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Sept. 24, 2019 **Lab 3**

Sect. 11.5 #2. Line 1: $\begin{cases} 2x+3y+4z=6 \\ x-2y+z=3 \end{cases}$

Line 2: $\frac{2x-1}{22}=\frac{y+2}{2}=\frac{1-z}{7}=t$ Q-n: find eq-n of plane containing both lines.

Lines 1 and 2 intersect each other iff the system

$$\begin{cases} 2x+3y+4z=6 \\ x-2y+z=3 \\ 2x-1=22t \\ y+2=2t \\ 1-z=7t \end{cases}$$ is consistent.

System of linear eq-s with 5 eq-s and 4 unknowns.

$$\begin{cases} x=\frac{22t+1}{2}=11t+\frac{1}{2} \\ y=2t-2 \\ z=1-7t \\ 22t+1+3(2t-2)+4(1-7t)=6 \\ 11t+\frac{1}{2}-2(2t-2)+1-7t=3 \end{cases}$$

Is there t satisfying both eq-s.

$-1=6$ $\therefore$ The system is inconsistent.

$\therefore$ Lines do not intersect each other.

Line 1: $\bar{n}_1=(2,3,4)$, $\bar{n}_2=(1,-2,1)$

Line 1 $\parallel$ $\bar{n}_1\times\bar{n}_2$

$$\bar{n}_1\times\bar{n}_2=\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 3 & 4 \\ 1 & -2 & 1 \end{vmatrix}=(11,2,-7)$$

Line 2: $\dfrac{x-\frac{1}{2}}{11} = \dfrac{y+2}{2} = \dfrac{z-1}{-7}$

Line 2 $\parallel (11, 2, -7)$ $\therefore$ Line 1 $\parallel$ Line 2.

$P(\frac{1}{2}, -2, 1)$ is on Line 2.

$Q(3, 0, 0)$ is on Line 1.

$\overline{PQ} = (3-\frac{1}{2}, 2, -1) = (\frac{5}{2}, 2, -1) \parallel$ plane.

$\therefore (5, 4, -2) \parallel$ plane.

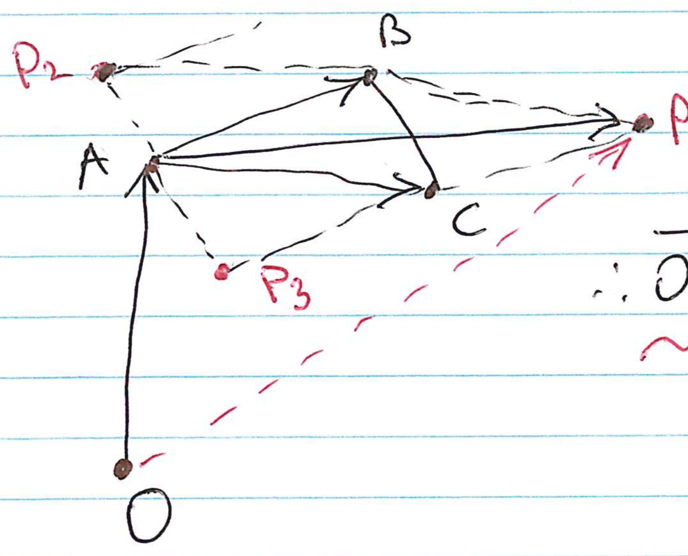
$\therefore$ Normal $(5,4,-2) \times (11,2,-7) =$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 5 & 4 & -2 \\ 11 & 2 & -7 \end{vmatrix} = (-24, 13, -34)$$

$$\therefore \boxed{-24(x-3) + 13y - 34z = 0}$$

use that plane passes through $Q(3,0,0)$.

Sect. 11.6 # 6.



$\overline{AP_1} = \overline{AB} + \overline{AC}$

$\overline{OP_1} = \overline{OA} + \overline{AP_1}$

$\therefore \overline{OP_1} = \overline{OA} + \overline{AB} + \overline{AC}$

Sect. 11.6 #4. $P(1,-1,2)$

$x+2y-5z=6$. Plane Q

$\bar{n}=(1,2,-5)$

$x+2y-5z+D=0$, $D\in\mathbb{R}$.

are $\parallel$ to plane Q for any D.

$\text{dist}(P,\ x+2y-5z+D=0)=1$

find all D satisfying this eq-n.

$$LHS=\frac{|x_0+2y_0-5z_0+D|}{\sqrt{1+4+25}}=\frac{|1-2-10+D|}{\sqrt{30}}=\frac{|D-11|}{\sqrt{30}}$$

$$\therefore\ \frac{|D-11|}{\sqrt{30}}=1 \iff |D-11|=\sqrt{30} \iff D-11=\pm\sqrt{30}$$

$$\iff D_1=11-\sqrt{30},\quad D_2=11+\sqrt{30}.$$

Eq-s: $x+2y-5z+(11\pm\sqrt{30})=0$ (2 planes)

Sect. 11.9, #2 $f(t)=t^2+1$

$$\bar{v}(t)=\left(e^t, \frac{t}{(t^2+1)^3}, -t\sqrt{t^2+1}\right)$$

$\int f(t)\,\bar{v}(t)\,dt$ - ?

$$\int\left((t^2+1)e^t, \frac{t}{(t^2+1)^2}, -t(t^2+1)^{\frac{3}{2}}\right)dt=$$

$$=\left(\int(t^2+1)e^t\,dt, \int\frac{t}{(t^2+1)^2}dt, -\int t(t^2+1)^{\frac{3}{2}}dt\right)$$

$$\int (t^2+1)e^t\,dt = \int (t^2+1)(e^t)'\,dt =$$

$$= (t^2+1)e^t - \int 2t(e^t)'\,dt =$$

$$= (t^2+1)e^t - \left[2te^t - \int 2e^t\,dt\right]$$

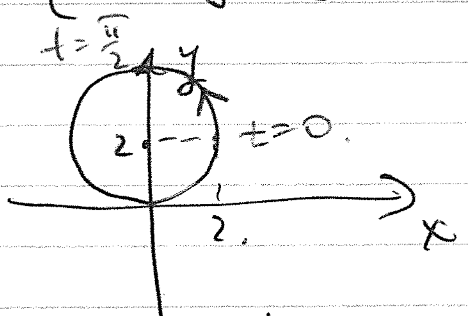
$$= (t^2+1)e^t - 2te^t + 2e^t + C =$$

$$= (t^2-2t+3)e^t + C.$$

<u>Sect. 11.10, #5</u>. $\begin{cases} z = x^2+y^2 \\ x^2+y^2-4y=0 \end{cases}$ Clockwise from far up z-axis.

$\Leftrightarrow$

$$\begin{cases} z = 4y \\ x^2+y^2-4y=0. \end{cases} \Leftrightarrow \begin{cases} z = 4y \\ x^2+(y-2)^2 = 4. \end{cases}$$



$$\begin{cases} x = 2\cos t \\ y = 2+2\sin t \\ z = 8+8\sin t, \quad t \in \mathbb{R} \end{cases}$$

Wrong orientation (!). $\therefore$ Replace t by $-t$. etc.