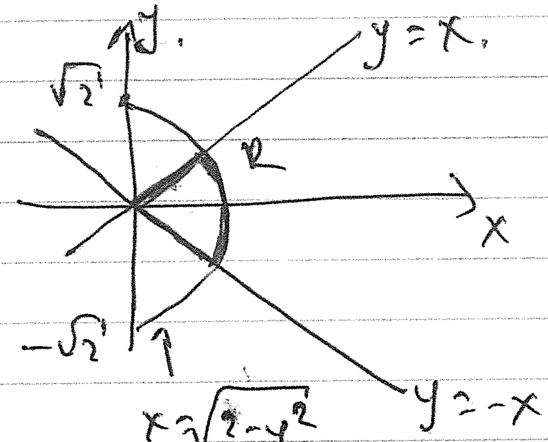
Lab 11. NOV. 26, 2019

#13, p. 939 Find the centroid: bounded by $y = x$, $y = -x$, $x = \sqrt{2 - y^2}$.



$\bar{y} = 0$ by symmetry.

$M\bar{x} = \iint_R x\, dA$, where

$M = \iint_R 1\, dA = \frac{1}{4} \cdot \pi (\sqrt{2})^2 = \frac{\pi}{2}$.

$$\iint_R x\, dA = \left\{ \begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \\ R = \{(r,\theta) \mid -\frac{\pi}{4} \le \theta \le \frac{\pi}{4},\ 0 \le r \le \sqrt{2}\} \end{array} \right\}$$

$$= \int_{-\frac{\pi}{4}}^{\pi/4} \int_0^{\sqrt{2}} r\cos\theta \cdot r\, dr\, d\theta =$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos\theta\, d\theta \cdot \int_0^{\sqrt{2}} r^2\, dr =$$

$$= \sin\theta \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cdot \frac{1}{3} r^3 \Big|_0^{\sqrt{2}} = \left[\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)\right] \cdot \frac{1}{3} \cdot 2\sqrt{2} =$$

$$= \sqrt{2} \cdot \frac{2}{3}\sqrt{2} = \frac{4}{3}$$

$\therefore \bar{x} = \frac{4}{3(\frac{\pi}{2})} = \frac{8}{3\pi}$.

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#18, p. 939 Find the area of $z = xy$ inside $x^2+y^2=9$.

$$S = \iint_D \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\, dA, \text{ where}$$

$$D = \{(x,y) \mid x^2+y^2 \le 9\}$$

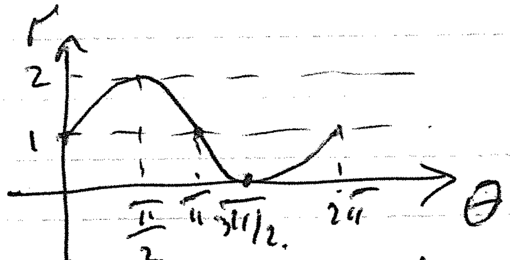
$$\therefore S = \iint_D \sqrt{1+y^2+x^2}\, dA = \left\{ \begin{array}{l} \text{Polar coord:} \\ x = r\cos\theta \\ y = r\sin\theta \\ D = \{(r,\theta) \mid \begin{array}{l} 0 \le \theta \le 2\pi \\ 0 \le r \le 3 \end{array}\} \end{array} \right\}$$

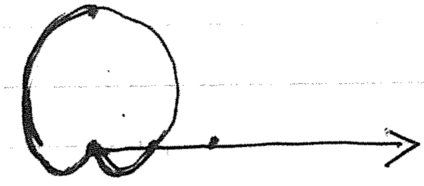
$$= \int_0^{2\pi}\int_0^3 \sqrt{1+r^2}\, r\, dr\, d\theta = 2\pi \int_0^3 r\sqrt{1+r^2}\, dr$$

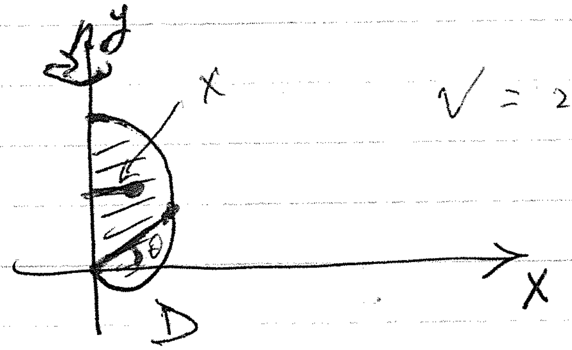
$$= 2\pi \cdot (1+r^2)^{\frac{3}{2}} \cdot \frac{2}{3} \cdot \frac{1}{2} \Big|_0^3 = \frac{2\pi}{3}\left[10^{\frac{3}{2}} - 1\right] =$$

$$= \frac{2\pi}{3}\left(10\sqrt{10} - 1\right).$$

#22, p. 939 Find the volume of the solid of revolution: region is bounded by $r = 1+\sin\theta$, about y-axis.







$$V = 2\pi x\, dA$$

Volume: $V = \iint_D 2\pi x \, dA \;=$

$$= \left\{ \begin{array}{l} \text{Polar coord.:} \\ x = r\cos\theta,\ y = r\sin\theta \\ D = \{(r,\theta) \mid -\frac{\pi}{2} \le \theta \le \frac{\pi}{2},\quad 0 \le r \le 1+\sin\theta\} \end{array} \right\}$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{1+\sin\theta} 2\pi \cdot r\cos\theta \cdot r \, dr \, d\theta =$$

$$= 2\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta \cdot \left[\frac{1}{3} r^3 \Big|_{r=0}^{r=1+\sin\theta} \right] d\theta =$$

$$= \frac{2\pi}{3} \int_{-\frac{\pi}{2}}^{\pi/2} \cos\theta \, (1+\sin\theta)^3 \, d\theta = \left\{ \begin{array}{l} u = 1+\sin\theta \\ du = \cos\theta \, d\theta \\ \text{---} \end{array} \right\}$$

$$= \ldots \text{ HW.}$$

Note: $r = 1+\sin\theta$ in Cartesian coordinates:

$$r^2 = r + \underbrace{r\sin\theta}_{y}$$

$$\boxed{x^2+y^2 = \sqrt{x^2+y^2} + y.}$$