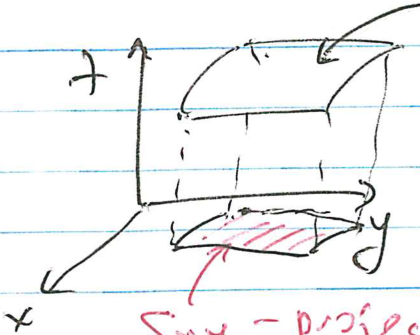
Nov. 19, 2019

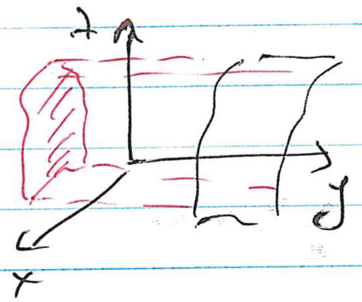


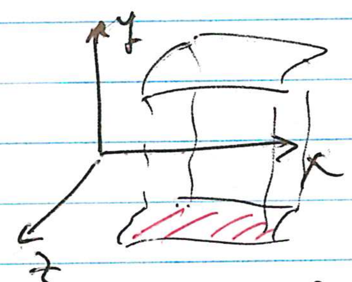
Surface area $= \iint_{S_{xy}} \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\, dA$

Remark: $y = f(x,z)$

Surface area $= \iint_{S_{xz}} \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2}\, dA$

$S_{xz} \leftarrow$ projection onto the xz-plane.





<u>#1, p. 930</u> Find the area of $2x + 3y + 6z = 1$ in the first octant.

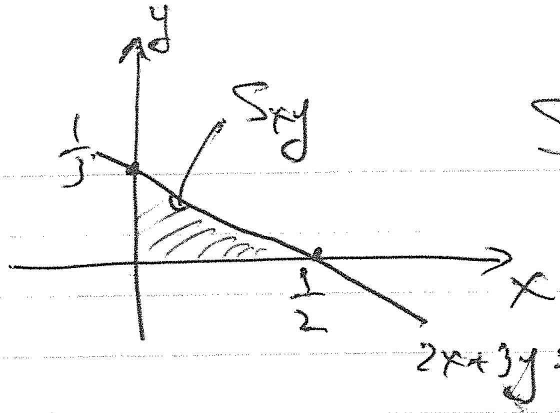
Projection onto the xy-plane:

$S_{xy} = \{(x,y) \mid x \geq 0, y \geq 0$, there is z on the surface corresp. to (x,y) and s.t. $z \geq 0\}$

$= \left\{(x,y) \mid x \geq 0, y \geq 0,\ z = \frac{1-2x-3y}{6} \text{ and } z \geq 0\right\}$

$= \{(x,y) \mid x \geq 0, y \geq 0 \text{ and } 1 - 2x - 3y \geq 0\}$

15.7

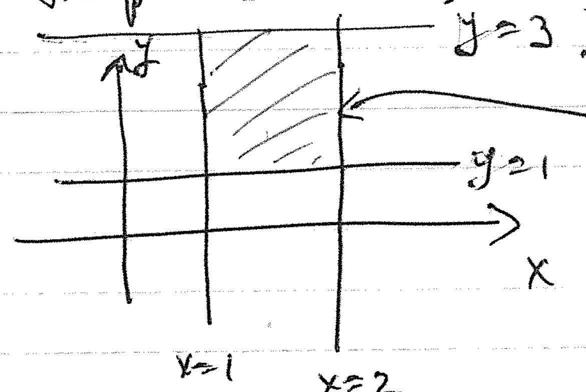


$$S = \iint_{S_{xy}} \sqrt{1+\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2}\, dA = \iint_{S_{xy}} \sqrt{1+\left(-\frac{1}{3}\right)^2+\left(-\frac{1}{2}\right)^2}\, dA$$

$$\left\{ z = \frac{1-2x-3y}{6} = f(x,y) \right\} \quad = \iint_{S_{xy}} \sqrt{1+\frac{1}{9}+\frac{1}{4}}\, dA =$$

$$= \sqrt{\frac{36+4+9}{36}}\ \text{Area}(S_{xy}) = \frac{7}{6}\cdot\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{2} = \ldots$$

#4, p. 930 The area of $z=\sqrt{2xy}$ cut out by the planes $x=1$, $x=2$, $y=1$, $y=3$.



$$S_{xy} = \{(x,y) \mid 1 \le x \le 2,\ 1 \le y \le 3\}$$

$$S = \iint_{S_{xy}} \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\, dA =$$

$$= \iint_{S_{xy}} \sqrt{1+\left(\frac{\sqrt{2y}}{2\sqrt{x}}\right)^2+\left(\frac{\sqrt{2x}}{2\sqrt{y}}\right)^2}\, dA =$$

$$= \iint_{S_{xy}} \sqrt{1+\frac{y}{2x}+\frac{x}{2y}}\, dA = \iint_{S_{xy}} \sqrt{\frac{2xy+y^2+x^2}{2xy}}\, dA =$$

$$= \iint_{S_{xy}} \sqrt{\frac{(x+y)^2}{2xy}}\, dA = \iint_{S_{xy}} \frac{x+y}{\sqrt{2xy}}\, dA =$$

$$= \int_1^2 \int_1^3 \left(\frac{x}{\sqrt{2xy}} + \frac{y}{\sqrt{2xy}} \right) dy\, dx =$$

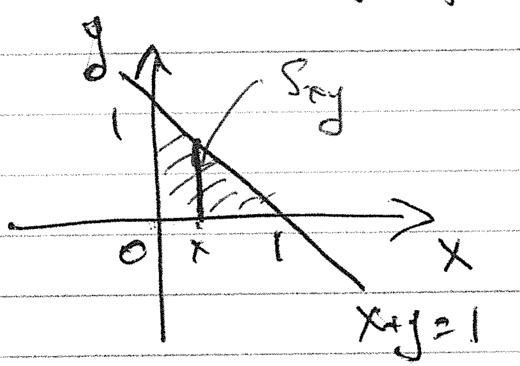
$$= \frac{1}{\sqrt{2}} \int_1^2 \int_1^3 \left(\frac{\sqrt{x}}{\sqrt{y}} + \frac{\sqrt{y}}{\sqrt{x}} \right) dy\, dx =$$

$$= \frac{1}{\sqrt{2}} \int_1^2 \left[\sqrt{x} \cdot 2\sqrt{y} + \frac{1}{\sqrt{x}} \cdot \frac{2}{3} y^{3/2} \right] \Bigg|_{y=1}^{y=3} dx =$$

$$= \frac{1}{\sqrt{2}} \int_1^2 \left[2\sqrt{x}\sqrt{3} + \frac{1}{\sqrt{x}} \cdot \frac{2}{3} 3^{3/2} - 2\sqrt{x} - \frac{2}{3} \frac{1}{\sqrt{x}} \right] dx$$

$$= \ldots$$

<u>#6, p. 930</u> The area of $z = x^{3/2} + y^{3/2}$ in the first octand cut off by the plane $x+y=1$.



$$S = \iint_{S_{xy}} \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$$

$$= \iint_{S_{xy}} \sqrt{1 + \left(\frac{3}{2} x^{\frac{1}{2}}\right)^2 + \left(\frac{3}{2} y^{\frac{1}{2}}\right)^2}\, dA =$$

$$= \iint_{S_{xy}} \sqrt{1 + \frac{9}{4} x + \frac{9}{4} y}\, dA =$$

$$= \int_0^1 \int_0^{1-x} \sqrt{1 + \frac{9}{4} x + \frac{9}{4} y}\; dy\, dx = \ldots \text{HW}$$

#7, p. 930 The area of $x^2+y^2+z^2=2$ inside the cone $z=\sqrt{x^2+y^2}$.

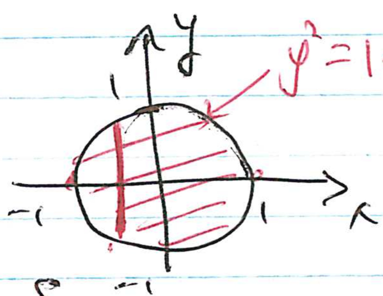
$S_{xy}=\{(x,y)\mid$ there is z corresp to (x,y) on the surface $x^2+y^2+z^2=2$ and $z\geq\sqrt{x^2+y^2}\}=$

$=\{(x,y)\mid z=\sqrt{2-x^2-y^2}$ and $\sqrt{2-x^2-y^2}\geq\sqrt{x^2+y^2}\}$

$\{2-x^2-y^2\geq x^2+y^2 \Leftrightarrow 2\geq 2x^2+2y^2 \Leftrightarrow x^2+y^2\leq 1\}$

$=\{(x,y)\mid x^2+y^2\leq 1$ and "$z=\sqrt{2-x^2-y^2}$ exists."$\}$

$=\{(x,y)\mid x^2+y^2\leq 1\}$



$$S=\iint_{S_{xy}}\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dA=$$

$$=\iint_{S_{xy}}\sqrt{1+\left(\frac{-2x}{2\sqrt{2-x^2-y^2}}\right)^2+\left(\frac{-2y}{2\sqrt{2-x^2-y^2}}\right)^2}\,dA$$

$$=\iint_{S_{xy}}\sqrt{1+\frac{x^2}{2-x^2-y^2}+\frac{y^2}{2-x^2-y^2}}\,dA=\iint_{S_{xy}}\sqrt{\frac{2}{2-x^2-y^2}}\,dA$$

$$=\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\sqrt{\frac{2}{2-x^2-y^2}}\,dy\,dx$$