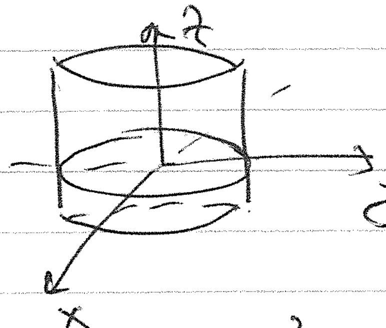
Sept. 10, 2019.

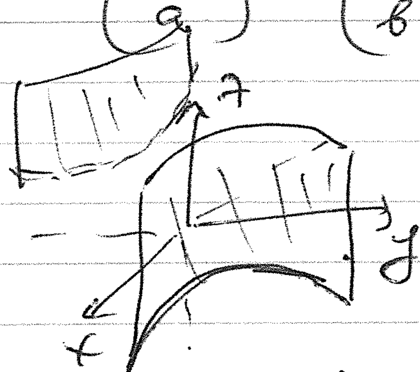
1. (*) $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ $\left\{ \begin{array}{l} Ax^2 + By^2 = C \\ A, B, C \text{ have the same sign} \end{array} \right\}$

$S = \{ (x, y, z) \in \mathbb{R}^3 \mid (*) \text{ holds} \}$.



Elliptic cylinder.

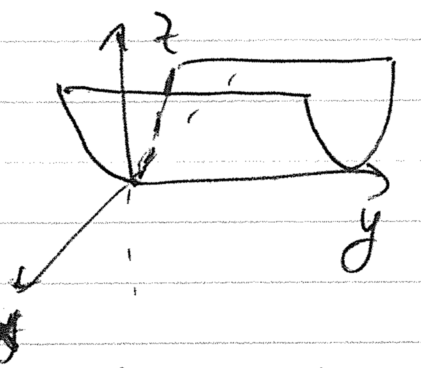
2. $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$ $\left\{ \begin{array}{l} Ax^2 + By^2 = C \\ AB < 0 \end{array} \right\}$



Hyperbolic cylinder.

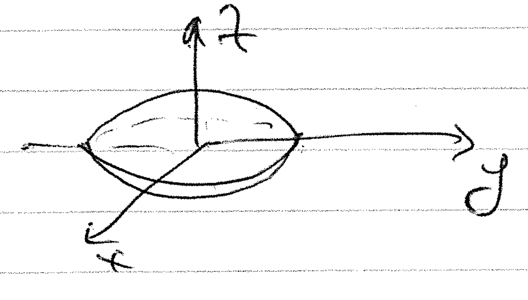
3. $z = Ax^2$

Parabolic cylinder.

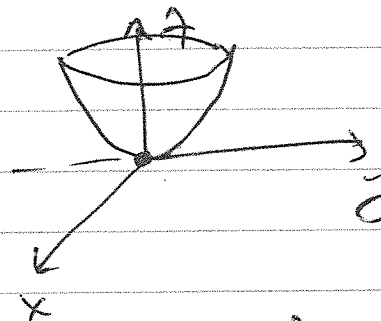


4. $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$

Ellipsoid



5. $z = \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2$



Elliptical paraboloid.

6. $z^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

$z = k =$ const

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = k^2$

Symmetry about x, y, z-axes, etc

$y = kx$

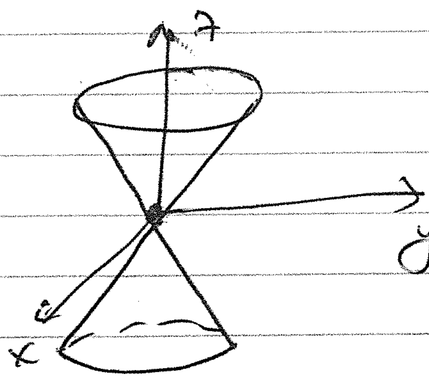
$z^2 = \frac{x^2}{a^2} + \frac{k^2}{b^2}x^2 \Leftrightarrow$

$\Leftrightarrow z^2 = \left(\frac{1}{a^2} + \frac{k^2}{b^2}\right)x^2$

$\Leftrightarrow z = \pm\sqrt{\frac{1}{a^2} + \frac{k^2}{b^2}}\, x$

2 lines

Elliptical cone.

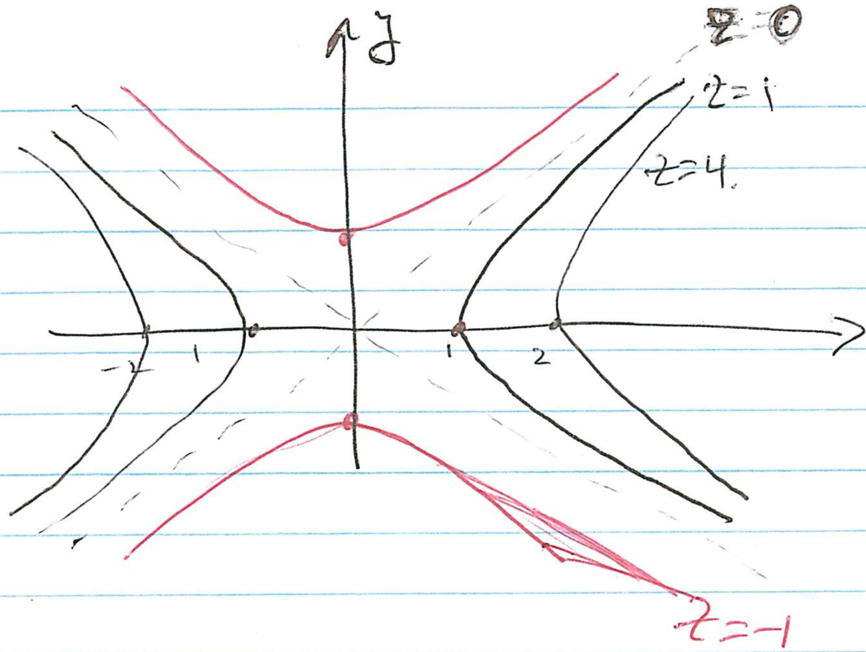


7. $z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$

Consider the following example:

$z = x^2 - y^2$

Level curve:



$z=0$: $y^2=x^2 \Leftrightarrow y=\pm x$

$z=1$: $x^2-y^2=1$

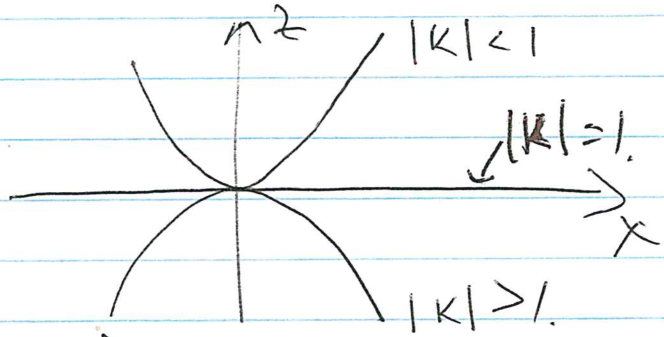
$z=-1$: $x^2-y^2=-1$

$\Leftrightarrow$ $y^2=x^2+1$

$z=4$: $x^2-y^2=4$.

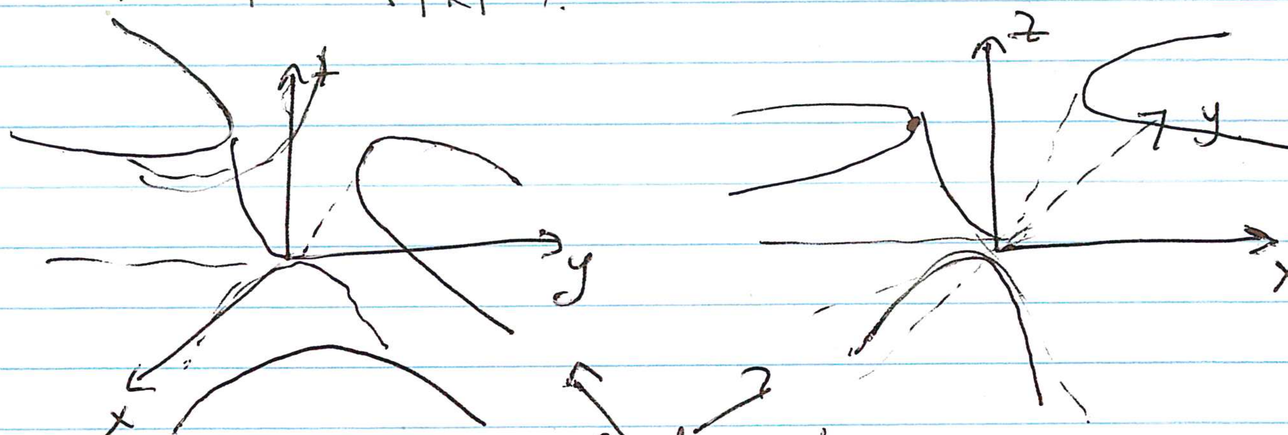
Cross-sections by the planes $y=kx$:

$z=x^2-k^2x^2 \Leftrightarrow z=(1-k^2)x^2$



if $|k|<1$: $z=ax^2$ with $a>0$.

if $|k|>1$: $z=bx^2$ with $b<0$



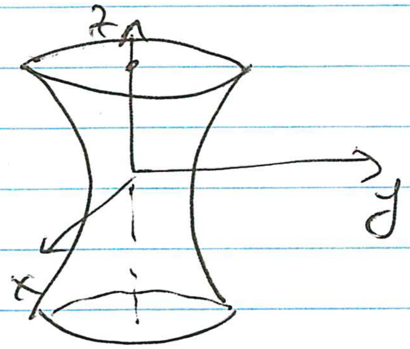
bad pictures.

Hyperbolic paraboloid

8. $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

if $z=k$ - const ; $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1+\frac{k^2}{c^2}$

$z=0$: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



Elliptic hyperboloid.
(one sheet)

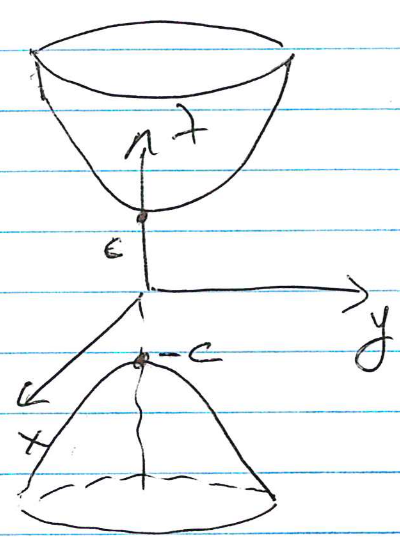
9. $\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

if $z=k$ - const : $\frac{k^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\Updownarrow$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{k^2}{c^2}-1$

Ellipses if RHS > 0.



if $z=0$: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=-1$

no points

Elliptic hyperboloid of two sheets.