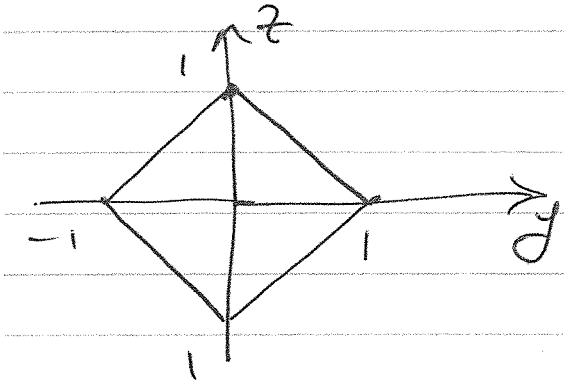
Lab 2 Sept. 17, 2019.

Sect. 11.2 (#5) $|z|+|y|=1$

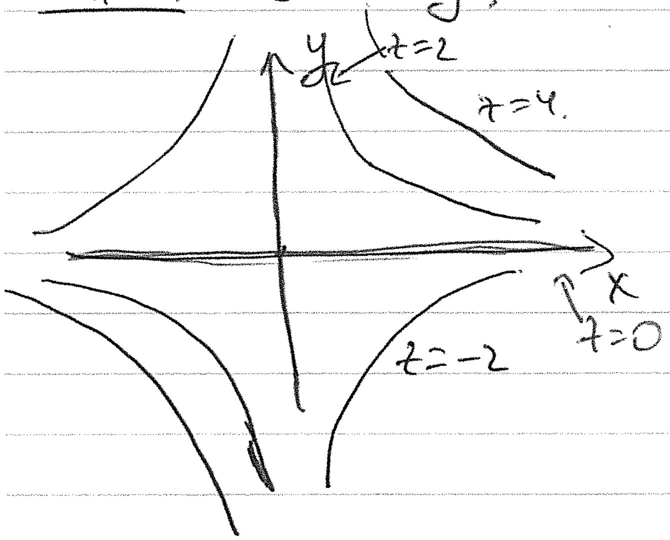
Sketch the curve $|z|+|y|=1$ in the yz-plane.



if $y \geq 0$ and $z \geq 0$, then

$z+y=1 \iff z=1-y$

#2. $z=2xy$.



Level sets.

if $z=0$: $x=0$ or $y=0$

$z=2$: $xy=1 \iff y=\frac{1}{x}$

$z=4$: $xy=2 \iff y=\frac{2}{x}$

$\{ y=x: \ z=2x^2 \}$

$z=-2$: $xy=-1 \iff y=-\frac{1}{x}$.

$\{ y=-x: \ z=-2x^2 \}$

#3. $z=|x+y|$ $\{ z=x+y \}$

#8. $x^2+y^2=2x-4y+5$

$\Updownarrow$

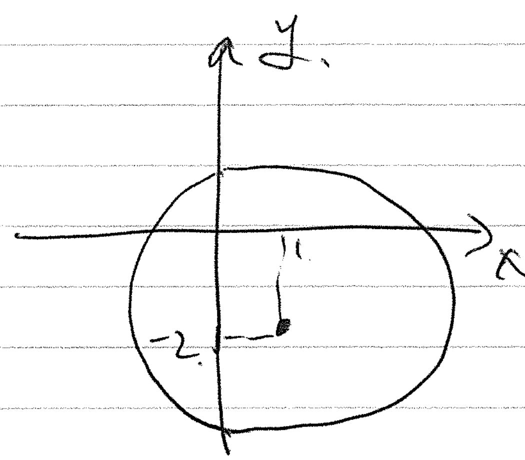
$x^2-2x+y^2+4y=5$

$\Updownarrow$

$(x-1)^2-1+(y+2)^2-4=5$

$\Updownarrow$

$(x-1)^2+(y+2)^2=10$



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#9 $x^2+y^2=2x-4y-5$

$\Updownarrow$

$(x-1)^2+(y+2)^2-5=-5$

$\Updownarrow$

$(x-1)^2+(y+2)^2=0 \iff \begin{cases} x=1 \\ y=-2 \end{cases}$

$S=\{(x,y,z) \mid x=1,\ y=-2,\ z\in\mathbb{R}\}$.

#17. $\begin{cases} z=x^2+y^2 \\ 2z=x^2 \end{cases}$ ⟵ curve.

$\Updownarrow$

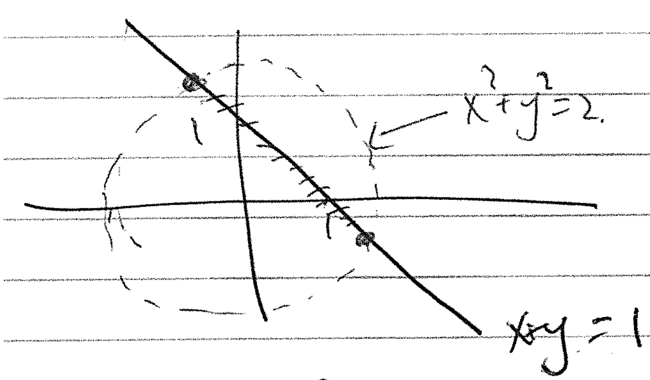
$\begin{cases} z=\frac{x^2}{2} \\ \frac{x^2}{2}=x^2+y^2 \end{cases} \iff \begin{cases} z=\frac{x^2}{2} \\ \frac{x^2}{2}+y^2=0 \end{cases} \iff \begin{cases} z=0 \\ x=y=0 \end{cases}$

$S=\{(x,y,z) \mid z=x^2+y^2 \text{ and } 2z=x^2\}$

$=\{(x,y,z) \mid x=y=z=0\}=\{(0,0,0)\}$.

#15. $\begin{cases} x^2+y^2+2z^2=2 & (*) \\ x+y=1 \end{cases}$

Projection onto **xy-plane**: $S=\{(x,y) \mid (*) \text{ has a sol-n for } z\}$



$2z^2=2-x^2-y^2$

$\Updownarrow$

$z=\pm\sqrt{\frac{2-x^2-y^2}{2}}$

exists if $2-x^2-y^2\geq 0$.

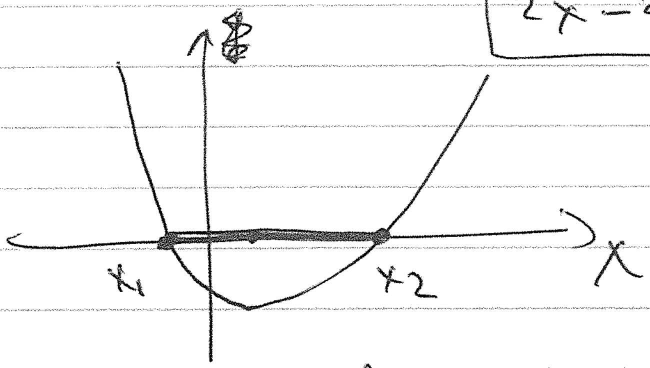
$S=\{(x,y) \mid x+y=1 \text{ and } x^2+y^2\leq 2\}$

$y = 1 - x \quad \therefore \quad x^2 + (1-x)^2 \le 2$

$x^2 + x^2 - 2x + 1 \le 2$

$\boxed{2x^2 - 2x - 1 \le 0} \iff x_1 \le x \le x_2,$

where x_1 and x_2 are solns of $2x^2 - 2x - 1 = 0$ with $x_1 < x_2$.



$$2x^2 - 2x - 1 = 0 \iff x = \frac{2 \pm \sqrt{4 + 4 \cdot 2}}{2 \cdot 2} = \frac{2 \pm \sqrt{12}}{4} = \frac{1 \pm \sqrt{3}}{2}.$$

$x_1 = \frac{1 - \sqrt{3}}{2}$ and $x_2 = \frac{1 + \sqrt{3}}{2}$.

$$S = \left\{ (x, y) \,\middle|\, y = 1 - x \text{ and } \frac{1-\sqrt{3}}{2} \le x \le \frac{1+\sqrt{3}}{2} \right\}.$$

Projection onto xz-plane.

$S_1 = \{(x, z) \mid (*) \text{ has a soln for } y\}.$

$$\begin{cases} x^2 + y^2 + 2z^2 = 2 \\ x + y = 1 \end{cases} \iff \begin{cases} y = 1 - x \\ x^2 + (1-x)^2 + 2z^2 = 2 \end{cases}$$

$\therefore \; S_1 = \{(x, z) \mid x^2 + (1-x)^2 + 2z^2 = 2\}$

$x^2 + 1 + x^2 - 2x + 2z^2 = 2 \iff 2x^2 - 2x + 2z^2 = 1$

$\iff x^2 - x + z^2 = \frac{1}{2} \iff \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + z^2 = \frac{1}{2} \iff$

$\iff \left(x - \frac{1}{2}\right)^2 + z^2 = \frac{3}{4}.$