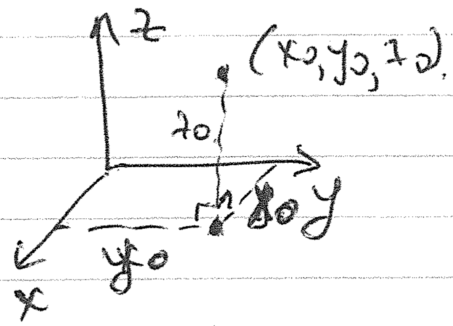
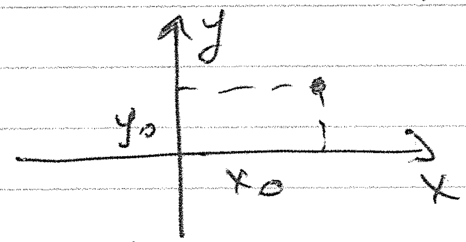
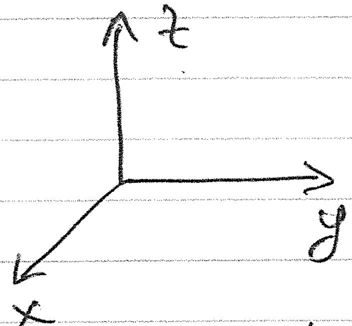
Sept. 5, 2019

11.1. Rectangular Coordinates

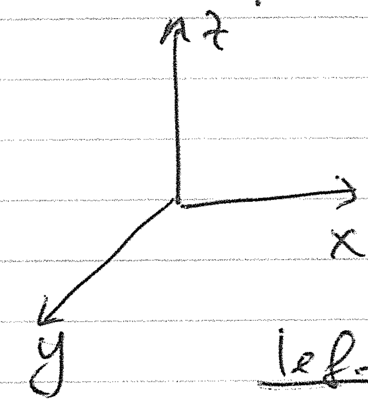


Cartesian coordinates.



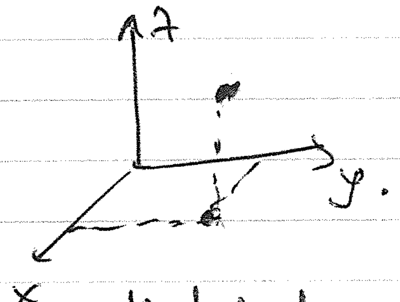


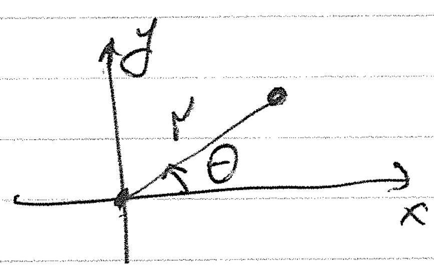
right coord. system



left coord. system.

Other coordinate systems:



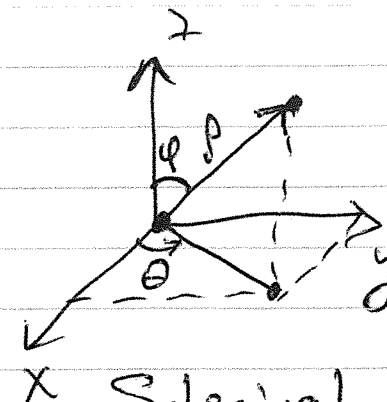


Polar coordinates
$[r, \theta]$

Cylindrical coord:

$(\underbrace{r, \theta}, z)$

polar coord. of the projection of a pt. onto the xy-plane.

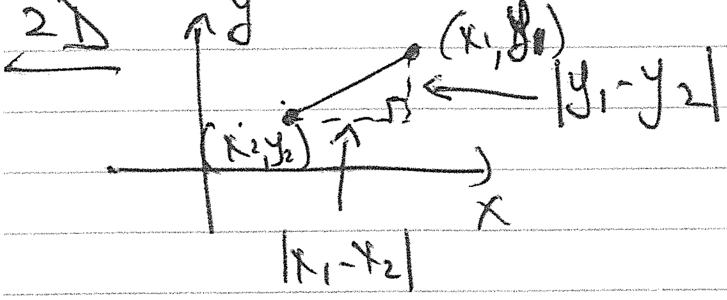


Spherical coordinates.

(ρ, θ, φ)

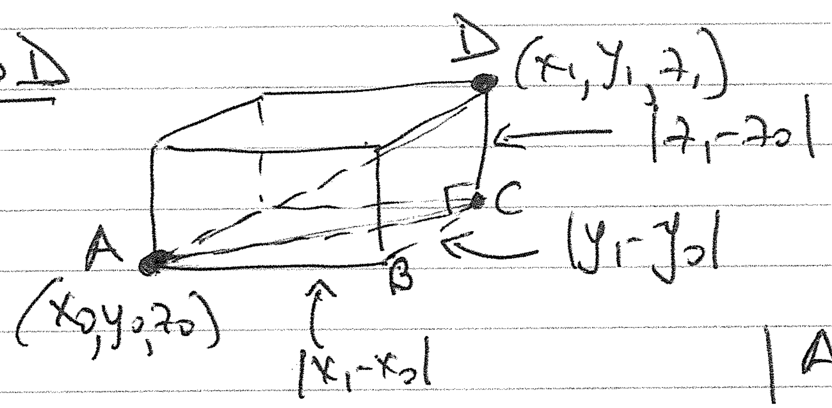
Distance between (x_0, y_0, z_0) and (x_1, y_1, z_1).

2D



$$\text{dist} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

3D



$|AB| = |x_1 - x_0|$

$|BC| = |y_1 - y_0|$

$|CD| = |z_1 - z_0|$

$|AC|$ (hypotenuse of the right triangle $\triangle ABC$)

is $|AC| = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$

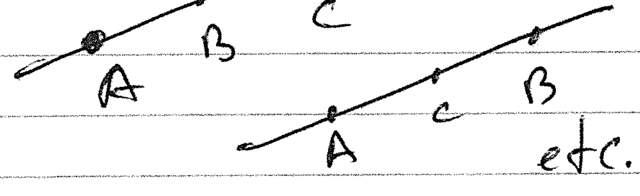
$\triangle ADC$: $|AD| = \sqrt{|AC|^2 + |DC|^2} =$

$$= \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

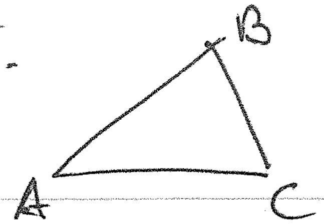
$\therefore$ Distance between (x_0, y_0, z_0) and (x_1, y_1, z_1) is

$$\boxed{\text{dist} = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}}$$

#10, p. 697 Prove that $A(1,3,5)$, $B(-2,0,3)$ and $C(7,9,9)$ are collinear.

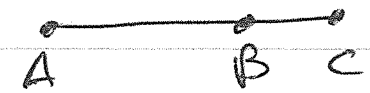


I.



$$|AB| + |BC| = |AC|$$

$$\Downarrow$$



$$|AB| = \sqrt{(-2-1)^2 + (0-3)^2 + (3-5)^2} = \sqrt{9+9+4} = \sqrt{22}$$

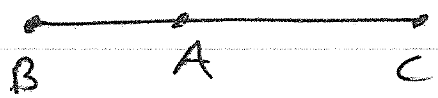
$$|AC| = \sqrt{(7-1)^2 + (9-3)^2 + (9-5)^2} = \sqrt{36+36+16} =$$

$$= \sqrt{9 \cdot 4 + 9 \cdot 4 + 4 \cdot 4} = 2\sqrt{9+9+4} = 2\sqrt{22}$$

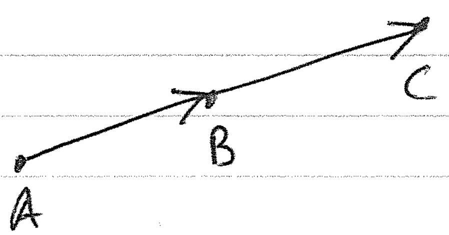
$$|BC| = \sqrt{(7+2)^2 + 9^2 + 6^2} = \sqrt{9^2 + 9^2 + 6^2} =$$

$$= \sqrt{9(9+9+4)} = 3\sqrt{22}.$$

$$\therefore \quad |AB| + |AC| = |BC|$$



II.



$$\overrightarrow{AB} \overset{?}{\parallel} \overrightarrow{AC}$$

$$\overline{AB} = (-2-1, 0-3, 3-5)$$
$$= (-3, -3, -2)$$

$$\overline{AC} = (7-1, 9-3, 9-5) = (6, 6, 4)$$

$\therefore$ Vectors $\overline{u}$ and $\overline{v}$ are parallel if $\overline{u} = t\overline{v}$, for some $t \in \mathbb{R}$.

In our case, $\overline{AC} = -2 \cdot \overline{AB}$, i.e.,

$\overline{AC} \parallel \overline{AB} \Rightarrow A, B, C$ are collinear.