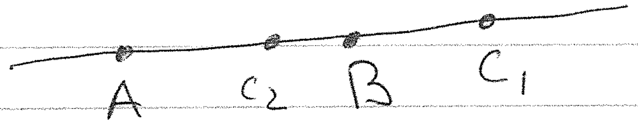
Lab 1: Sept. 10 2019.

#5. $A(1, 2, 3)$, $B(3, -2, -1)$

Find all C s.t. 1) C is on the line AB

2) $|AC| = 2|BC|$



Method 1: Let $C(a, b, c)$.

$$|AC| = \sqrt{(a-1)^2 + (b-2)^2 + (c-3)^2} \qquad |AC| = 2|BC|$$

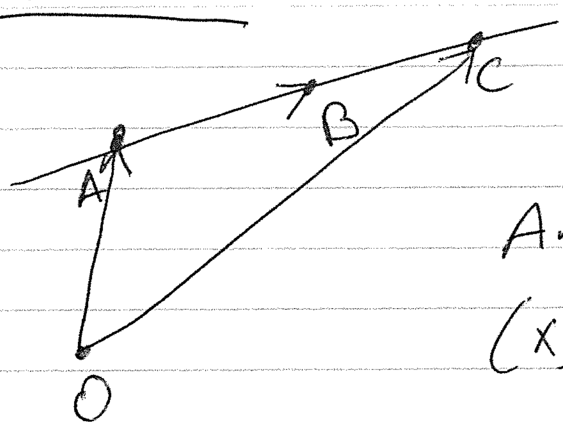
$$|BC| = \sqrt{(a-3)^2 + (b+2)^2 + (c+1)^2}$$

C_1: $|AC_1| = |AB| + |BC_1|$, i.e.,

$$\sqrt{(a-1)^2 + (b-2)^2 + (c-3)^2} = 6 + \sqrt{(a-3)^2 + (b+2)^2 + (c+1)^2}$$

$$\left\{ |AB| = \sqrt{(3-1)^2 + (-2-2)^2 + (-1-3)^2} = \sqrt{4 + 16 + 16} = 6 \right\}$$

Method 2:



$$\overline{OA} = (1, 2, 3)$$

$$\overline{AB} = (2, -4, -4)$$

Any point on the line AB:

$$(x, y, z) = \overline{OA} + t\,\overline{AB}$$

$$= (1, 2, 3) + t(2, -4, -4), \quad t \in \mathbb{R}$$

$$C: (x, y, z) = (1 + 2t, 2 - 4t, 3 - 4t), \quad t \in \mathbb{R}.$$

$$|AC|^2 = \| t(2,-4,-4) \|^2 = t^2(4+16+16) = 36t^2$$

$$|BC|^2 = (1+2t-3)^2 + (2-4t+2)^2 + (3-4t+1)^2 =$$
$$= (2t-2)^2 + (4-4t)^2 + (4-4t)^2$$
$$= 4(t-1)^2 + 32(1-t)^2 = 36(1-t)^2$$

$$|AC| = 2|BC| \iff |AC|^2 = 4|BC|^2$$

$36t^2 = 4 \cdot 36(1-t)^2 \iff t^2 = 4(1-t)^2$ ... solve for t.

<u>Sect 11.2</u> tutorial

<u>#14.</u> $z = 2x^2 + 4y^2, \quad y+z=1.$

Curve $= \{(x,y,z) \in \mathbb{R}^3 \mid z = 2x^2+4y^2,\ y+z=1\}$.

$$\begin{cases} z = 2x^2+4y^2 \\ y+z=1 \end{cases} \iff \begin{cases} z = 1-y \\ 1-y = 2x^2+4y^2 \end{cases}$$

$$\begin{cases} \underline{4y^2+y}+2x^2-1=0 \\ 4\left(y^2+\frac{1}{4}y\right) = 4\left(\left(y+\frac{1}{8}\right)^2 - \frac{1}{8^2}\right) = 4\left(y+\frac{1}{8}\right)^2 - \frac{1}{16} \end{cases}$$

$$\therefore \quad 4\left(y+\frac{1}{8}\right)^2 - \frac{1}{16} + 2x^2 - 1 = 0$$

$$\overset{?}{4}\left(y+\frac{1}{8}\right)^2 + 2x^2 = \frac{17}{16} \qquad (*)$$

$\therefore$ Curve $= \{(x,y,z) \in \mathbb{R}^3 \mid (*) \text{ is valid and } y+z=1\}$,

i.e., the curve is an ellipse.

Projection of this curve onto the xy-plane.

Projection $= \{ (x,y) \in \mathbb{R}^2 \mid$ for (x,y) there is z s.t. (x,y,z) is on the curve $\}$

$$= \{ (x,y) \in \mathbb{R}^2 \mid 1-y = 2x^2 + 4y^2 \}$$

$$= \{ (x,y) \in \mathbb{R}^2 \mid 4\left(y+\tfrac{1}{8}\right)^2 + 2x^2 = \tfrac{17}{16} \}.$$