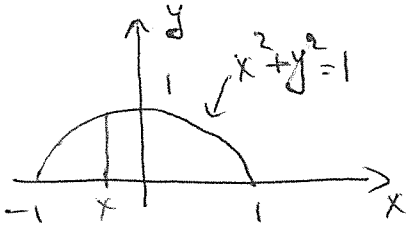
Name: **Student ID:**

3 pts. 1. Evaluate $\iint_D x^2 y\,dA$, where D is the part of the disk $x^2+y^2 \le 1$ above the x-axis.

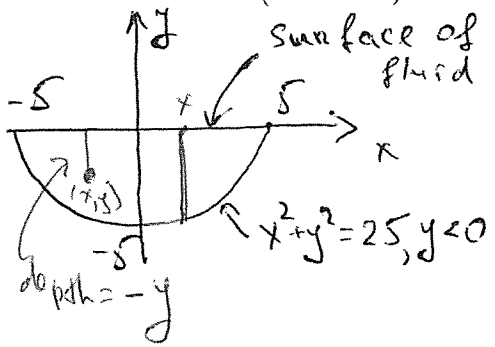


$$\iint_D x^2 y\,dA = \int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} x^2 y\,dy\,dx = \int_{-1}^{1} x^2\left[\left(\tfrac{1}{2}y^2\right)\Big|_{y=0}^{y=\sqrt{1-x^2}}\right]dx$$

$$= \int_{-1}^{1} \tfrac{1}{2}x^2(1-x^2)\,dx = \{\text{by symmetry + } x^2(1-x^2) \text{ is even}\}$$

$$= \int_0^1 x^2(1-x^2)\,dx = \int_0^1 (x^2-x^4)\,dx = \left(\tfrac{1}{3}x^3 - \tfrac{1}{5}x^5\right)\Big|_0^1 = \tfrac{1}{3}-\tfrac{1}{5} = \tfrac{2}{15}$$

3 pts. 2. A thin plate is submerged vertically in a fluid with density ρ. Find the force due to fluid pressure on one side of this plate. The plate: A semicircle of radius 5 with the (diameter) base in the surface.



$$F = \iint_D g\rho(-y)\,dA = \int_{-5}^{5}\int_{-\sqrt{25-x^2}}^{0} g\rho(-y)\,dy\,dx =$$

$$= g\rho\int_{-5}^{5}\left[-\tfrac{1}{2}y^2\Big|_{-\sqrt{25-x^2}}^{0}\right]dx =$$

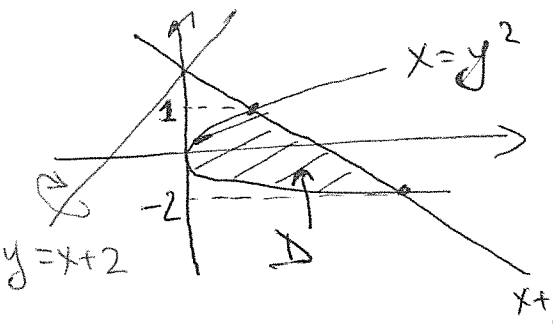
$$= g\rho\int_{-5}^{5}\tfrac{1}{2}(25-x^2)\,dx = g\rho\int_0^5 (25-x^2)\,dx =$$

$$= g\rho\left(25x-\tfrac{1}{3}x^3\right)\Big|_0^5 = g\rho\left(5^3-\tfrac{1}{3}\cdot 5^3\right) = \tfrac{2}{3}g\rho\cdot 5^3 = \tfrac{250}{3}\rho g$$

turn over ...

4 pts. 3. Let I be the second moment of area of the region bounded by $x = y^2$, $x + y = 2$ about $y - x = 2$.

2 pts. (a) Draw a picture and set up a double integral for I.



Pts of intersection: $y^2 + y - 2 = 0 \Rightarrow y = \frac{-1 \pm 3}{2}$, i.e., $y = -2$ or $y = 1$

$\text{Dist}\left((x,y), y-x-2=0\right) = \frac{|y-x-2|}{\sqrt{2}}$

$\therefore I = \iint_D \frac{(y-x-2)^2}{2} dA$, where

$D = \{(x,y) \mid -2 \le y \le 1 \text{ and } y^2 \le x \le 2-y\}$

1 pts. (b) Write the double integral from part (a) as an iterated double integral.

$$I = \int_{-2}^{1} \int_{y^2}^{2-y} \frac{1}{2}(y-x-2)^2 \, dx \, dy$$

1 pts. (c) Evaluate the inner integral in part (b), i.e., represent I is a univariate definite integral.

$$I = \int_{-2}^{1} \left[-\frac{1}{2} \cdot \frac{(y-x-2)^3}{3} \Big|_{x=y^2}^{x=2-y} \right] dy =$$

$$= -\frac{1}{6} \int_{-2}^{1} \left[(y-2-2+y)^3 - (y-y^2-2)^3 \right] dy =$$

$$= \frac{1}{6} \int_{-2}^{1} \left[(y-y^2-2)^3 - (2y-4)^3 \right] dy$$