Analysis of high-speed interconnects using efficient multipoint Padé approximation

L.Y. Li, G.E. Bridges and I.R. Ciric

An efficient method is presented for analysis of large interconnect circuits including lossy coupled transmission lines. Based on multipoint Padé approximations and a new expansion point search algorithm, the method can obtain single close-form frequency-domain and transient solutions of the interconnect problem with a minimum of frequency expansion points.

Introduction: The efficient simulation of large interconnect circuits has been greatly enhanced by using moment-matching techniques such as asymptotic waveform evaluation (AWE) [1, 2]. AWE generates the moments of a linear circuit through a Taylor series expansion of its frequency response about a single frequency point \( s = 0 \). The moments are then matched to a reduced-order model in the form of a rational Padé approximation. To improve the accuracy of the AWE method at high frequencies, multipoint moment-matching techniques have been introduced in which the Taylor series expansions are performed at selected frequency points [3, 4]. The expansion points are typically chosen on the imaginary axis using a binary search algorithm. Although multipoint techniques can produce accurate approximations over a specified frequency range, the associated computational effort is proportional to the number of frequency-expansion points, and is thus relatively expensive. In this Letter, we propose an efficient method for the analysis of large interconnect circuits, which can include lossy coupled transmission lines. This method is based on multipoint Padé approximations for all moment sets at all updated expansion points. An improved algorithm for the selection of expansion points is developed, resulting in a smaller number of expansion points being required for the same accuracy and yielding reduction in CPU cost.

Formulation and algorithm: The modified nodal admittance matrix equation for a network that contains linear lumped components and distributed transmission line systems can be written in the frequency domain as

\[
Y(s)X(s) = E
\]

(1)

where \( Y(s) \) is a matrix formed by linear lumped and distributed components, \( X(s) \) is a vector containing unknown output variables, and \( E \) is a vector representing an impulse input excitation. A Taylor series expansion of \( X(s) \) about a complex frequency point \( s_k \) is given by

\[
X(s) = \sum_{n=0}^{\infty} M_k(n) (s - s_k)^n
\]

(2)

Here \( M_k(n) \) is the \( n \)th moment vector about \( s = s_k \) and can be calculated using a recursive relationship [1, 2].

We expand \( X(s) \) at \( K \) expansion points \( s_k \) and truncate the expansion (eqn. 2) at each expansion point to the first 2\( N \) moment vector terms. For a specified output \( Y(s) \), \( K \) moment sets are generated, comprising 2\( KN \) moments \( m_{k,n} = [M_k]_n \), \( n = 0, 1, 2, \ldots, 2N - 1 \). Using a multipoint Padé approximation, the \( K \) moment sets are then matched to a rational network transfer function of order \( q = NK \) in the form

\[
\frac{\sum_{j=0}^{2N-1} a_j s^j}{1 + \sum_{j=1}^{K} b_j s^j} = \sum_{n=0}^{\infty} m_k(n) (s - s_k)^n \quad k = 1, 2, \ldots, K
\]

(3)

This results in a 2\( q \) \( \times \) 2\( q \) matrix equation whose solution yields the coefficients of the rational transfer function.

In implementing multipoint moment matching for large circuits, computational cost is dominated by the moment-generation stage, which requires the solutions of the network matrix eqn. 1 at different frequency-expansion points. To minimise the number of frequency-expansion points, an efficient algorithm for expansion point selection is developed by combining a bisection search technique with a multipoint Padé approximation for all the moment sets available from all updated expansion points. As a result, the number of expansion points required can be reduced compared with previous techniques [3, 4].

To illustrate the algorithm, assume that there are currently \( K \) expansion points within the frequency range \([0, f_{\text{max}}]\). Let \( H_{\text{exp}}(s) \) denote the rational Padé transfer function generated by the moments of all \( K \) expansion points. If we specify a middle frequency point \( f \) between two consecutive expansion points, let \( H_{\text{med}}(s) \) denote the rational Padé transfer function generated from the moments of \( K - 1 \) expansion points that are the closest to that mid-frequency point. The search algorithm is described below, starting with two expansion points:

**Step 1:** Set \( f_L = 0 \) and \( f_H = f_{\text{max}} \).

**Step 2:** Expand the network impulse response at \( f_L \) and \( f_H \) using eqn. 2.

**Step 3:** Construct \( H_{\text{med}}(s) \) from moments in the expansions at \( f_L \) and \( f_H \) using eqn. 3.

**Step 4:** Construct \( H_{\text{med}}(s) \) corresponding to the mid-point \( f = (f_L + f_H)/2 \).

**Step 5:** If \( |H_{\text{med}}(2\pi f) - H_{\text{med}}(2\pi f)| < \epsilon \), where \( \epsilon \) is a specified error tolerance, proceed to step 6. Otherwise, expand at the mid-point and update \( H_{\text{med}}(s) \) to include the moment information from all existing expansion points.

**Step 6:** If no mid-point expansion is needed between any two consecutive expansion points, end the algorithm. Otherwise, repeat steps 4-5 using the new subintervals, with \( f_L \) and \( f_H \) corresponding to each new subinterval.

Once the search process is completed, a network transfer function in the form of the rational function (eqn. 3) is automatically generated which is accurate over \([0, f_{\text{max}}]\). From the frequency-domain transfer function, the pole-residue form of the impulse transient response of the network can be derived using a procedure consisting of stable-pole extraction, conjugate duplication and least-square approximation. The resultant impulse transient response corresponding to \( q \) stable poles \( \rho_i \) and residues \( k_i \) is written as

\[
h(t) = \sum_{j=1}^{q} k_j e^{\rho_j t}
\]

(4)

**Fig. 1 Interconnect circuit with lossy multiconductor transmission lines**

[3]

**Numerical results and discussion:** The proposed method was applied to the interconnect circuit example shown in Fig. 1 [3]. The circuit contains two lossy coupled transmission lines and was excited by a 1 V pulse with 0.4 ns rise/fall time and 5 ns duration. The frequency response at the load capacitor was calculated using the proposed method with \( f_{\text{max}} = 2.5 \) GHz. Using ten moments at each expansion point, we found that four frequency expansion points were required at 0.625, 1.25 and 2.5 GHz. Fig. 2 shows the comparison of the approximate frequency response with the actual response obtained by solving eqn. 1 at 1024 frequency points. The resulting transient waveform is shown in Fig. 3 and compared with the result obtained by applying an inverse fast Fourier transform (IFFT) to the actual response. We also applied the CFH multipoint moment-matching technique [3] to this circuit. With \( \epsilon \) moments employed at each expansion point, the CFH required seven frequency expansion points to achieve the same accuracy. A similar improvement was observed for other cir-
Fig. 2 Frequency response of circuit in Fig. 1

- actual response
- proposed method

Fig. 3 Transient response of circuit in Fig. 1

- IFFT
- proposed method

© IEE 2001
Electronics Letters Online No: 20010615
DOI: 10.1049/el:20010615

L.Y. Li, G.E. Bridges and I.R. Ciric (Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, R3T 5V6 Canada)
E-mail: bridges@ee.umanitoba.ca

References