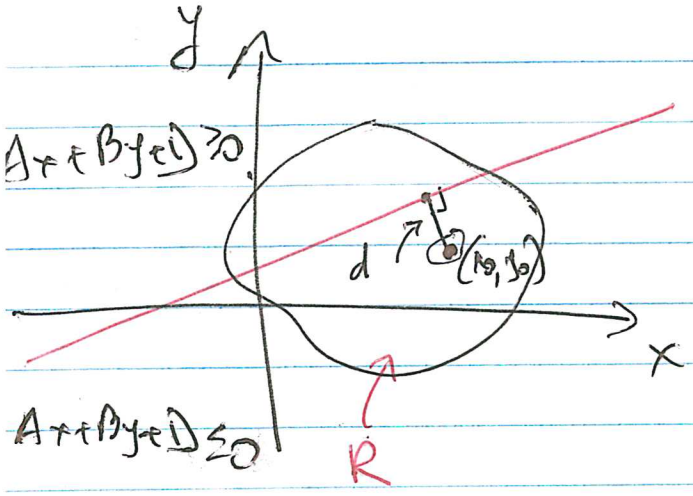
Lab 10 Nov. 19, 2019



$$d = \left\{ \begin{array}{l} Ax+By+0\cdot z+D=0 \\ \text{plane} \\ \text{point } d=(x_0,y_0,0) \end{array} \right\}$$

$$= \frac{|Ax_0+By_0+0\cdot 0+D|}{\sqrt{A^2+B^2+0^2}} =$$

$$= \frac{|Ax_0+By_0+D|}{\sqrt{A^2+B^2}}$$

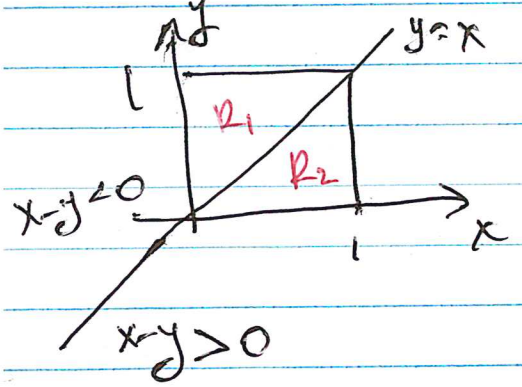
signed distance

$$M \approx \sum_{i,j} \rho(x_0,y_0)\, A_{ij}(x_0,y_0] \cdot d(x_0,y_0) = \frac{Ax_0+By_0+D}{\sqrt{A^2+B^2}}$$

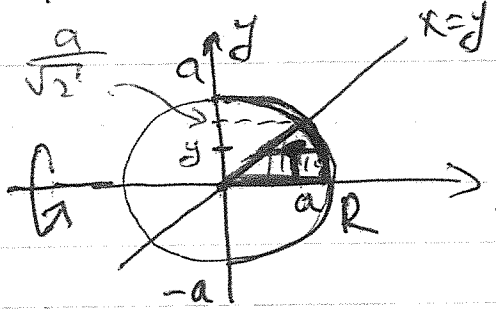
$$M = \iint_R \rho(x,y) \cdot \frac{(Ax+By+D)}{\sqrt{A^2+B^2}}\, dA$$

#42, p. 906

$$\int_0^1 \int_0^1 |x-y|\, dy\, dx = \iint_{R_1} [-(x-y)]\, dA + \iint_{R_2} (x-y)\, dA$$



Sect. 13.5, #4 Edges: $x=\sqrt{a^2-y^2}$, $y=x$, $y=0$



$$M = \iint \rho\, y \quad dA =$$

$$= \int_0^{\frac{a}{\sqrt{2}}} \int_y^{\sqrt{a^2-y^2}} \rho y \quad dx\,dy \in$$

$$\begin{cases} x=\sqrt{a^2-y^2} \\ x=y \end{cases} \Rightarrow \quad y=\sqrt{a^2-y^2}, \; y>0$$

$$y^2 = a^2-y^2 \Leftrightarrow 2y^2=a^2 \Leftrightarrow y^2=\frac{a^2}{2}$$

$$\Leftrightarrow y=\frac{a}{\sqrt{2}}.$$

$$\in \int_0^{\frac{a}{\sqrt{2}}} \rho y\left(\sqrt{a^2-y^2}-y\right) dy =$$

$$= \rho \int_0^{\frac{a}{\sqrt{2}}} \left[\left(y\sqrt{a^2-y^2}\right)-y^2\right] dy =$$

$$= \rho\left[\frac{2}{3}\left(a^2-y^2\right)^{\frac{3}{2}}\left(-\frac{1}{2}\right) - \frac{1}{3}y^3\right]\Bigg|_0^{\frac{a}{\sqrt{2}}} =$$

$$= \rho\left[-\frac{1}{3}\left[\left(\frac{a^2}{2}\right)^{\frac{3}{2}} - \left(a^2\right)^{\frac{3}{2}}\right] - \frac{1}{3}\left(\frac{a}{\sqrt{2}}\right)^3\right]$$

$$= \rho a^3\left[-\frac{1}{3\cdot 2\sqrt{2}} + \frac{1}{3} - \frac{1}{3\cdot 2\cdot\sqrt{2}}\right] =$$

$$= \rho a^3\left[\frac{1}{3} - \frac{1}{3\sqrt{2}}\right] = \frac{\rho a^3}{3\sqrt{2}}\left(\sqrt{2}-1\right).$$