^{ex}_a}{q} J_a, \quad p = 1, 2, \ldots, n, \]  

(14)

which we have expressed in terms of the Kronecker delta \( \delta_{pq} \), the identity operator \( I \) and the surface integral operators \( \frac{p \mathcal{E}^{ex}_a}{q} \) and \( \frac{p \mathcal{E}^{em}_a}{q} \).
defined from
\begin{equation}
p_q^e E_a^e x = \frac{-\omega \mu_a}{4} \int_{S_q} x(\vec{r}') H_0^{(2)}(k_a R) d\vec{l}', \quad \vec{r} \in S_p, \tag{15}\end{equation}
\begin{equation}
p_q^m E_a^m x = \frac{-j}{4} \int_{S_q} x(\vec{r}') \frac{\partial}{\partial n'} H_0^{(2)}(k_a R) d\vec{l}', \quad \vec{r} \in S_p, \tag{16}\end{equation}

where the left-hand side superscript and subscript indicate the field surface and the source surface, respectively, the right-hand side subscript provides the index of the material constants $k_a$ and $\mu_a$ in $V_\alpha'$, the right-hand side superscript indicates the nature of the source current ($e$: electric, $m$: magnetic), and (16) contains the principal value of the integral. Solving the system of equations (14) satisfied by the currents $J_q$ in terms of the unknown current density $J_a$ yields a linear mapping from $J_a$ to the single source current density on each included surface,
\begin{equation}
J_q = L_q J_a, \quad q = 1, 2, \ldots, n. \tag{17}\end{equation}
The tangential components of the actual fields on the outer surface $S_a$ can now be expressed exclusively in terms of the unknown electric current $J_a$, that is, (see (12) and (13)) $E_z(\vec{r}) = E_a^{ex} J_a$ and $H_t(\vec{r}) = H_a^{ex} J_a$, $\vec{r} \in S_a$, where $E_a^{ex}$ and $H_a^{ex}$ are the exclusive operators for the surface $S_a$ and are obtained from (7–11) by expressing the electric and magnetic field intensities tangent to the scattering surfaces, (12) and (13), respectively, in terms of $J_a$ by way of the source mapping (17), as
\begin{equation}
E_a^{ex} = a^e E_a^e + \sum_{q=1}^n \left( a^e E_a^e H_q^{ex} + a^e E_a^m E_q^{ex} \right) L_q, \tag{18}\end{equation}
\begin{equation}
H_a^{ex} = -\frac{1}{2} I + a^e H_a^e + \sum_{q=1}^n \left( a^e H_a^e H_q^{ex} + a^e H_a^m E_q^{ex} \right) L_q, \tag{19}\end{equation}

where $p_q^e H_a^e$ and $p_q^m H_a^m$ are the integral operators that provide the contributions to $H_t$ on $S_p$ due to electric and magnetic current densities, respectively, on the surface $S_q$.
\begin{equation}
p_q^e H_a^e x = \frac{j}{4} \int_{S_q} x(\vec{r}') \frac{\partial}{\partial n'} H_0^{(2)}(k_a R) d\vec{l}', \quad \vec{r} \in S_p, \tag{20}\end{equation}
The multiply-nested body is now said to be reduced from the point of view of treating electromagnetic scattering by single source surface integral equations. By this process, we have formed an equivalent surface representation consisting of the exclusive operators \( \mathcal{E}_a^{ex} \) and \( \mathcal{H}_a^{ex} \) associated with the source mapping operators \( L_q, \quad q = 1, 2, \ldots, n. \) Note that this equivalent surface representation is independent of both the material and the illumination in the region outside the enveloping surface \( S_a, \) and is thus independent of the orientation and location of the body considered, i.e., invariant under rotation and translation. This invariance is obvious since, throughout the entire reduction procedure, the field point and the source point are only related to each other through the relative position vector \( \bar{r} - \bar{r}', \) independently of the origin. The process is clearly recursive since the exclusive operators for each inclusion must be generated (in the same manner) prior to generating the exclusive operators for the enveloping surface.

In a similar manner, we reduce the other bodies \( V_b \) through \( V_m \) and proceed to solve a linear system for the unknown currents \( J_i, \quad i = a, b, \ldots, m, \) in terms of the known incident field \( E_{inc}^e, \)

\[
\sum_{i=a,b}^m \left( p \mathcal{E}_i^e \mathcal{H}_i^{ex} + \left( -\frac{\delta p_i}{2} I + p \mathcal{E}_i^m \right) \mathcal{E}_i^{ex} \right) J_i = -E_{inc}^e \bigg|_{S_p}, \quad p = a, b, \ldots, m.
\]  

(22)

The free-space scattered field \( E_z^{sc} \) may now be expressed by using the Kirchhoff integral representation in terms of the fields tangent to each scattering surface \( E_z(\bar{r}) = \mathcal{E}_i^{ex} J_i, \quad H_t(\bar{r}) = \mathcal{H}_i^{ex} J_i, \quad \bar{r} \in S_i, \quad i = a, b, \ldots, m, \) as

\[
E_z^{sc}(\bar{r}) = \sum_{i=a,b}^m \left( i \mathcal{E}_i^e \mathcal{H}_i^{ex} + i \mathcal{E}_i^m \mathcal{E}_i^{ex} \right) J_i, \quad \bar{r} \in V_0,
\]  

(23)

where we have used (15) and (16), omitting the left-hand side superscripts in order to indicate that the field point is not necessarily located on a particular surface.
3. FIELDS INSIDE THE MULTIPLY-NESTED BODIES

It is important to note that the reduction of the multiply-nested body to an equivalent outer surface representation does not preclude the calculation of the fields at interior points. On the contrary, the operators generated and archived during the course of the reduction provide a window through which to view the fields in any isolated interior subregion without the necessity of calculating the fields throughout the entire body. The electric field within any homogeneous subregion is expressed exclusively in terms of the source of the local incident field in that region, i.e., the unknown electric current density distributed on the enveloping surface. For example,

\[ E_z(\vec{r}) = \left( a \mathcal{L}_a + \sum_{q=1}^{n} \left( q \mathcal{L}_a \mathcal{H}_q^{ex} + q \mathcal{L}_a \mathcal{E}_q^{ex} \right) \mathcal{L}_q \right) J_a, \quad \vec{r} \in V_a', \quad (24) \]

where \( V'_a \) surrounds the bodies \( V_q, q = 1, 2, \ldots, n \). Fortunately, in the modeling procedure presented, it is not necessary to determine this single source density on all surfaces simultaneously. Having formed the equivalent surface representation for the entire heterogeneous body, and having solved for the single unknown electric current density on the outermost surface, the single source density on any internal surface is obtained by following a chain of hierarchical mappings from the enveloping surface to a particular included surface. In order to formalize this backward recursion, it is necessary to renumber the inclusions along the path of the mapping. The interior body within which we require the electric and magnetic field values is renumbered \( V_{b_0} \) having an outer surface \( S_{b_0} \) on which is distributed the unknown current density \( J_{b_0} \). Our target body \( V_{b_0} \) is embedded within the overall scattering system as \( V_{b_0} \subset V_{b_1} \subset V_{b_2} \subset \cdots \subset V_{b_l} \), where \( V_{b_l} \) is a body on which the single source current density \( J_{b_l} \) is known. In this manner, the target body \( V_{b_0} \) is uniquely identified even if it appears as an inclusion within a body that is defined only as a translated or translated and rotated copy of another body. The single source current \( J_{b_0} \) on the surface of the target body is determined recursively from

\[ J_{b_i} = \mathcal{L}_{b_i} J_{b_{i+1}}, \quad i = l - 1, l - 2, \ldots, 0, \quad (25) \]

where the mapping operators \( \mathcal{L}_{b_i}, i = 0, 1, \ldots, l - 1 \), have been determined and saved during the course of the reduction algorithm. The
field components tangent to $S_{b_0}$ are written directly as $E_z(\vec{r}) = E_{b_0}^{ex} J_{b_0}$ and $H_{t}(\vec{r}) = H_{b_0}^{ex} J_{b_0}$, $\vec{r} \in S_{b_0}$. Through this selective backward recursion, the fields in a particular region are reconstructed without the need to calculate the fields throughout the entire body.

4. SUMMARY OF THE RECURSIVE PROCEDURE

The proposed recursive reduction procedure is summarized in the following steps:

1. The complex body is decomposed into a hierarchical multiply-nested structure as shown, as an example, in Fig. 1.
2. Starting at the innermost level of nesting, a pair of exclusive operators are formed for each unique, homogeneous, dielectric inclusion by way of (18) and (19) with $n = 0$. These inclusions are now said to be reduced. The exclusive operators thus obtained are also applied to duplicate inclusions, obtained through translation or translation and rotation, without repeating the reduction calculations.
3. The single source surface integral equation system (14) is solved for each unique instance where a set of reduced inclusions (one or more) is embedded within a surrounding homogeneous dielectric material region, to obtain the mappings (17).
4. The source mappings (17) are applied to (18) and (19) to yield the exclusive operators for each enveloping surface. Each of the nested regions (for which (14) was solved in Step 3) is now said to be reduced and may be treated as an inclusion as required in subsequent steps. The exclusive operators thus obtained are also applied to duplicate regions, obtained through translation or translation and rotation, without repeating the reduction calculations.
5. Steps 3 and 4 are repeated until the exclusive operators (18) and (19) have been determined for the outermost surface of each multiply-nested body, that is, the surface illuminated in free space by the given incident field.
6. The single source surface integral equation (22) is solved for a fictitious electric current density distributed on the outermost surface of each multiply-nested body in terms of the known incident field.
7. Free space fields are expressed by way of (23). Source mappings and exclusive operators archived during the reduction procedure are recalled, as required, for the selective reconstruction of interior fields via (25) and (24).
Figure 3. Multiply-nested test dielectric cylinder \((\mu = \mu_0)\) containing twin elliptical inhomogeneities, illuminated by a TM plane wave of wavelength \(\lambda\). The relative permittivity of the identical homogeneous regions \(V_4\) and \(V_5\) is \(\varepsilon_r = 8\), while that of the homogeneous multiply-connected regions \(V_2'\) and \(V_1'\) is \(\varepsilon_r = 4\) and \(\varepsilon_r = 2\), respectively. \(V_3\) is a translated and rotated copy of \(V_2\).

5. ILLUSTRATIVE TESTS

The recursive reduction algorithm presented above was implemented in a simple method of moments code. The cross sections of the cylindrical surfaces are discretized into curved segments over which the unknown single source current is considered to have a constant surface density. Our integral operators are thus transformed into matrices where the number of rows and columns are given by the number of segments on the field surface and source surface, respectively. In this manner, the size of the matrix to be inverted at any step in the reduction process is given by the total number of segments on the scattering surfaces within only the body to be reduced.

As a numerical test example, we solve the problem of the TM plane wave scattering by the multiply-nested dielectric cylinder shown in Fig. 3. This circular dielectric cylinder contains twin elliptical cylindrical inclusions, each of which contains its own inclusions, namely.
two identical circular dielectric cylinders. This structure was selected specifically to highlight the invariance under rotation and translation of the equivalent surface representations obtained through application of the recursive multiply-nested reduction algorithm. The invariance properties are exploited first in the course of the reduction algorithm itself, from the innermost inclusions to the outer surface of the whole body, and again in the recovery of the interior fields, using the backward recursion from the outer surface to the surface of the inclusions. Having generated the exclusive operators for the body $V_4$, we simply copy these operators to the surface of $V_5$ without repeating the calculation; similarly (and with greater computational savings), the exclusive operators obtained for the body $V_2$ are copied directly to the surface of $V_3$ which is a translated and rotated copy of $V_2$. We are now free to construct the equivalent surface representation for the encasing body $V_1$ and solve for the single source density $J_1$ on the surface $S_1$ of $V_1$ in terms of the specified TM plane wave illumination.

The normalized bistatic radar cross section in the far-field obtained by the recursive algorithm is compared in Fig. 4 with that obtained by direct simultaneous solution of $E_z$ and $H_t$ on all surfaces by applying the electric field integral equation method (EFIE). Using identical surface discretization (i.e., roughly ten curved segments per local wavelength), the two methods yield practically the same solution, but a five-fold decrease in computation time is attributed to the recursive multiply-nested algorithm presented. It should be mentioned that when the single source surface integral equation is formulated without the proposed recursive algorithm and invariance properties due to the exclusive operators, only a decrease of less than 50% in the computation time is achieved and the storage space is substantially larger.

In this reduction procedure, the inner structure of $V_3$ was never explicitly used; rather, it was imposed implicitly by copying to $V_3$ the exclusive operators determined for $V_2$, since the body $V_3$ is a translated and rotated copy of $V_2$, and as such it contains copies of $S_4$ and $S_5$, namely $S_4^*$ and $S_5^*$, respectively, with the associated internal mapping operators $L_4$ and $L_5$ obtained for $V_2$. As an illustration of field computation at points inside the multiply-nested body, we exploit again the invariance properties in order to obtain the electric and magnetic fields tangent to the surface $S_5^*$ of the homogeneous inclusion $V_5^*$ in $V_3$, by the backward recursion (25), only following the chain of mappings associated with the embedding $V_5^* \subset V_3 \subset V_1$. The electric
Figure 4. Bistatic RCS of the multiply-nested cylinder shown in Figure 3. CPU time on a Power Macintosh 5200 personal computer is 53.7s for the recursive algorithm and 280s for the EFIE, using 92 segments on $S_1$, 56 segments on both $S_2$ and $S_3$, and 24 segments on both $S_4$ and $S_5$.

and magnetic field intensities are plotted in Fig. 5, and are graphically identical to those obtained by the EFIE method. Only 1.3 seconds of CPU time (i.e., 2.4% of the total time) is required in order for the recursive algorithm to retrieve the relevant matrices from the hard-disk and perform the four matrix-vector multiplications required to execute the recursive source mapping $J_5^* = \mathcal{L}_5 J_3 \leftarrow J_3 = \mathcal{L}_3 J_1 \leftarrow J_1$ and recover the surface fields via the exclusive operators $\mathcal{E}_5^{\text{ex}}$ and $\mathcal{H}_5^{\text{ex}}$. It should be noticed that due to the invariance properties of the operators involved, the position of $V_3$ relative to $V_2$, in $V_1$, and of $V_5$ relative to $V_4$, in $V_2$, as well as of $V_5^*$ relative to $V_4^*$, in $V_3$, is not relevant to the total amount of computation and storage required.
Figure 5. Intensity of (a) electric and (b) magnetic fields tangent to the surface $S_5^*$ of the inclusion $V_5^*$ in $V_3$, computed by copying the operators from $S_5$ in $V_2$ and using the backward recursion corresponding to $V_5^* \in V_3 \in V_1$. $\theta$ is defined in Figure 3 and $|E_z^{inc}| = 1V/m$. 
6. CONCLUSION

By formulating the field problem within a multiply-nested body in terms of a local scattering problem we have constructed a recursive reduction algorithm that allows an equivalent surface representation of the multiply-nested body which is independent of both the material and the illumination in the region outside the body, and is therefore invariant under rotation and translation. As a consequence, the relevant computations are not repeated for identical bodies which are obtained from each other by translation or by translation and rotation. A body thus reduced may be placed alongside other reduced bodies to form a system of complex scatterers, or embedded (along with other such bodies) as an inclusion in an enclosing multiply-nested body. Although the body has been reduced to an equivalent single surface representation, the fields at any interior point can be generated in terms of the single source density on the outermost surface through a selective backward recursion which entails a short sequence of matrix-vector multiplications. This is important in numerous practical problems, for instance in the case of the human body, where one is interested in computing the field reaching only one specific part of the body and not the field everywhere inside the body. It should be remarked that in the method presented the matrix storage requirements do not increase with the overall detail of the problem, but rather are limited to those matrices required to represent only the largest set of inclusions in any one body. Thus very large, very detailed problems can easily be handled on a small personal computer. Significant savings in computation time are demonstrated for a sample problem. In the special case of wave scattering by only a single, homogeneous, dielectric cylinder, the proposed method reduces to a previously published single source surface integral equation [2].

REFERENCES


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