THE ROLE OF MULTISCALE ANALYSIS AND COMPLEXITY MEASURES APPLIED TO CLIMATE CHANGE DATA

W. Kinsner, A. Faghfouri, and M. Potter

Signal and Data Compression Laboratory
Department of Electrical and Computer Engineering
University of Manitoba
Winnipeg, MB, Canada R3T 5V6
and the Institute of Industrial Mathematical Sciences
and Telecommunications Research Laboratories, TRLabs
<w.kinsner@ieee.org>

UNIVERSITY OF MANITOBA

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Outline

- Motivation & Discussion
  - Some definitions
  - Evidence
    (temp, sea level, CO_2, precipitation, soil moisture, vegetation)
  - Modelling (Canadian and others)
  - **Single-scale analysis is incomplete**
- Study 1: Lightning Strike Maps
  - Spatial & temporal inhomogeneities
  - Multiscale multifractal measures
  - Feature extraction
- Study 2: FM & Cyclostationary Signals
  - Generating true time series
  - Translation from feature space
  - Incompleteness of data

Definitions (1/4)

- Weather is related to the Earth's atmosphere.
- Atmosphere is the 800-km blanket of air that covers the Earth.
- The blanket consists of several layers with different densities, pressures, and temperatures:
  - Troposphere, 18 km
  - Stratosphere, 50 km
  - Mesosphere, 80 km
  - Ionosphere (aurora), 350 km
  - Thermosphere, 690 km
  - Exosphere, 800 km
- The average temperature at the surface of the Earth is 14º C.
- The average atmospheric pressure at the sea level is about 101.3 kpascal (14.7 pound/sq in).
- About 50% of the total atmospheric mass is within a layer of 5 km above the sea level, and 99.99999% below 108 km.

Definitions (2/4)

- The main composition of the atmosphere is (99.9 % total):

– Nitrogen, N_2,	78.084 %	780,840.0	ppmv
– Oxygen, O_2,	20.946 %	209,460.0	ppmv
– Argon, Ar,	0.9340 %	9,340.0	ppmv
– Neon, Ne		18.18	ppmv
– Helium, He		5.24	ppmv
– Krypton, Kr		1.14	ppmv
– Hydrogen, H		0.55	ppmv
– Water, H_2O	0 to 7 %		ppmv

- "Greenhouse" gasses (0.1% total):

– Carbon dioxide, CO_2,	0.01 to 0.1 %	350.0	ppmv
– Methane, CH_4,		1.745	ppmv
– Nitrous oxide, N_2O,		0.5	ppmv
– Ozone, O_3,		0 to 0.07	ppmv

NOTE: ppmv stands for "parts per million by volume"

Definitions (3/4)

- Weather is the fluctuating state of the atmosphere around us, characterized by factors such as
  - temperature,
  - wind,
  - cloudiness, and
  - precipitation.
- Weather is the result of rapidly developing and decaying weather systems such as mid-latitude low/high pressure systems with their associated
  - frontal zones,
  - showers, and
  - tropical cyclones/hurricanes.
- Weather has only limited predictability.

Definitions (4/4)

- Climate is defined as the long-term average of weather conditions, such as
  - weather parameters
    (temperature, wind, cloudiness, precipitation),
  - sea level,
  - snow/ice cover,
  - circulation, and
  - extremes.
- Climate change is characterized by trends in these conditions for decades or longer.
- Climate varies in space and time (from season to season, year to year, decade to decade, or even Ice Ages).
- Various climate regimes found on Earth are classified and described by classical climatology.
- The following few diagrams will illustrate the the problem of climate change.

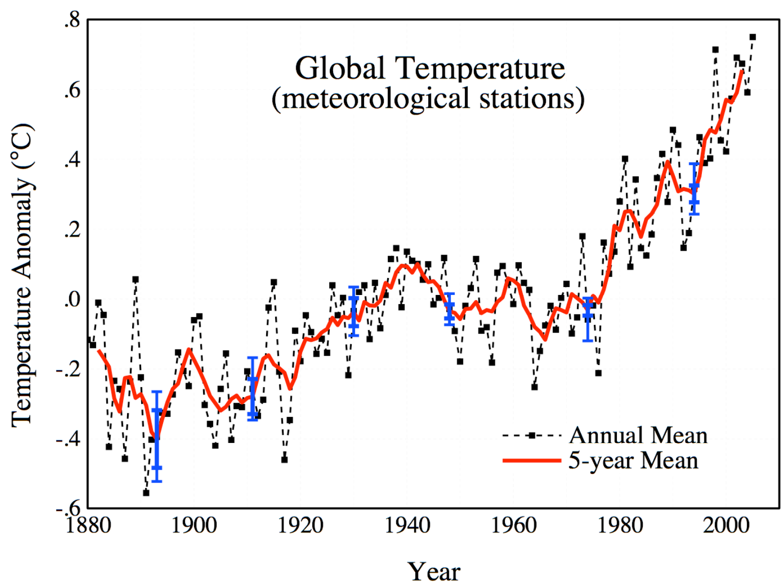
Temperature Heterogeneity In Time

- Evidence of global warming trend is based on the increase in temperature during the past century (by 0.7º C or 1.4º F), particularly since 1978 (by 0.5º C or 0.9º F), as shown in the next plot [Godd06], [NaAc06, p. 4], [HRSI01].

- The plot shows global annual-mean surface air temperature (based on measurements from meteorological stations, ships, and satellites), relative to the 1951-1980 mean. Error bars are calculated for 2σ (95%) confidence.

- This increase was not due to changes in the Sun output.
- The data are consistent with other evidence of warming [HRSI01], including
  - ocean levels,
  - ocean temperature,
  - shrinking mountain glaciers, and
  - decreasing polar ice cover.

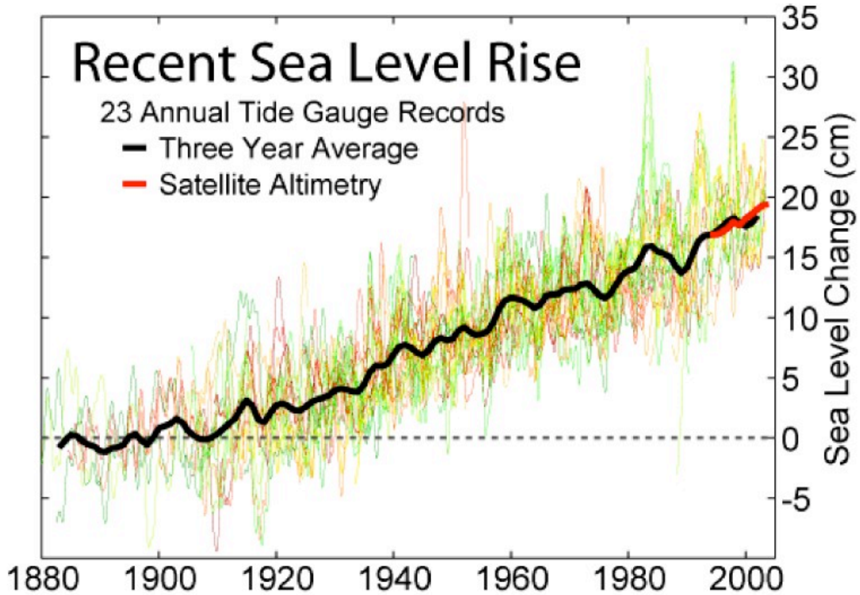
Temperature Change



Sea Level Heterogeneity In Time

- The next graph shows the change in the annually-averaged sea-level, as collected at 23 geologically-stable tide gauge sites.
- The long-term records have been assembled by Douglas [Doug97] from the Permanent Service for Mean Sea Level (PSMSL) [Perm06] .
- The thick dark line is a 3-year average of the instrument data.
- The red line is the recent (since 1992) confirmation from the annually-averaged TOPEX/Poseidon satellite altimetry data from, and its successor, Jason (since 2001) with an uncertainty of 3-4 mm, as assembled by the Colorado Center for Astrodynamics Research [Colo06].
- It is seen that the sea level has increased about 18.5 cm from 1900 to 2000.

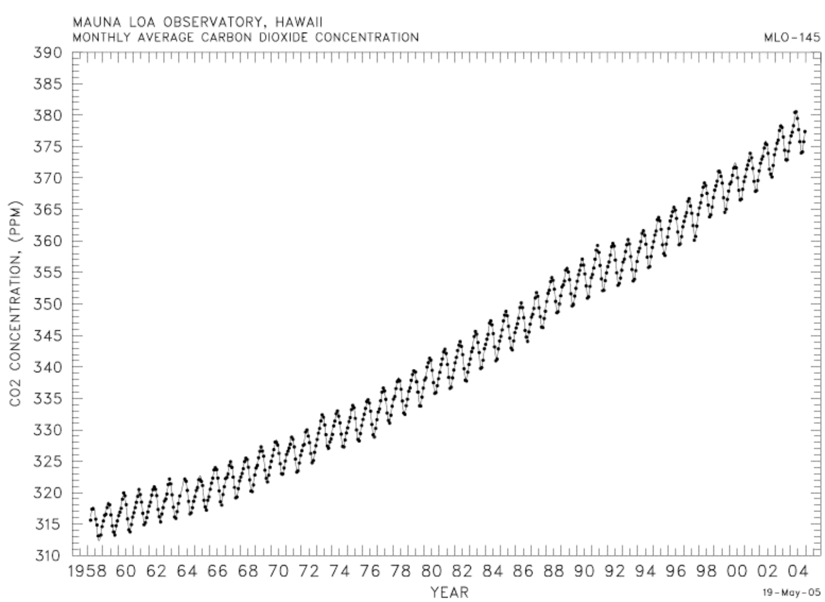
Sea Level Changes



Carbon Dioxide Heterogeneity In Time

- Although the level of CO2 in the atmosphere is very small (not exceeding 0.1 %), its impact on temperature is critical.
- The next graph shows the annual mean concentration of CO2 at the middle of the troposphere (as recorded by the Mauna Loa Observatory, Hawaii, 3397 m above the mean sea level (MSL), 19°32' N, 155°35' W) [KeWh06].
- The record shows a 19.4 % increase from 1959 (315.98 ppmv) to 2004 (377.38 ppmv), with an average rate of 1.4 ppmv/year.

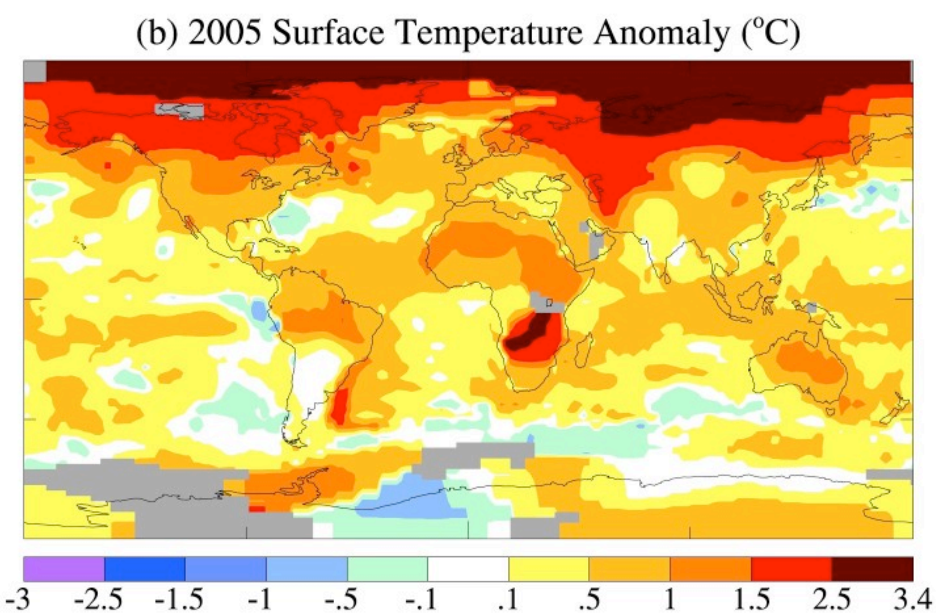
CO_2 Increase



Temperature Heterogeneity In Space

- The next figure shows the distribution of temperature anomalies for 2005 [Godd05], with the grey areas indicating a lack of station data within 1200 km.
- It is seen that the major increase is localized in the Northern Hemisphere, and that the distribution is not uniform. In fact, there are concentrations of elevated values.
- Although this spatial distribution appears to be random, it is correlated, but with a hetero-geneous pattern that is very complicated.

Spatial Distribution of Temperature



PRECIPITATION: Introduction (1/4)

- Precipitation is any form of air moisture
  (mist, rain, hail, freezing rain, sleet, snow)
  that becomes too heavy to remain suspended in the air.
- Precipitation is one of the critical components of the Earth hydrological cycle in that it provides fresh water to sustain life, and affects life fundamentally.
- Precipitation systems evolve rapidly and appear to be random, with pronounced local impacts.
- Understanding precipitation is essential to unravelling many uncertainties about the Earth climate.

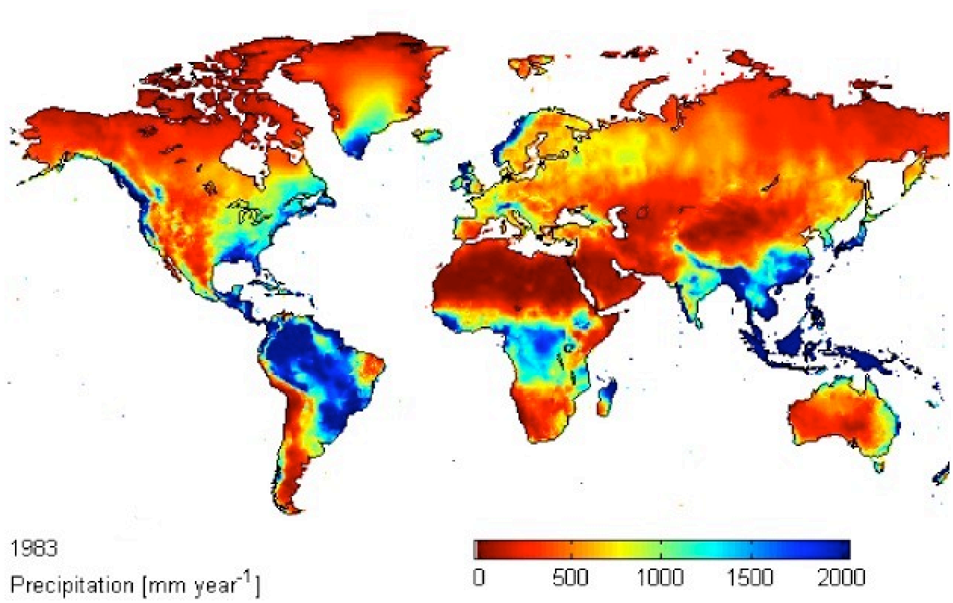
PRECIPITATION: TRMM (2/4)

- Precipitation (i.e., its amount, distribution, rates, and associated heat release) can be measured both
  - locally (ground-based measurements using rain gauges and radars, including Doppler), and
  - globally on the planetary scale (space-based remote sensing using radiation sensors (passive) and weather satellite radar (active).
- The passive techniques capture the temperature of the cloud tops from which rainfall can be inferred (not very accurate).
- The passive techniques use multifrequency emissions (e.g., 6,10, 19, 23, 36, 50-60, 89, 150/166, 183.3 GHz [HuHo06]), and measure the scattered energy from the clouds, as well as any raindrops, and the vertical distribution of the rain.
- Since November 1992, these techniques have been used by the US-Japanese **Tropical Rainfall Measuring Mission (TRMM)** with the world's first spaceborne precipitation radar and four other sensors.

PRECIPITATION: GPM (3/4)

- The TRMM is being expanded to the Global Precipitation Measurement (GPM) system [HuHo06] capable of measuring rain rates as small as 0.5 mm per hour to 4 inch per hour, as well as sizes of particles, snow and rain.
- The GPM can cover 80% of the globe within 3 hours, and deliver the data to users within 3 hours thereafter (latency).
- GPM Equipment
  - Dual-frequency Precipitation Radar (DPR)
  - GPM Microwave Imager (GMI) which uses high-resolution (6 km [HuHo06]) multichannel passive microwave (PMW) rain radiometer; and
  - Calibration reference system for a heterogeneous constellation of eight satellites in the Core.
- GPM applications include prediction of
  - floods;
  - droughts;
  - fresh water resources; and
  - crop conditions.
- The next slide shows an example of the global precipitation map [OwHJ04, Fig. 1b].

PRECIPITATION: Global Map (4/4)



SOIL MOISTURE: Measurements (1/4)

- Soil moisture is the water content per volume of soil [m^3/m^3].
- It can be estimated from
  - Direct measurements of the soil (local); and
  - Remote satellite microwave brightness temperature observations (global).
- Many parameters affect the accuracy of the estimates, including the roughness of the Earth surface.

SOIL MOISTURE: Modelling (2/4)

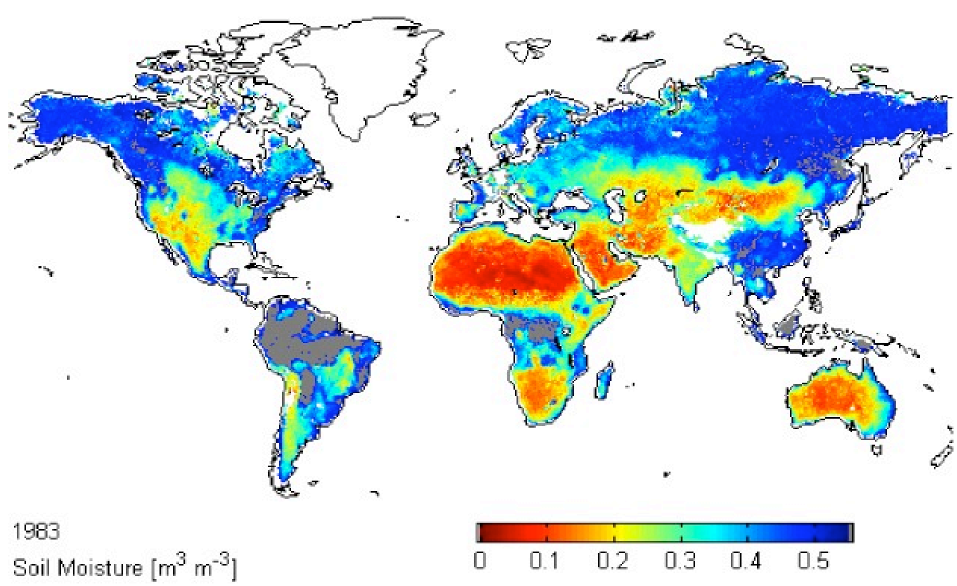
- The satellite estimates can be done as frequently as every hour.

  Average monthly and annual soil moisture maps can be produced either globally or for a particular region (e.g., Africa [OwHJ04]).
- Soil moisture is the most important parameter influencing the atmospheric circulation over land during the summer, as well as in energy balance and radiative transfer based applications such as
  - Climate (long-term) prediction models (to improve accuracy of the of the large-scale surface-atmospheric interaction models, and
  - Numerical weather (short-term) forecasting models.
- Regular global-scale estimates of soil moisture from satellite microwave observations improve all weather and climate models by providing
  - Improved initial conditions;
  - Continuous bias corrections, and
  - Validation datasets.

SOIL MOISTURE: Discussion (3/4)

- The next slide shows the mean annual soil moisture derived from brightness temperature obtained from the Scanning Multichannel Microwave Radiometer (SMMR) that flew on-board the Nimbus-7 satellite.
- The daily data were collected for nine years (1978 to 1987) at five frequencies and two polarizations.
- The distribution is for 1983 [OwHJ04; Fig. 1a].

SOIL MOISTURE: Global Map (4/4)



VEGETATION: NDVI (1/4)

- Global vegetation maps show the density of plant growth on the Earth.
- Vegetation can be estimated remotely by observing radiation reflected from plants. More specifically, the pigment of plant leaves (chlorophyll) absorbs visible light (0.4 to 0.7 µm) for photosynthesis, but reflects near infrared radiation (0.7 to 1.1 µm).
- The most common metric for vegetation is the **Normalized Difference Vegetation Index (NDVI)** defined as

$$NDVI = (W_{NIR} – W_R) / (W_{NIR} + W_R)$$

where W_{NIR} is the near-infrared band (e.g., Band 4: 0.75 to 0.90 µm on the Landsat ETM), and W_R is the red band (e.g., Band 3: 0.63 to 0.69 µm on the same satellite)

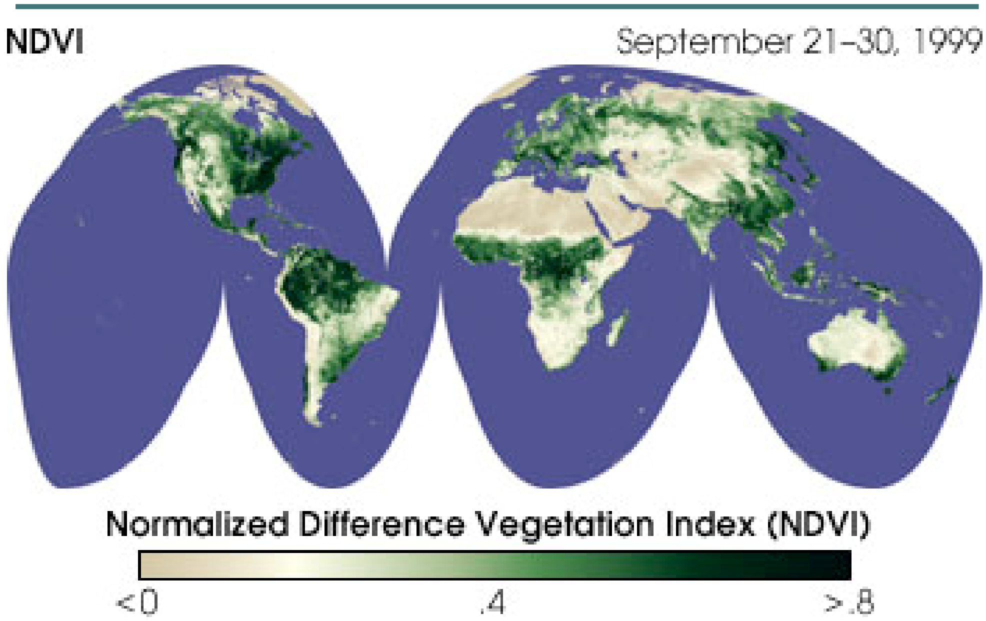
VEGETATION: Discussion (2/4)

- Although NDVI can range from –1 to +1, the vegetation values range between 0.1 to 0.7, with the low values (0.1 and below) corresponding to barren areas of rock, sand, snow or clouds, while moderate values (0.2 to 0.3) representing shrubs and grasslands, and the high values (0.6 to 0.8) representing temperate and tropical rainforests.
- NDVI is a good indicator of
  - Climate variables (e.g., El Niño Southern Oscillations, ENSO [GBGSL00];
  - Precipitation [ScKa00];
  - Relative biomass and greenness [BGSL00];
  - Primary production [RiAP99];
  - Dominant species; and
  - Stocking rates.

VEGETATION: FASIR NDVI (3/4)

- An example of recent green-vegetation data is the dataset from the NOAA-7, NOAA-9 and NOAA-11 satellites, using the Advanced Very High Resolution Radiometer (AVHRR) collected over a period from 1982 to 1998.
- The resolution is 1 square kilometer per pixel, and have been corrected to remove artifacts such as
  - Sensor degradation;
  - Volcanic aerosol effects;
  - Cloud contamination
  - Short-term atmospheric effects (e.g., water vapour and aerosol effects);
  - Sun angle variations; and
  - Missing data (due to glares and other factors).
- The corrected data are called **Fourier Adjusted Solar-zenith-angle corrected, Interpreted and Reconstructed (FASIR NDVI).**
- Other data are also being collected using the Moderate-resolution Imaging Spectroradiometer (MODIS) in the form of **Enhanced Vegetation Index (EVI)** with a resolution of 250 m.
- The next slide shows the NDVI for September 21-30, 1999 [WeHe06].

VEGETATION: Global Map (4/4)



Canadian MODELS: GEM & GCM2 (1/2)

- Since September 1995, the Canadian Meteorological Centre of Environment Canada has been producing three-months outlooks for Canada regarding
  - surface air temperature (SAT), and
  - precipitation (PSPN) anomalies.
- The temperature or precipitation anomalies are calculated as the difference between the predicted values and the mean forecasts.
- Two models have been used to produce six runs from each:
  - the **Global Environmental Multiscale (GEM)** model from Recherche en Prévision Numérique [CGMP98], and
  - the **Global Climate Model (GCM2)** of the Canadian Centre for Climate Modelling and Analysis [MBBL92].
- The models have been formulated using a knowledge-driven approach, combined with a data-driven scheme (the concepts described in, for example, [ToDz06]).

Canadian MODELS: GEM & GCM2 (2/2)

- The models have the same hydrostatic equations, but differ through their use of the surface force fields [Envi06], [CGMP98, Sec. 6]:
  - sea surface temperature (SST),
  - sea ice , and
  - snow cover.
- The climatic drift is removed from the forecasts [KhZw01], [KhZG01].
- While the horizon of predictability of weather prediction models is around 4 to 10 days [PIGa00], the climate predictions are much longer term.
- There are many weather and climate models in use today around the world (e.g., the Australian CSIRO Mark 2 coupled global climate model with its atmospheric, oceanic, biospheric, and sea-ice components [HuEl06]).

Discussion: Modelling & Simulation (1/5)

- The above diagrams were presented to point to a serious problem that the Earth is facing due to changing climate, including
  - a real rapid increase in the temperature and its consequences,
  - possibly linked to the CO_2 emissions.
- Many linear and nonlinear methods have been applied to forecast the future behaviour of the trends. (Some predict that, at the current melting rate of the polar ice, there will be none left by 2050.)
- So, how can the problem be studied?
- **Key questions**:
  - How can one simulate such patterns?
  - How can one compare such measured patterns with simulated ones?
  - What are the suitable metrics for such a comparison?
  - What are the suitable features that could be used for classification of the patterns in the distribution?
- Study 1 will address the questions.

Discussion: Modelling & Simulation (2/5)

- Climate studies are not just a collection of long-term weather statistics based on temporal or spatial data [HDGN01].
- The important concept that has emerged from climate science is that Earth's natural climate includes complex interactions (or couplings) between the following system components [Lore93] [HDGN01]
  - the atmosphere (fast changing),
  - the ocean (hydrosphere, 70 % of Earth's surface),
  - the land surface,
  - ice cover (cryosphere),
  - ecosystems (forced or influenced by external mechanisms), and
  - human life.
  - Climate models reflect this view by first modelling each system component separately, and then linking them to account for the couplings.
- The uncertainties incorporated in the models include:
  - future population growth,
  - economic development,
  - energy use, and
  - policy choices (e.g., technology sharing, support of the Kyoto Protocol).

Discussion: Modelling & Simulation (3/5)

- Such models represent **nonlinear dynamical systems** that can exhibit not only the standard behaviours (i.e., either the point, cycle, or torus stabilities, or instability), but also the fourth stable state, the chaotic state with its strange attractor.

- When such dynamical systems operate far from equilibrium, they can exhibit behaviours that have long-range dependencies described by power-law **self-affinity** (or a simpler self-similarity) [Kins05].

- Examples of such patterns are described in Study 1 of this paper, and other weather related phenomena such as wildfires [RADD01], earthquakes, avalanches, and various ecosystem properties [BaTa89], [SoSo89], [JoMN98], and Earth fluctuations in the range of 0.04 to 0.3 Hz [CUMV06].

- The horizon of predictability of the future behaviour in such systems is always short, and little can be done about it.

Discussion: Modelling & Simulation (4/5)

- The problem is compounded by a single-scale analysis (monoscale) of measured and simulated data which can provide only partial information about the patterns because it cannot reveal the power-law self-afine (or self-similar) dependencies in the data.
- For example, the Canadian GEM model can compute at different temporal and spatial scales, but one scale at a time.

- To extract the power-law dependencies from the data, a **multiscale multifractal modelling and analysis** must be performed which requires that the computations be done “simultaneously” (which means either “parallel” computation, or “one after the other,” but within the same analysis).

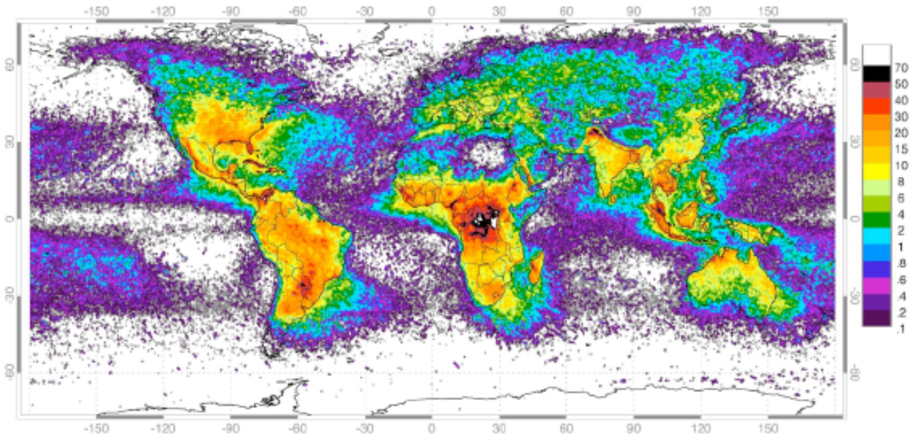
- This approach applies not only to one dimensional (1D) data (i.e., time series) as described in Study 2, but also to 2D maps (Study 1), as well as to 3D and 4D data such as the volumetric weather Doppler radar data [KLSW94].

Discussion: Modelling & Simulation (5/5)

- Furthermore, the evaluation of the differences between the measured and simulated data is often based either on energy (e.g., mean-squared error) or on some energetic features. This approach is inadequate because many permutations of the patterns can produce the same outcome.
- Instead, multiscale multifractal characterization of errors or features should replace the monoscale analysis [Kins05].
- In addition, characterization of errors or features should be based on information or entropy contained in the measured and simulated data.
- **Study 1** will show how it can be done.
- **Study 2** will address additional questions pertinent to the long-term data found in the study of climate change.

Study 1: Lightning and Power Lines

Global Lightning Strike Activities

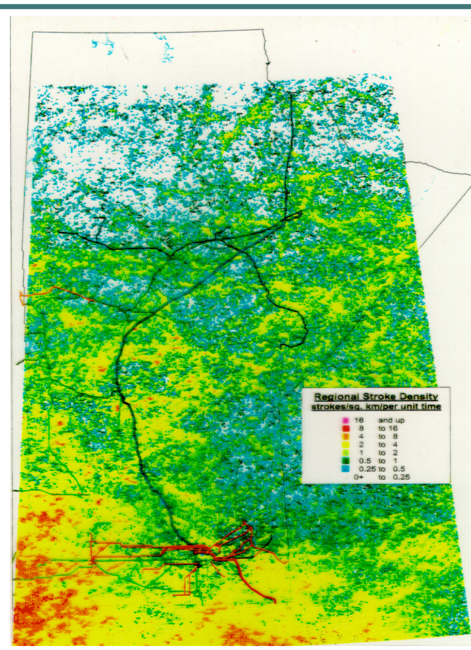


High Resolution Full Climatology Annual Flash Rate

Global distribution of lightning April 1995-February 2003 from the combined observations of the NASA OTD (4/95-3/00) and LIS (1/98-2/03) instruments

Lightning strikes are a global phenomenon.
The pattern changes with temperature.

Lightning Strike Map of Manitoba (2001)

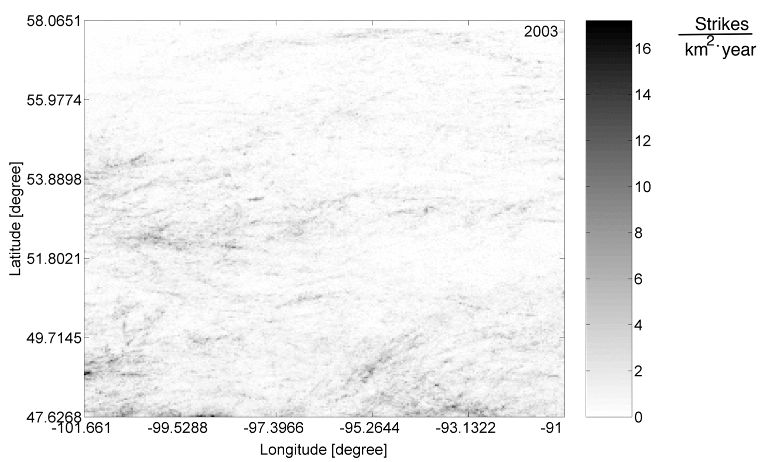


High Resolution Data

- The Canadian Lightning Detection Network (CLDN) is a network consisting of 83 lightning detection sensors strategically located throughout Canada.
- CLDN provides:
  - Real time data
  - Archived lightning information.
- The network collects all the lightning strikes over time, and it has the best achievable location accuracy of 500 m, and time error less than 1.5 microseconds [Glob99].
- In each data file, 14 different data specifications of the lightning strikes are stored, such as
  - Date and time
  - Latitude
  - Longitude
  - Peak current
- The data can then be extracted for any region and time period, and the selected discharges can be represented as an image or map.

Lightning Strike Maps (Ensemble Behavior)

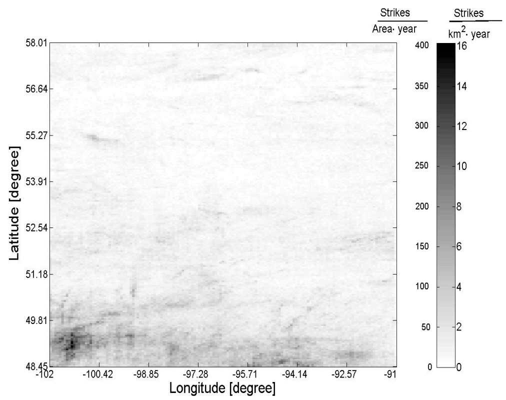
- **Why lightning strike maps?**
  - Lightning strokes have a stochastic distribution and thus, tracking the individual strokes is very complicated
- **Definition**
  - The time and/or space distribution of the strikes can be illustrated as *lightning strike maps* (LSMs).

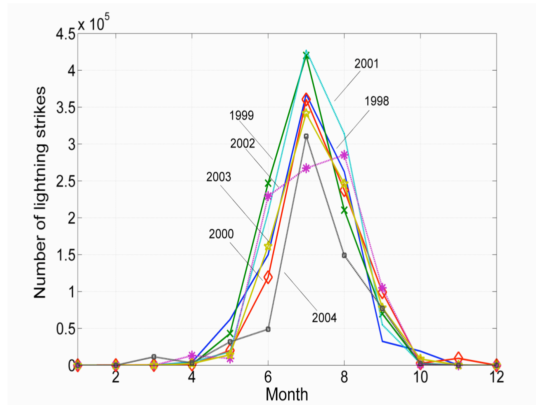


Lightning strike map of Manitoba for the year 2003.

Lightning Strike Maps

- **Specifications**
  - Stochastic and highly nonstationary in time and space
  - Local and global characteristics are different



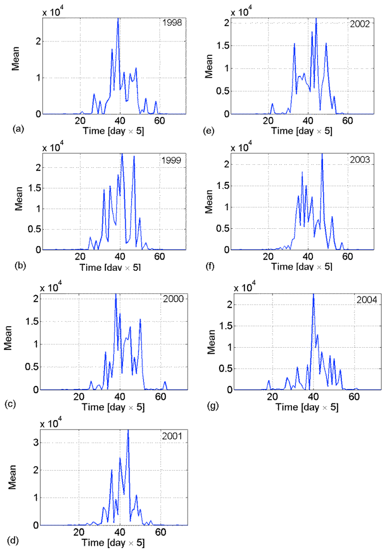


Lightning strike map of Manitoba for the year 2001.

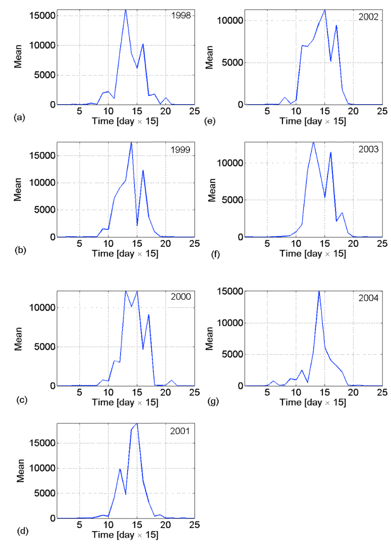
Data Exploration: Physics & Statistics

- **How to explore stochastic data such as the LSMs?**
  - Exploring the high resolution data, not relying on the existing models
  - Statistical methods
- **Physics of the LSMs**
  - Similar clusters of densities with different sizes can be observed in the map
    - Is there any relationship between patterns in different sizes (scales)?
- Higher order statistics
  - Multiscale methods
  - Statistical moments

Nonstationarity Over Time

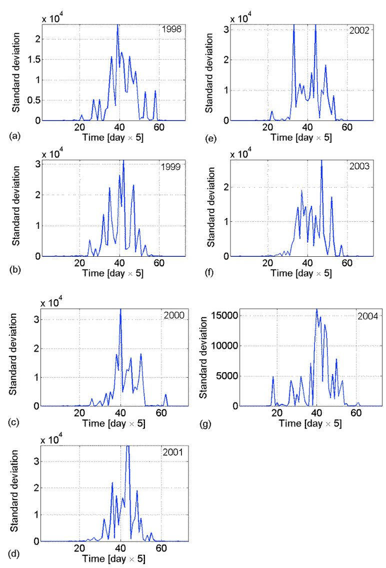


Mean of the number of strikes over 5 day non-overlapping windows for the years: (a) 1998, (b) 1999, (c) 2000, (d) 2001, (e) 2002, (f) 2003, (g) 2004.

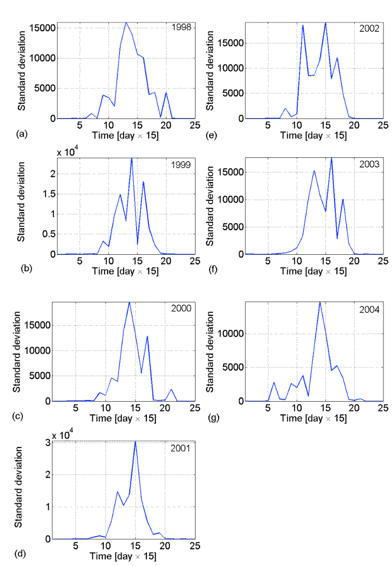


Mean of the number of strikes over 15 day non-overlapping windows for the years: (a) 1998, (b) 1999, (c) 2000, (d) 2001, (e) 2002, (f) 2003, (g) 2004.

Nonstationarity Over Time



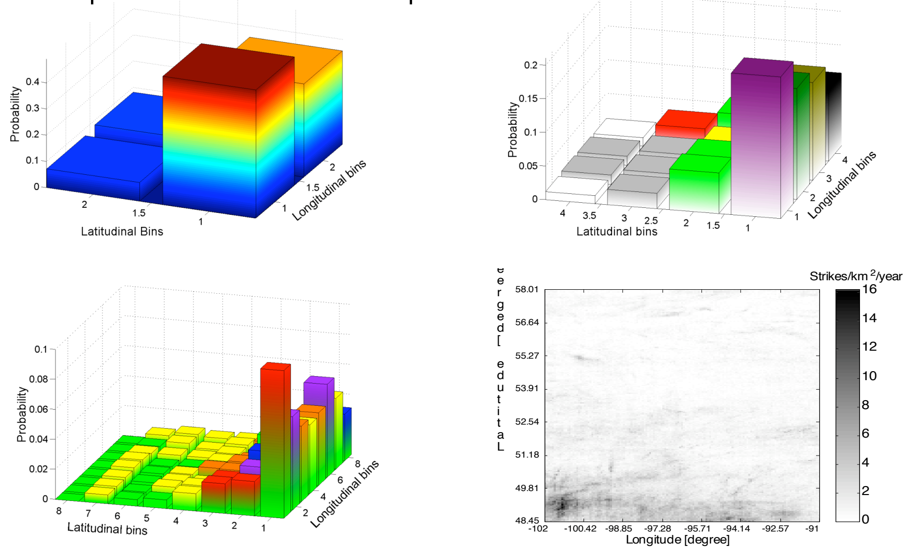
Standard deviation of the number of strikes over 5 day non-overlapping windows for the years: (a) 1998, (b) 1999, (c) 2000, (d) 2001, (e) 2002, (f) 2003, (g) 2004.



Standard deviation of the number of strikes over 15 day non-overlapping windows for the years: (a) 1998, (b) 1999, (c) 2000, (d) 2001, (e) 2002, (f) 2003, (g) 2004.

Multiscale PDF Estimation

Is it possible to find a relationship between PDFs of different scales?



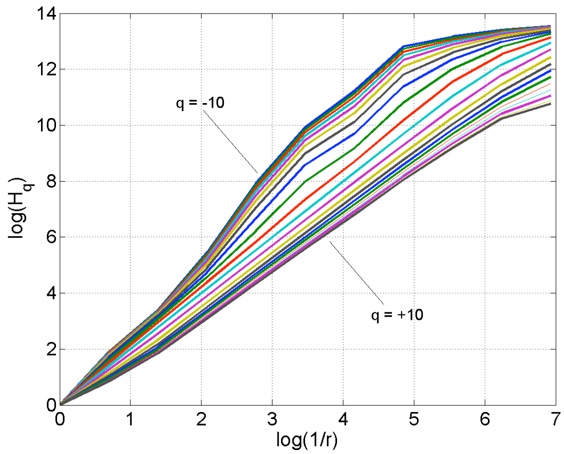
Multiscale histograms of the lightning strike map of Manitoba for the year 2001.

Entropy

- How to study a possible relationship between patterns at different scales?
  - Through their probability density functions (PDFs)
- How to measure PDFs?
  - Entropy, as a level of disorder in the object is meaningful physically, and can be used for measuring PDFs and comparing them to one another.
- Study of the pattern through PDFs using entropy
  - Multiple scales
  - Multiple statistical moments

Rényi Entropy (Generalized Entropy)

- Entropy over multiple scales and moments
  - Rényi entropy $$H_q(r) = \frac{1}{1-q}\log\sum_{j=1}^{N_r} p_{rj}^q \qquad (1)$$



Power law relationship of the entropies over multiple scales:
Multifractality [Kins03]

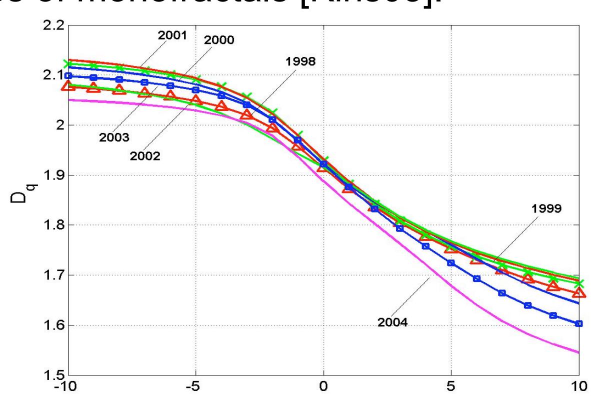
[FaKS04a] [FaKS04b] [FaKS05]

Fractals & Multifractals

- Repetition of patterns in multiple scales leads to fractality.
- Monofractals are self similar objects that their entropies hold power-law relationship over several scales.
- Multifractals are mixtures of monofractals [Kins03].

Dimensions of multifractal Object

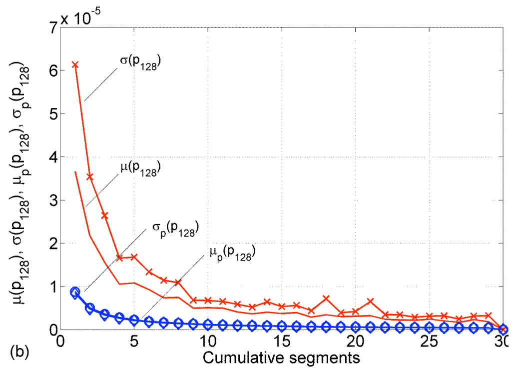
$$D_q = \lim_{r\to 0}\frac{H_q(r)}{\log(1/r)} \qquad (2)$$



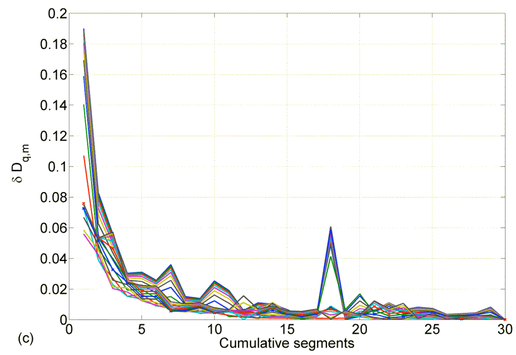
Rényi fractal dimension spectrum

Consistency of the Features

- Consistency of the features over time
  - Convergence of the features



Based on PDF



Based on the multifractal features

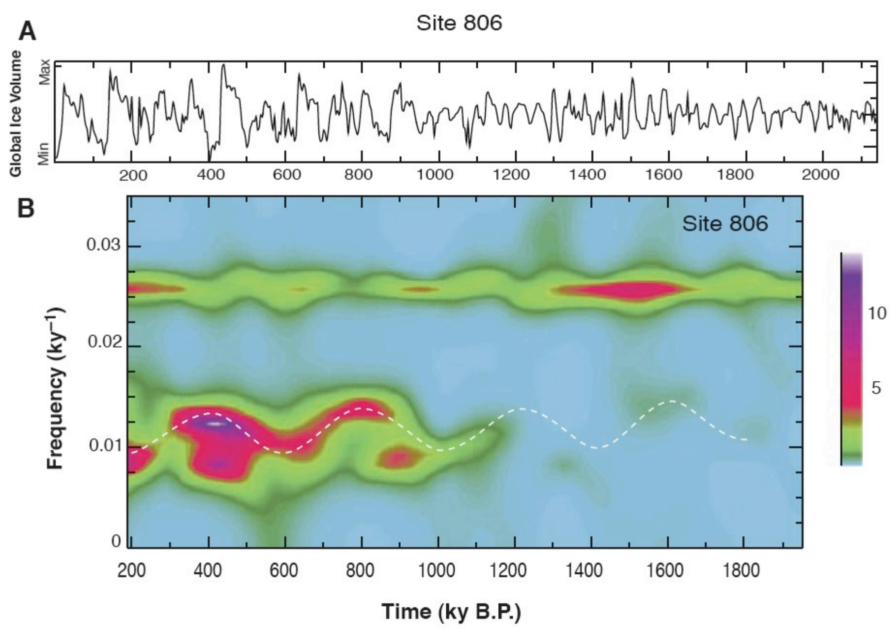
Conclusions (Study 1)

- Multiscale analysis of complicated phenomena (such as the LSMs) can extract information about the inter-dependence of their patterns, thus leading to richer features;
- Only 7 years of the data are enough to demonstrate a convergence in distribution for the multifractal features and thus, existence of an ultimate spatial distribution for the LSMs;
- Multifractal properties of the LSMs motivate using multifractal models such as percolation or cellular automata for modeling the LSMs;
- Both the ultimate spatial distribution and the rate of convergence can be used for tuning the multifractal model (i.e., either percolation, or cellular automata) parameters;
- Due to the universality of the strokes, the same approach can be used for them; and
- The same methods and techniques can be used for other similarly complex phenomena.

Study 2: Long-Term Data Analysis

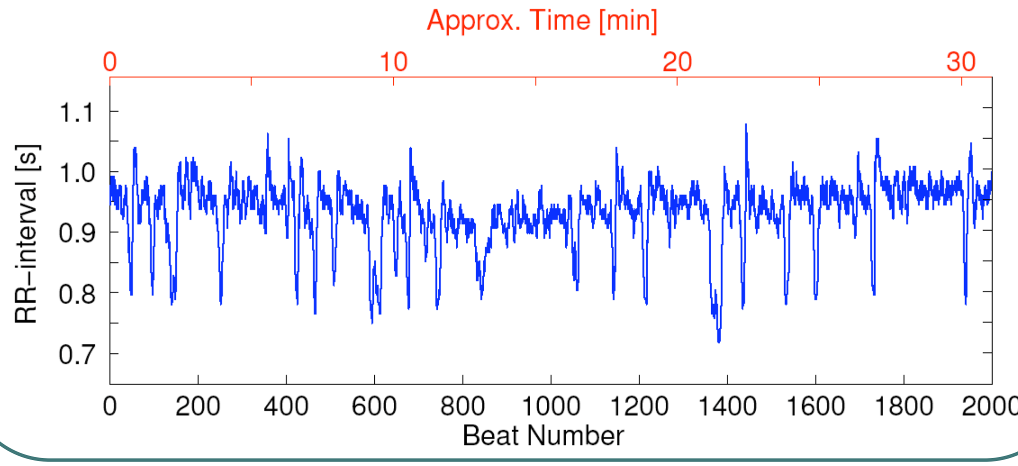
- Figure A shows the $\delta^{18}O$ record of the Ocean Drilling Program (ODP) Site 806 over a period of the last 2.1 million years (1 ky = 1,000 years). This record is time series obtained from a deep-sea sediments measured through the oxygen isotope. The records are a proxy for the global ice volume [Rial99].
- The data show that, during the last one million years, the Earth has gone through at least 10 major glaciations due to Earth's orbital eccentricity, axial tilt, and the longitude perihelion (distance from the Sun).
- Figure B shows a spectrogram obtained from a moving-window Fourier transform.
- The diagram suggests that the 100-ky eccentricity signal is modulated by the longer-period 413-ky signal.
- Study 2 shows how the Rial model could be improved.
- This approach could also be used to generate a time series either from partial data, or from features representing a time series.

Ice Volume Changes (2.1 m yrs)



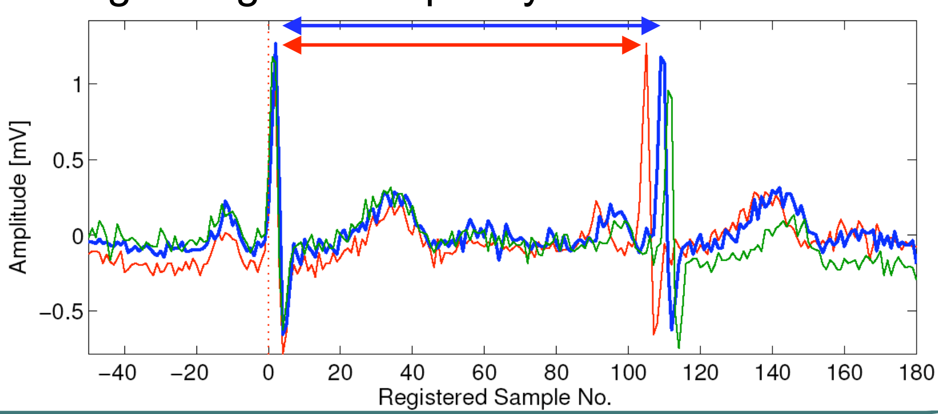
Study 2: Irregular Intervals in Data

- What if the data is not a uniformly sampled representation of a continuous time function?
  - *e.g.*, Inter-event intervals



Inter-Event Intervals

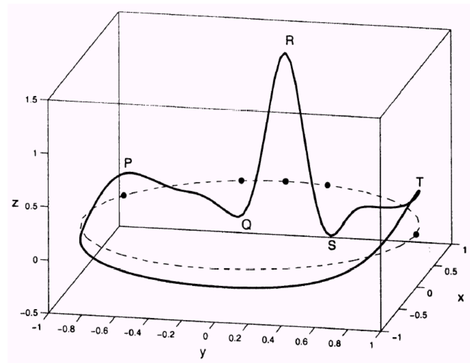
- Cyclic variables can exhibit variations in cycle length
- *Inter-event intervals* measure this variability using a single event per cycle



Cyclostationarity Inverse Model (1/3)

- A *cyclostationary* time function can be reconstructed from inter-event intervals by *frequency-modulated interpolation* [PoKi06]
- The marker events occur at the same phase θ modulo 2π
- The cyclic behaviour can be represented by a nonlinear dynamical system with a characteristic limit cycle z(θ)

$$z(t) = z \circ \theta(t)$$

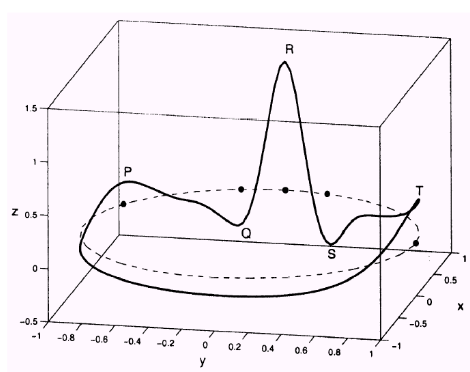


(From [MCTS03, 2003])

Cyclostationarity Inverse Model (2/3)

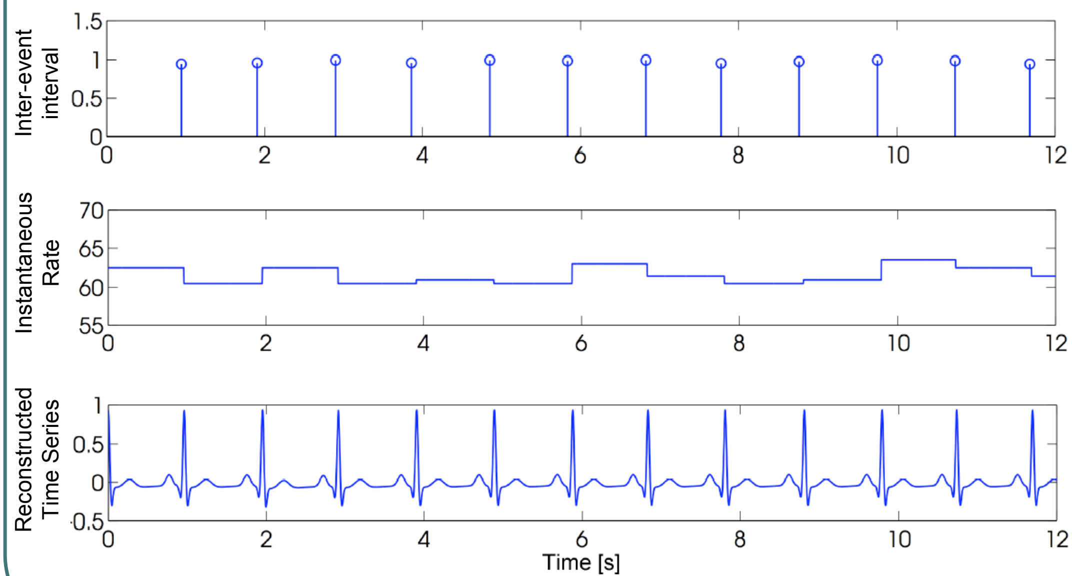
- The cumulative sum of inter-event interval times represent interpolation points for the function θ(t)
- The interpolation method can be optimized to account for the systematic properties of θ(t)
  - e.g., smoothness
- Gradient of the interpolation provides instantaneous rate for nonlinear dynamical system

$$z(t) = z \circ \theta(t)$$



(From [MCTS03, 2003])

Cyclostationarity Inverse Model (3/3)



Study 2: Conclusions

- Cyclic time series can be generated from data with irregular intervals using *frequency-modulated interpolation*
- Method applies to any cyclostationary data with irregularly spaced intervals
  - Climate change data
- Enables the benefits of well-sampled time series to apply

Closing Remarks

- Concerns related to climate change call for improvements of data analysis, as proposed in this paper.
- The improvements can be achieved by employing:
  - multiscale multifractal characterization of temporal and spatial data,
  - intelligent (relevant) feature extraction,
  - classification of long-range-dependence patterns in the data.
- Although the present weather and climate models are very impressive, they could also benefit from multiscale multifractal considerations.

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