Oct. 15, 2019 Lab 6

#3, Sect. 12.6 $\nabla f(x^2-y^2)\cdot\nabla g(xy)=0$.

$$\frac{\partial f}{\partial x}=f'(x^2-y^2)\cdot\frac{\partial}{\partial x}(x^2-y^2)$$ $f=f(s)$

$$=f'(x^2-y^2)\cdot 2x$$ $g=g(t)$.

$$\frac{\partial f}{\partial y}=f'(x^2-y^2)\cdot(-2y)$$

$$\nabla f(x^2-y^2)=f'(x^2-y^2)(2x,-2y)$$

$$\frac{\partial g}{\partial x}=g'(xy)\cdot y$$

$$\frac{\partial g}{\partial y}=g'(xy)\cdot x$$

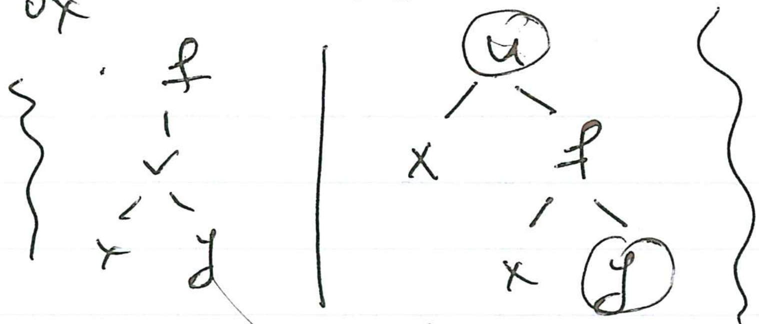
$$\nabla g(xy)=g'(xy)(y,x)$$

$$\therefore\ \nabla f(x^2-y^2)\cdot\nabla g(xy)=f'(x^2-y^2)g'(xy)\cdot\underbrace{(2x,-2y)\cdot(y,x)}_{=2xy-2yx=0}=0.$$

#30, p. 832 $f=f(v)$, $u(x,y)=x^2f\left(\frac{y}{x}\right)$

Show: $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u$

$$\frac{\partial u}{\partial x} = 2x f\left(\frac{y}{x}\right) + x^2 \cdot f'\left(\frac{y}{x}\right) \cdot \frac{-y}{x^2} =$$



$$= 2x f\left(\frac{y}{x}\right) - y f'\left(\frac{y}{x}\right)$$

$$\frac{\partial u}{\partial y} = x^2 f'\left(\frac{y}{x}\right) \cdot \frac{1}{x} = x f'\left(\frac{y}{x}\right),$$

$$\underline{x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}} = 2x^2 f\left(\frac{y}{x}\right) \cancel{- xy f'\left(\frac{y}{x}\right)}$$

$$\cancel{+ xy f'\left(\frac{y}{x}\right)} = 2x^2 f\left(\frac{y}{x}\right) = \underline{2u.}$$

<u>#20, p. 839</u> $\begin{cases} x = u^2 - v^2 \\ y = 2uv \end{cases}$ define u and v as f-s of x and y

Find $\frac{\partial^2 u}{\partial x^2}$.

$$F(x, y, u, v) = x - u^2 + v^2 \qquad \begin{cases} F = 0 \\ G = 0 \end{cases}$$

$$G(x, y, u, v) = y - 2uv$$

$$\frac{\partial u}{\partial x} = - \frac{\frac{\partial(F, G)}{\partial(x, v)}}{\frac{\partial(F, G)}{\partial(u, v)}} = - \frac{\begin{vmatrix} 1 & 2v \\ 0 & -2u \end{vmatrix}}{\begin{vmatrix} -2u & 2v \\ -2v & -2u \end{vmatrix}} =$$

$$= - \frac{-2u}{4u^2 + 4v^2} = \frac{u}{2(u^2 + v^2)}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left[\frac{u}{2(u^2+v^2)}\right] =$$

$$= \frac{1}{2} \cdot \frac{\frac{\partial u}{\partial x} \cdot (u^2+v^2) - u \cdot \left(2u\frac{\partial u}{\partial x} + 2v\frac{\partial v}{\partial x}\right)}{(u^2+v^2)^2} =$$

$$= \frac{1}{2(u^2+v^2)^2}\left[(v^2-u^2)\underbrace{\frac{\partial u}{\partial x}}_{\text{known}} - 2uv\underbrace{\frac{\partial v}{\partial x}}_{?}\right]$$

Need to find $\frac{\partial v}{\partial x}$ (we'll use method 2 for a change)

$$\begin{cases} 1 = 2u\frac{\partial u}{\partial x} - 2v\frac{\partial v}{\partial x} \\ 0 = 2v\frac{\partial u}{\partial x} + 2u\frac{\partial v}{\partial x} \end{cases}$$

$$\begin{cases} v\frac{\partial u}{\partial x} + u\frac{\partial v}{\partial x} = 0 \\ u\frac{\partial u}{\partial x} - v\frac{\partial v}{\partial x} = \frac{1}{2}. \end{cases}$$

$$\frac{dv}{dx} = -\frac{v}{u}\frac{\partial u}{\partial x} = -\frac{v}{u} \cdot \frac{u}{2(u^2+v^2)} =$$

$$= -\frac{v}{2(u^2+v^2)}.$$

$$\therefore \frac{\partial^2 u}{\partial x^2} = \frac{1}{2(u^2+v^2)^2}\left[(v^2-u^2)\frac{u}{2(u^2+v^2)} + \right.$$

$$\left. + 2uv \cdot \frac{v}{2(u^2+v^2)}\right] = \ldots$$

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#22, p. 839 $\begin{cases} x^2-2y^2s^2t-2st^2=1 \\ x^2+2y^2s^2t+5st^2=1 \end{cases}$ $\Leftrightarrow$

define s and t as f-s of x and y.

Find $\dfrac{\partial^2 t}{\partial y^2}$.

$\Leftrightarrow \begin{cases} 2x^2+3st^2=2 \\ 4y^2s^2t+7st^2=0 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 2x^2+3st^2=2 \\ st(4y^2s+7t)=0 \end{cases}$

$s\equiv 0$ and $t\equiv 0$ are NOT sol-s

$\therefore \begin{cases} 2x^2+3st^2=2 \\ 4y^2s+7t=0 \end{cases}$

$F(x,y,s,t)=2x^2+3st^2-2$

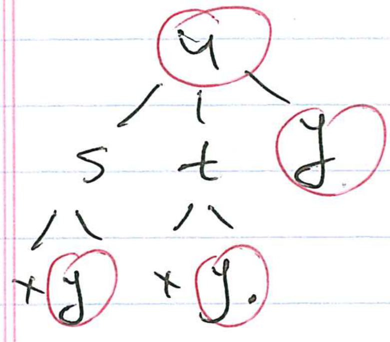
$G(x,y,s,t)=4y^2s+7t$

$$\frac{\partial t}{\partial y}=-\frac{\frac{\partial(F,G)}{\partial(s,y)}}{\frac{\partial(F,G)}{\partial(s,t)}} = -\frac{\begin{vmatrix} 3t^2 & 0 \\ 4y^2 & 8ys \end{vmatrix}}{\begin{vmatrix} 3t^2 & 6st \\ 4y^2 & 7 \end{vmatrix}}$$

$$= -\frac{24st^2y}{21t^2-24y^2st}$$

2)

$$\frac{\partial^2 t}{\partial y^2} = \frac{\partial}{\partial y}\left[- \frac{24 s t^2 y}{\underbrace{21 t^2 - 24 y^2 s t}_{u}} \right].$$



$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial s}\cdot\frac{\partial s}{\partial y} + \frac{\partial u}{\partial t}\cdot\frac{\partial t}{\partial y} + \left.\frac{\partial u}{\partial y}\right)_{s,t}$$