Oct. 1, 2019

Sect. 11.11 #5. Curve: $\begin{cases} x^2+y^2 = z^2-4 \\ x+y=4 \end{cases}$

$P_1(4, 0, 2\sqrt{5})$, $P_2(2, 2, 2\sqrt{3})$.

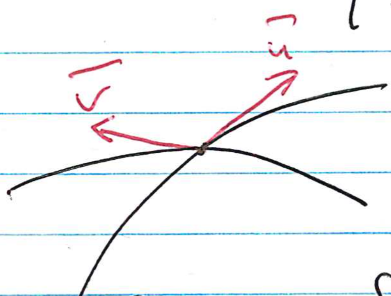
$\begin{cases} x^2+(4-x)^2 = z^2-4 \\ y = 4-x \end{cases}$ $\left\{ \begin{array}{l} x^2+16-8x+x^2+4 = z^2 \\ z^2 = 2x^2-8x+20 \\ z = \pm\sqrt{2x^2-8x+20} \\ z = \sqrt{2x^2-8x+20}, \quad z \geq 0 \end{array} \right\}$.

Curve: $\begin{cases} x = t \\ y = 4-t \\ z = \sqrt{2t^2-8t+20} \end{cases}$ $2 \leq t \leq 4.$

$$L = \int_2^4 \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\, dt = \ldots$$

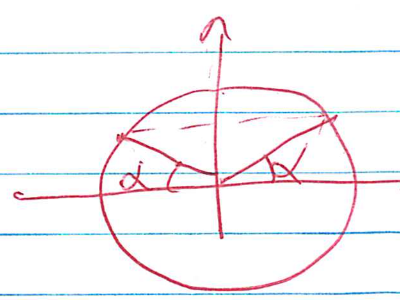
Sect. 11.4 #2. Curve 1: $\begin{cases} x^2+y^2 = z+4 \\ x+2y=5 \end{cases}$

Curve 2: $\begin{cases} x+y^2 = 5 \\ 2x+3y+4z=4. \end{cases}$



$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cos\alpha.$

$$\alpha = \cos^{-1} \frac{|\vec{u} \cdot \vec{v}|}{|\vec{u}| \cdot |\vec{v}|}.$$



Curve 1: $\begin{cases} x = t \\ y = \frac{5-t}{2} \\ z = t^2 - \frac{5-t}{2} - 4 \quad \ldots \end{cases}$

Sect. 11.11 #1. $\begin{cases} x^2+z^2=4 \\ x+y=1 \end{cases}$ $P(\sqrt{2}, 1-\sqrt{2}, \sqrt{2})$

$$\begin{cases} x = 2\cos t \\ z = 2\sin t \\ y = 1-2\cos t, \ t \in \mathbb{R}. \end{cases} \qquad t = \frac{\pi}{4} \leftarrow P$$

$$\bar{r}(t) = (2\cos t, 2\sin t, 1-2\cos t)$$

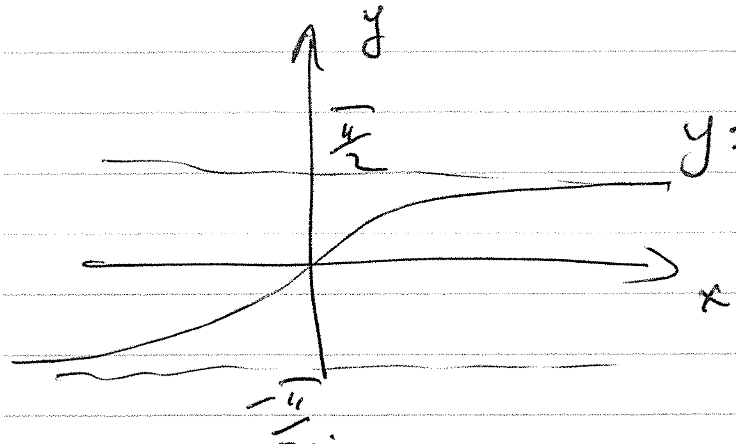
$$\bar{r}'(t) = (-2\sin t, 2\cos t, 2\sin t)$$

Unit tangents $= \pm \dfrac{\bar{r}'(t)}{|\bar{r}'(t)|} = \pm \dfrac{(-\sin t, \cos t, \sin t)}{\sqrt{2\sin^2 t + \cos^2 t}}$

at $t = \frac{\pi}{4}$: $\hat{T}\left(\frac{\pi}{4}\right) = \pm \dfrac{\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)}{\sqrt{1+\frac{1}{2}}} =$

$$= \pm\sqrt{\frac{2}{3}}\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \pm\frac{1}{\sqrt{3}}(-1, 1, 1).$$

Sect. 12.2 #7.



$$\lim_{(x,y)\to(0,1)} \mathrm{Tan}^{-1}\left|\frac{y}{x}\right| = \left\{ \left|\frac{y}{x}\right| \to +\infty \right\} = \lim_{z\to+\infty} \mathrm{Tan}^{-1}|z| = \frac{\pi}{2}.$$

Sect. 12.1 #2. (i) Radius $= C$: $x^2+y^2 = C^2 \ (z = C)$

$z = f(x,y)$ where $z = C$.

$$x^2+y^2 = z^2 \Rightarrow z = \sqrt{x^2+y^2}$$

$$f(x,y) = \sqrt{x^2+y^2}$$

$$\left\{ \begin{array}{l} f(x,y) = -\sqrt{x^2+y^2} \\ -\sqrt{x^2+y^2} = C, \ C < 0 \end{array} \right| \ x^2+y^2 = C^2, \ C<0 \Big\}$$

(ii) Radius $= C^2$: $x^2+y^2 = (C^2)^2$

$$C = \sqrt[4]{x^2+y^2}, \ f(x,y) = \sqrt[4]{x^2+y^2}$$

(iv) Radius $= \ln C$: $x^2+y^2 = (\ln C)^2$

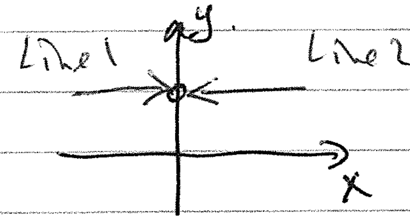
Solve for C: $\ln C = \sqrt{x^2+y^2} \Leftrightarrow C = e^{\sqrt{x^2+y^2}}$.

$$\therefore f_1(x,y) = e^{\sqrt{x^2+y^2}}$$

$$\ln C = -\sqrt{x^2+y^2} \Rightarrow C = e^{-\sqrt{x^2+y^2}}$$

$$f_2(x,y) = e^{-\sqrt{x^2+y^2}}.$$

Ex: $\lim_{(x,y)\to(0,1)} \mathrm{Tan}^{-1}\left(\frac{y}{x}\right)$.



1) $\lim_{\substack{(x,y)\to(0,1) \\ y=1 \\ x<0}} \mathrm{Tan}^{-1}\left(\frac{y}{x}\right) = \lim_{x\to 0^-} \mathrm{Tan}^{-1}\left(\frac{1}{x}\right)$

$$= \lim_{z\to-\infty} \mathrm{Tan}^{-1}(z) = -\frac{\pi}{2}$$

2) $\lim_{\substack{(x,y)\to(0,1) \\ y=1 \\ x>0}} \mathrm{Tan}^{-1}\left(\frac{y}{x}\right) = \lim_{x\to 0^+} \mathrm{Tan}^{-1}\left(\frac{1}{x}\right) = \lim_{z\to+\infty} \mathrm{Tan}^{-1}(z) = \frac{\pi}{2}$

$\frac{\pi}{2} \neq -\frac{\pi}{2}$ $\therefore$ DNE

Sect. 12.2 #5.

$$\lim_{(x,y)\to(2,-3)} \frac{x^2-4x-y^2-6y-5}{x^2-4x+y^2+6y+13} =$$

$$= \lim_{(x,y)\to(2,-3)} \frac{(x-2)^2-(y+3)^2}{(x-2)^2+(y+3)^2} = \left\{ \begin{array}{l} x_1 = x-2 \\ y_1 = y+3 \end{array} \right\} =$$

$$= \lim_{(x_1,y_1)\to(0,0)} \frac{x_1^2-y_1^2}{x_1^2+y_1^2} = \lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}.$$

Sect. 12.2 #6 $\displaystyle\lim_{(x,y)\to(3,2)} \frac{\sin(2x-3y)}{2x-3y}$ =

$$\lim_{(x,y)\to(3,2)} (2x-3y) = 0$$

= $\boxed{\displaystyle\lim_{z\to 0} \frac{\sin z}{z} = 1}$