Lab 7 Oct. 27, 2019

$$D_{\bar v} f(a,b,c) = \nabla f(a,b,c)\cdot \bar v, \text{ where } |\bar v| = 1$$

$$= |\nabla f(a,b,c)|\cdot \underbrace{|\bar v|}_{=1} \cos\alpha$$

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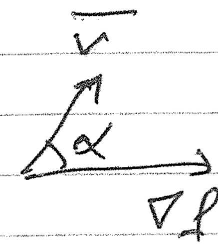
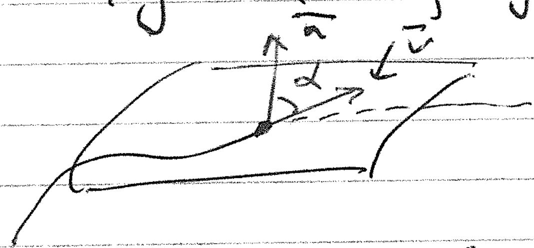


#3, Sect. 12.9

$x+z=3$ ← surface

$xy^3z+z^3=6,\ xy+yz=-3$ ← curve



$F(x,y,z) = x+z-3$

$F(x,y,z)=0$ ← surface.

$\bar n = \nabla F = (1,0,1)$

Find pt. of intersection:

$$\begin{cases} x+z=3 \\ xy^3z+z^3=6 \\ xy+yz=-3 \end{cases}$$

Eq. 3: $y\underbrace{(x+z)}_{3 \leftarrow \text{eq. 1}}=-3$

$\therefore \boxed{y=-1}$

$$\begin{cases} y=-1 \\ x+z=3 \\ -xz+z^3=6 \end{cases}$$

$x = 3-z$

$\Downarrow$

$z^3-(3-z)z=6 \iff z^3+z^2-3z-6=0$

$\{\pm1, \pm2, \pm3, \pm6\}$ $\underline{z=2 \text{ is a root}}$

$z^3+z^2-3z-6=(z-2)(z^2+3z+3)$

$z^2+3z+3=0 \iff z=\dfrac{-3\pm\sqrt{9-4\cdot 3}}{2}$ ← <0

$\therefore$ No sol-s.

Point of intersection: $(1,-1,2)$.

$G(x,y,z)=xy^3z+z^3-6$ Curve: $\begin{cases} G=0 \\ H=0 \end{cases}$

$H(x,y,z)=xy+yz+3$

$\overline{n_1}=\nabla G(1,-1,2)$

Then $\overline{v} \parallel \overline{n_1\times n_2}$

$\overline{n_2}=\nabla H(1,-1,2)$

$\nabla G=(y^3z,\ 3xy^2z,\ xy^3+3z^2)$

$\nabla G(1,-1,2)=(-2,6,-1+12)=(-2,6,11)=\overline{n_1}$

$\nabla H=(y,\ x+z,\ y)$

$\nabla H(1,-1,2)=(-1,3,-1)=\overline{n_2}$

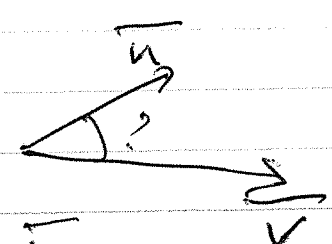
$$\overline{n_1}\times\overline{n_2}=\begin{vmatrix}\overline{i} & \overline{j} & \overline{k}\\ -2 & 6 & 11\\ -1 & 3 & -1\end{vmatrix}=(-6-33,\ -(2+11),\ -6+6)$$

$$=(-39,-13,0)=$$

$$=-13(3,1,0)$$

Take $\overline{v}=(3,1,0)$

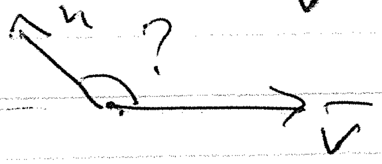
$\overline{n}=(1,0,1)$.



$\overline{n}\cdot\overline{v}=3$

$\parallel$

$|\overline{n}|\cdot|\overline{v}|\cos\alpha$



$3 = \sqrt{2} \cdot \sqrt{10} \cos\alpha \iff \cos\alpha = \frac{3}{\sqrt{20}} = \frac{3}{2\sqrt{5}}.$

$\therefore \quad \alpha = \cos^{-1}\left[\frac{3}{\sqrt{20}}\right].$

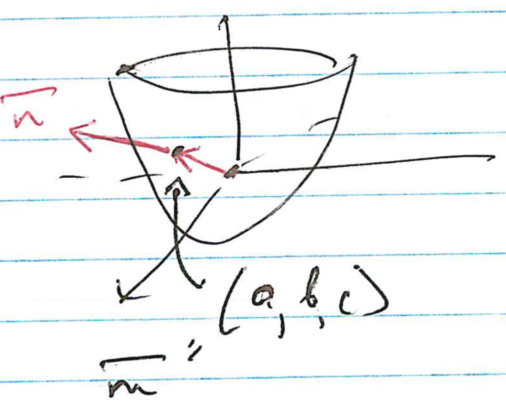
#38, p. 851 $\quad z = x^2 + y^2 - 1 \longleftarrow$ surface

Find (a, b, c) on this surface s.t.

$\overline{n} \parallel$ line from the origin to this point.

$F(x, y, z) = x^2 + y^2 - z - 1$

$F(x, y, z) = 0$ surface



$\overline{n} = \nabla F(a, b, c) =$

$= (2x, 2y, -1)\Big|_{(a,b,c)} =$

$= (2a, 2b, -1)$

Vector from the origin to (a, b, c) = position vector of this point: $\overline{m} = (a, b, c)$

Find all (a, b, c) s.t. $\boxed{a^2 + b^2 - c - 1 = 0}$

$\boxed{(2a, 2b, -1) \parallel (a, b, c).}$

$(2a, 2b, -1) = K(a, b, c), \ K \in \mathbb{R}$

$$\begin{cases} 2a = k \cdot a \\ 2b = k \cdot b \\ -1 = k \cdot c \\ a^2+b^2-c-1=0 \end{cases} \Leftrightarrow \begin{cases} a(k-2)=0 \\ b(k-2)=0 \\ k \cdot c = -1 \\ a^2+b^2-c-1=0 \end{cases}$$

$\underline{a=0}$

$$\begin{cases} b(k-2)=0 \\ k \cdot c=-1 \\ b^2-c-1=0 \end{cases} \Rightarrow \begin{cases} \underline{b=0} \\ k \cdot c = -1 \\ -c-1=0 \end{cases} \Leftrightarrow \begin{cases} b=0 \\ c=-1 \\ k=1 \end{cases}$$

One sol-n: $\begin{cases} a=0 \\ b=0 \\ c=-1 \end{cases}$

$$\Rightarrow \begin{cases} k=2 \\ c=-\frac{1}{2} \\ b^2+\frac{1}{2}-1=0 \end{cases} \Rightarrow \begin{cases} k=2 \\ c=-\frac{1}{2} \\ b^2=\frac{1}{2} \end{cases} \Rightarrow \begin{cases} k=2 \\ c=-\frac{1}{2} \\ b=\pm\frac{1}{\sqrt{2}} \end{cases}$$

More sol-s: $\begin{cases} a=0 \\ b=\pm\frac{1}{\sqrt{2}} \\ c=-\frac{1}{2} \end{cases}$

$k=2$ ...