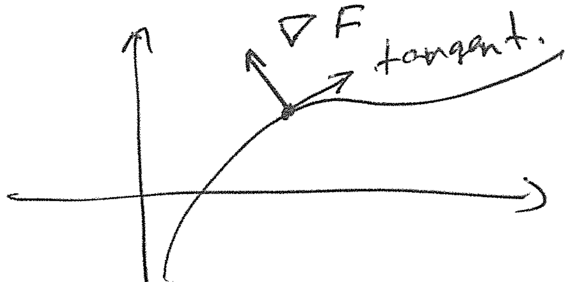
Oct. 8, 2019 Lab 5. 15

#5, Sect. 12.4: $F(x,y) = x^3y^2 + 3xy - 4$

$F(x,y) = 0$ ← curve in the xy-plane.



$$\nabla F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}\right) = (3x^2y^2 + 3y,\ 2x^3y + 3x)$$

$\bar{r}(t) = (x(t), y(t))$ parametric eq-s.

$$F(x(t), y(t)) \equiv 0$$

Tangent $\bar{r}'(t) = (x'(t), y'(t))$

Differentiate $x^3y^2 + 3xy - 4 = 0$ w/r t

(x, y are f-s of t).

$$3x^2y^2\, x' + 2x^3y \cdot y' + 3y\, x' + 3xy' = 0$$

$$(3x^2y^2 + 3y)\, x' + (2x^3y + 3x)\, y' = 0$$

$$\nabla F(x(t), y(t)) \cdot \underbrace{(x'(t), y'(t))}_{\bar{r}'(t)} = 0$$

$\nabla F(x,y) \perp$ curve at any (x,y) on the curve.

#2, Sect. 12.3 $\quad f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ 16

$$\lim_{\substack{(x,y)\to(0,0) \\ \text{along } y=mx}} f(x,y) = \lim_{x\to 0} f(x,mx) =$$

$$= \lim_{x\to 0} \frac{2x \cdot mx}{x^2+(mx)^2} = \lim_{x\to 0} \frac{2m}{1+m^2} = \frac{2m}{1+m^2}$$

depends on m $\quad \therefore \lim_{(x,y)\to(0,0)} f(x,y)$ DNE

$\therefore f$ is discontinuous at $(0,0)$

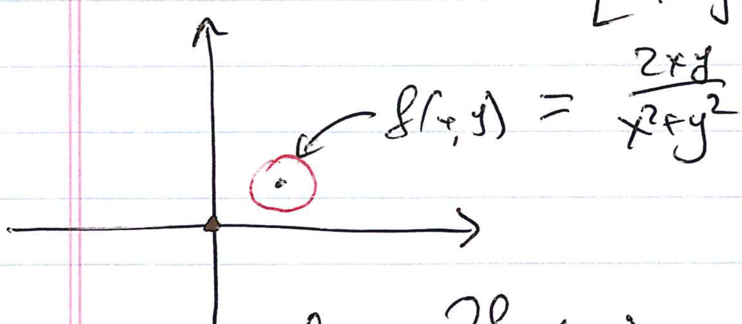
$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{f(0+h,0)-f(0,0)}{h} =$$

$$= \lim_{h\to 0} \frac{f(h,0)-0}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \ldots = 0.$$

If $(x,y) \neq (0,0)$, then $\frac{\partial f}{\partial x}(x,y)$ is

$$\frac{\partial f}{\partial x}(x,y) = \frac{\partial}{\partial x}\left[\frac{2xy}{x^2+y^2}\right] = 2y \frac{\partial}{\partial x}\left[\frac{x}{x^2+y^2}\right] =$$



$$= 2y \cdot \frac{x^2+y^2-x\cdot 2x}{(x^2+y^2)^2} =$$

$$= \frac{2y(y^2-x^2)}{(x^2+y^2)^2}.$$

$$\lim_{(x,y)\to(0,0)} \frac{\partial f}{\partial x}(x,y) = \lim_{(x,y)\to(0,0)} \frac{2y(y^2-x^2)}{(x^2+y^2)^2} = \text{DNE}$$

e.g. along $x=0$

#4, Sect. 12.4

$$\nabla F(x,y,z) = (2xy^3 + yz e^{xyz},\ 3x^2y^2 + xz e^{xyz},\ xy e^{xyz} + y).$$

Find F (if any).

$$\frac{\partial F}{\partial x} = 2xy^3 + yz e^{xyz}$$

Integrate wrt x:

$$F(x,y,z) = x^2y^3 + e^{xyz} + g(y,z)$$

$$\frac{\partial F}{\partial y} = \cancel{3x^2y^2 + xz e^{xyz}} + \frac{\partial g}{\partial y}(y,z)$$

$$\cancel{3x^2y^2 + xz e^{xyz}}$$

$$\frac{\partial g}{\partial y}(y,z) = 0$$

$\Downarrow$ Int. wrt y

$$g(y,z) = 0 + h(z) = h(z)$$

$$\therefore\ F(x,y,z) = x^2y^3 + e^{xyz} + h(z)$$

$$\frac{\partial F}{\partial z} = \cancel{xy e^{xyz}} + h'(z)$$

$$\cancel{xye^{xyz}} + y$$

$$\Downarrow$$

$$\underbrace{h'(z)} = y \quad \text{cannot happen}$$

Is a f-n of z only.

$\therefore$ Such F DNE.

18

#7, Sect. 12.5. $\mathrm{Tan}^{-1}\left(\frac{y}{x}\right)$ is harmonic if ~~$(x,y)\neq(0,0)$~~. $x\neq 0$.

$$\overset{f(x,y)}{\overset{\shortparallel}{}}$$

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \equiv 0$$

$$\frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{-y}{x^2} = -\frac{y}{x^2+y^2}.$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{y}{(x^2+y^2)^2} \cdot 2x = \frac{2xy}{(x^2+y^2)^2}.$$

$$\frac{\partial f}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x} = \frac{x}{x^2+y^2}$$

$$\frac{\partial^2 f}{\partial y^2} = -\frac{x}{(x^2+y^2)^2} \cdot 2y = -\frac{\partial^2 f}{\partial x^2}$$

$$\therefore \Delta f(x,y) \equiv 0 \text{ if } \cancel{(x,y)\neq(0,0)}\ x\neq 0.$$