Lab 8 October 29, 2019

#19, p. 867. $\begin{cases} x^2 - xy + y^2 - z^2 = 1 \\ x^2 + y^2 = 1 \end{cases}$ (*) Curve C.

Find pt. closest to $(0,0,0)$.

$P(x,y,z)$: $\text{Dist}^2 = x^2+y^2+z^2$

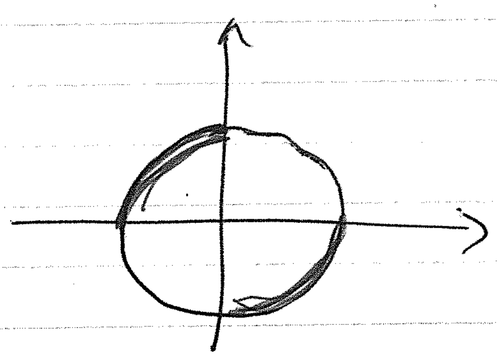
Q-n: minimize $\underbrace{(x^2+y^2+z^2)}_{f(x,y,z)}$ over all (x,y,z) s.t. (*) satisfied.

$f(x,y,z) = x^2+y^2+z^2 = z^2+1$

$\begin{cases} x^2 - xy + y^2 - z^2 = 1 \\ x^2+y^2=1 \end{cases} \iff \begin{cases} 1 - xy - z^2 = 1 \\ x^2+y^2=1 \end{cases} \iff$

$\iff \begin{cases} z^2 = -xy \\ x^2+y^2=1 \end{cases}$ i.e., $f(x,y,z) = 1 - xy$.

minimize over (x,y) s.t. $x^2+y^2=1$ and $xy \le 0$.



$g(x,y) = 1 - xy$

$\begin{cases} x = \cos t \\ y = \sin t \end{cases}$, $\frac{\pi}{2} \le t \le \pi$ or $-\frac{\pi}{2} \le t \le 0$.

$h(t) = g(\cos t, \sin t) = 1 - \sin t \cos t = 1 - \frac{1}{2}\sin 2t$

Find abs. minimum of $h(t) = 1 - \frac{1}{2}\sin 2t$ on $[\frac{\pi}{2}, \pi]$ and $[-\frac{\pi}{2}, 0]$.

#13, p. 867. Find the pt. on the part of $x+y+2z=4$ in the first octant that is closest to $(3,3,1)$.

$P(x,y,z)$: $\text{Dist}\{P,(3,3,1)\}^2 = (x-3)^2+(y-3)^2+(z-1)^2$ $= f(x,y,z)$

minimize subject to $\begin{bmatrix} x+y+2z=4 \\ x\geq 0,\ y\geq 0,\ z\geq 0. \end{bmatrix}$

$y = 4-x-2z$

$f(x, 4-x-2z, z) = (x-3)^2+(1-x-2z)^2+(z-1)^2 = g(x,z)$

minimize subject to
$x\geq 0,\ z\geq 0,\ 4-x-2z\geq 0.$

#13, p. 859. $f(x,y) = (1-x)(1-y)(x+y-1)$

$$\frac{\partial f}{\partial x} = (1-y)\{-(x+y-1)+1-x\} = (1-y)(2-2x-y)$$

$$\frac{\partial f}{\partial y} = (1-x)\{-(x+y-1)+1-y\} = (1-x)(2-x-2y).$$

Critical pts: $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$:

$$\begin{cases} (1-y)(2-2x-y)=0 \\ (1-x)(2-x-2y)=0 \end{cases}$$ $\leftarrow$ $y=1$ or $2x+y=2$

Case I: $y=1$ $\therefore (1-x)(-x)=0 \iff x=0$ or $x=1$.

$\boxed{(0,1),\ (1,1)}$ C.P.

Case 2: $2x+y=2 \iff y = 2-2x$

$$(1-x)\big(2-x-2(2-2x)\big)=0 \iff (1-x)(-2+3x)=0$$

$\therefore\ x=1$ or $x=\frac{2}{3}$

$\downarrow \qquad\qquad \downarrow$

$y=0 \qquad y = 2-\frac{4}{3} = \frac{2}{3}.$

$\therefore\ \boxed{(1,0),\ \left(\frac{2}{3},\frac{2}{3}\right)}$ C.P.

$$\frac{\partial^2 f}{\partial x^2} = \underbrace{(1-y)(-2)}_{=A} \qquad \frac{\partial^2 f}{\partial y^2} = (1-x)(-2) = C.$$

$$\frac{\partial^2 f}{\partial x \partial y} = -(2-2x-y)+(1-y)(-1) =$$

$$= -2+2x+y-1+y = \underbrace{2x+2y-3}_{=B}.$$

Critical pt.	A	B	C	B^2-AC	Nature
$(0,1)$	0	-1	-2	1	saddle pt
$(1,1)$	0	1	0	1	saddle pt
$(1,0)$	-2	-1	0	1	saddle pt
$\left(\frac{2}{3},\frac{2}{3}\right)$	$\underbrace{-\frac{2}{3}}_{<0}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$\underbrace{\frac{1}{9}-\frac{4}{9}=-\frac{1}{3}}_{<0}$	local max.

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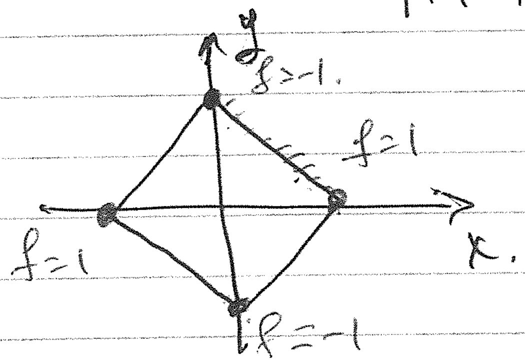
#24, p. 867. max/min? $f(x,y) = x^2 - y^2$ on $|x| + |y| = 1$.



$x, y \geq 0$: $x + y = 1$

$f(\pm x, \pm y) = f(x,y)$.

Consider $f(x,y) = x^2 - y^2$ on $x + y = 1$, $x \geq 0$, $y \geq 0$.

$y = 1 - x$, $\quad x \geq 0$, $x \leq 1$.

$f(x, 1-x) = x^2 - (1-x)^2 = 2x - 1$.

Q-n: find max/min of $g(x) = 2x - 1$ on $[0,1]$.

$g(0) = -1$, $g(1) = 1$.

Abs max value is 1 (attained at $(1,0)$ and $(-1,0)$)

— " — min — " — -1 (— " — at $(0,1)$ and $(0,-1)$).