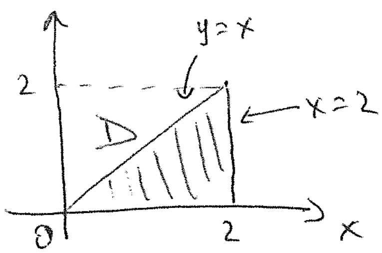
Name: **Student ID:**

Note: the total value of all problems is 13 points, but 10 points = 100% (hence, it is possible to receive 130% on this quiz).

3 pts. 1. Set up, but do not evaluate, a double iterated integral to find the area of the part of the surface $z = 2x^2 + 3y$ bounded by the planes $x = 2$, $y = 0$ and $y = x$.



$$\text{Surface area} = \iint_D \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\, dA =$$

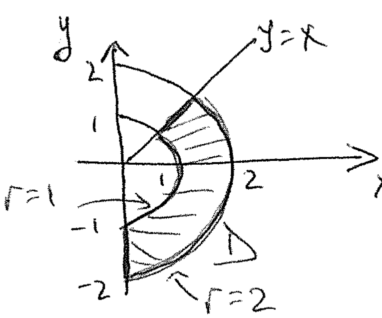
$$= \iint_D \sqrt{1+(4x)^2+3^2}\, dA = \iint_D \sqrt{10+16x^2}\, dA =$$

$$= \int_0^2 \int_0^x \sqrt{10+16x^2}\, dy\, dx$$

OR

$$= \int_0^2 \int_y^2 \sqrt{10+16x^2}\, dx\, dy$$

3 pts. 2. Evaluate $\iint_D \ln(x^2+y^2)dA$, where $D = \{(x,y) \mid 1 \le x^2+y^2 \le 4,\ x \ge 0,\ y \le x\}$.



D in polar coordinates: $\{(r,\theta) \mid 1 \le r \le 2, -\frac{\pi}{2} \le \theta \le \frac{\pi}{4}\}$

$$\iint_D \ln(x^2+y^2)dA = \int_{-\pi/2}^{\pi/4}\int_1^2 \ln(r^2)\, r\, dr\, d\theta =$$

$$= \left(\frac{\pi}{4}+\frac{\pi}{2}\right) I = \frac{3}{4}\pi I, \text{ where}$$

$$I = \int_1^2 r\ln(r^2)\,dr = \left\{\begin{matrix} u=r^2 \\ du = 2r\,dr\end{matrix}\right\} = \int_1^4 \frac{1}{2}\ln u\, du = \frac{1}{2}\int_1^4 u'\ln u\, du =$$

$$= \frac{1}{2}\left[u\ln u\Big|_1^4 - \int_1^4 1\cdot du\right] = \frac{1}{2}\left[4\ln 4 - \ln 1 - 3\right] = \frac{1}{2}\left[8\ln 2 - 3\right]$$

$$\therefore \text{Integral} = \frac{3}{4}\pi \cdot \frac{1}{2}(8\ln 2 - 3) = \frac{3}{8}\pi(8\ln 2 - 3)$$

turn over ...

3 pts. 3. Set up, but do not evaluate, a double iterated integral in polar coordinates for the area of that part of the surface $z = x^2 + y^2$ that is below the plane $z = 1$.

Surface $= \{(x,y,z) \mid z = x^2+y^2,\ z \le 1\} = \{(x,y,z) \mid z = x^2+y^2,\ x^2+y^2 \le 1\}$

$\therefore$ Surface area $= S = \iint_D \sqrt{1+(2x)^2+(2y)^2}\, dA$, where $D = \{(x,y) \mid x^2+y^2 \le 1\}$.

In polar coordinates, D is $\{(r,\theta) \mid 0 \le r \le 1,\ 0 \le \theta \le 2\pi\}$.

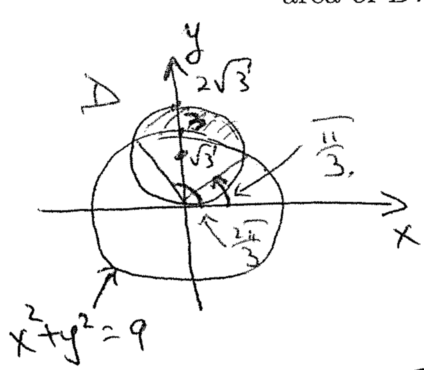
$$\therefore S = \int_0^{2\pi} \int_0^1 \sqrt{1+4r^2}\, r\, dr\, d\theta$$

4 pts. 4. Let D be the region outside $x^2 + y^2 = 9$ and inside $x^2 + y^2 = 2\sqrt{3}y$.

2 pts. (a) Sketch this region D.

2 pts. (b) Set up, but do not evaluate, a double iterated integral in polar coordinates for the area of D.

$x^2+y^2 = 2\sqrt{3}y \iff x^2 + (y-\sqrt{3})^2 = 3$



In polar coordinates:

Curve $x^2+y^2=9$: $r^2 = 9 \iff \boxed{r=3}$

Curve $x^2+y^2=2\sqrt{3}y$: $r^2 = 2\sqrt{3}\, r\sin\theta \iff \boxed{r = 2\sqrt{3}\sin\theta}$

Pts of intersection: $3 = 2\sqrt{3}\sin\theta \iff \sin\theta = \frac{\sqrt{3}}{2}$

$\iff \theta = \frac{\pi}{3}$ or $\theta = \frac{2\pi}{3}$.

$$\boxed{\text{Area} = \iint_D 1\, dA = \int_{\pi/3}^{2\pi/3} \int_3^{2\sqrt{3}\sin\theta} r\, dr\, d\theta}$$