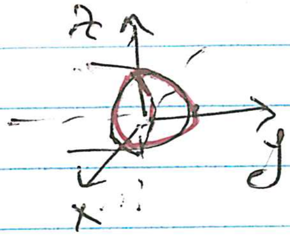
NOV. 21, 2019

#11, p. 930 The area of $y = 1 - x^2 - 3z^2$ to the right of the xz-plane.



"To the right of the xz-plane": $y \geq 0$.

Surface $= \{(x,y,z) \mid y = 1 - x^2 - 3z^2 \text{ and } y \geq 0\} \stackrel{?}{=}$

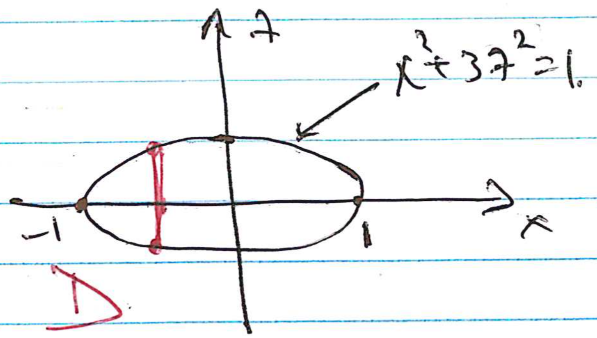
$\{\{(x,y,z) \mid y = g(x,z),\ \underline{(x,z) \in D}\}\}$?

$\stackrel{\circ}{=} \{(x,y,z) \mid \underbrace{y = 1 - x^2 - 3z^2}_{y = g(x,z)} \text{ and } \underbrace{1 - x^2 - 3z^2 \geq 0}_{D = \{(x,z) \mid x^2 + 3z^2 \leq 1\}}\}.$

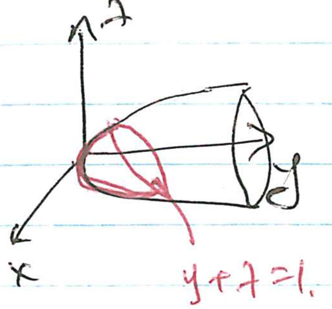
$$S = \iint_D \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2}\, dA =$$

$$= \iint_D \sqrt{1 + (-2x)^2 + (-6z)^2}\, dA =$$

$$= \int_{-1}^{1} \int_{-\sqrt{\frac{1-x^2}{3}}}^{\sqrt{\frac{1-x^2}{3}}} \sqrt{1 + 4x^2 + 36z^2}\, dz\, dx.$$



#15, p. 930 The area of $y = x^2 + z^2$ cut off by $y + z = 1$.

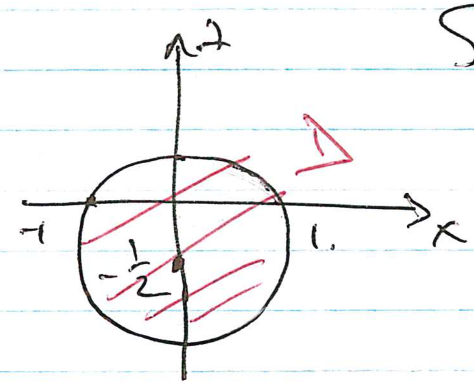


$$\text{Surface} = \{(x,y,z) \mid y = x^2 + z^2 \text{ and } y + z \le 1\}.$$

$$\begin{cases} \cancel{y + z \ge 1} \Rightarrow y \ge 1 - z \leftarrow \text{not bounded part of the surface.} \\ y + z \le 1 \end{cases}$$

$$= \{(x,y,z) \mid \underbrace{y = x^2 + z^2}_{y = g(x,z)} \text{ and } \underbrace{x^2 + z^2 + z \le 1}_{D = \{(x,z) \mid x^2 + z^2 + z \le 1\}}\}$$

$$= \{(x,y,z) \mid y = x^2 + z^2 \text{ and } x^2 + (z + \tfrac{1}{2})^2 \le \tfrac{5}{4}\}.$$



$S = \ldots$ HW.
(write as an iterated double integral).

#16, p. 930 The area of $y = z^2 + x$ inside $x^2 + y^2 = 1$.

$$\text{Surface} = \{(x,y,z) \mid y = z^2 + x \text{ and } x^2 + y^2 \le 1\}. =$$

$$\begin{cases} = \{(x,y,z) \mid z = f(x,y),\ (x,y) \in D_1\} \\ = \{(x,y,z) \mid y = g(x,z),\ (x,z) \in D_2\} \\ = \{(x,y,z) \mid x = h(y,z),\ (y,z) \in D_3\} \end{cases}$$

Q-n: which option to use?

Try Option 3: $\{ \{(x,y,z) \mid x = h(y,z),\ (y,z) \in D_3\}$.

$\Leftrightarrow \{(x,y,z) \mid x = y - z^2 \text{ and } \underbrace{(y-z^2)^2 + y^2 \le 1}_{D_3}\}$.

$D_3 = \{(y,z) \mid 2y^2 - 2yz^2 + z^4 \le 1\}$. ← Too complicated!!!

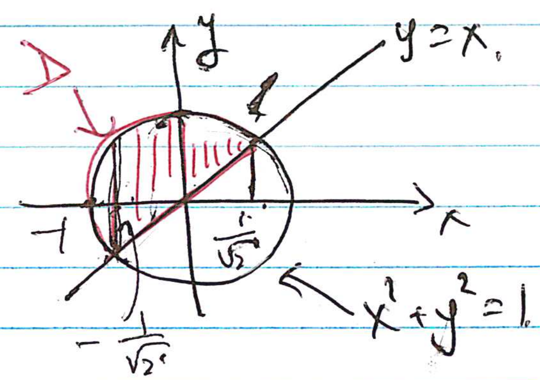
Option 1: $D_1 = \{(x,y) \mid x^2 + y^2 \le 1\}$.

Solve for z $(y = z^2 + x)$ for $(x,y) \in D_1$.

$$z^2 = y - x \Leftrightarrow z = \pm\sqrt{y-x}$$

$y - x \ge 0$, i.e., there is at least one z satisfying $y = z^2 + x$ only if $(x,y) \in D_1$ and $y - x \ge 0$.

$\therefore D = \{(x,y) \mid x^2 + y^2 \le 1,\ y \ge x\}$.



$\therefore$ Surface area $= S_1 + S_2$, where

$S_1 =$ surface area of $\{(x,y,z) \mid z = \sqrt{y-x} \text{ and } (x,y) \in D\}$

and $S_2 =$ surface area of

$$\{(x,y,z) \mid z = -\sqrt{y-x} \text{ and } (x,y) \in D\}.$$

Note: $S_1 = S_2$.

∴ Surface area $= 2S_1 =$

$$= 2\iint_D \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dA =$$

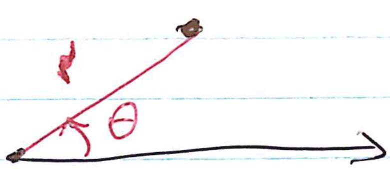
$$= 2\iint_D \sqrt{1+\left(\frac{-1}{2\sqrt{y-x}}\right)^2+\left(\frac{1}{2\sqrt{y-x}}\right)^2}\,dA =$$

$$= 2\left[\int_{-1}^{-\frac{1}{\sqrt{2}}}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\sqrt{1+\frac{1}{2(y-x)}}\,dy\,dx + \int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}}\int_{x}^{\sqrt{1-x^2}}\sqrt{1+\frac{1}{2(y-x)}}\,dy\,dx\right].$$

#17, p.930 HW.

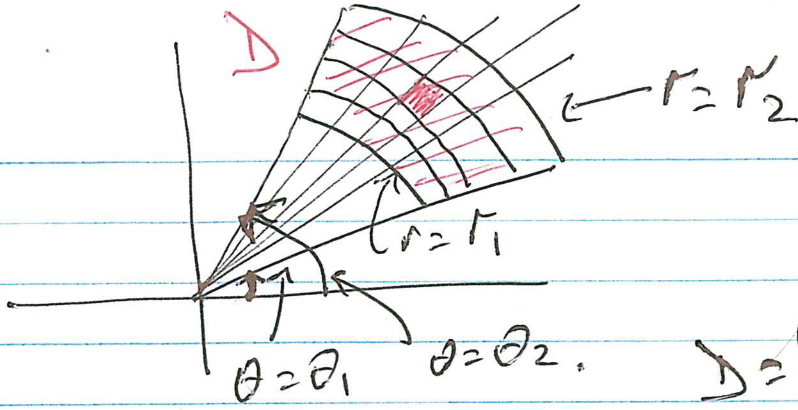
Sect. 13.7 Integrals in Polar Coordinates.

Polar coordinates: $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$



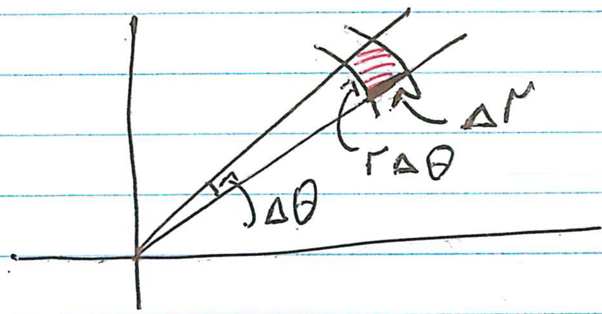
r = distance to the pole

∴ $r \geq 0$.



$$\iint_D f(x,y)\, dA$$

$$D = \{(r,\theta) \mid \theta_1 \le \theta \le \theta_2,\ r_1 \le r \le r_2\}.$$



$$r\Delta\theta \left\{ = \frac{2\pi r}{2\pi}\Delta\theta = r\Delta\theta \right\}$$

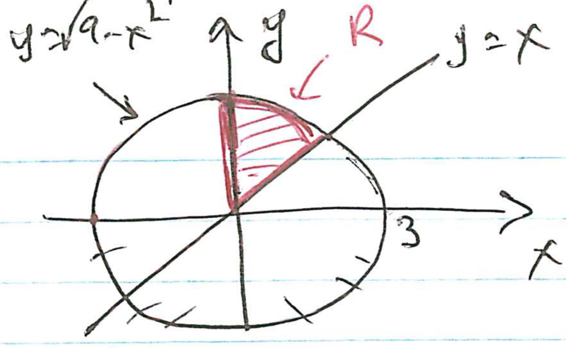
$$\boxed{dA = r\, d\theta\, dr.}$$

$$\left\{ \left| \frac{\partial(x,y)}{\partial(r,\theta)} \right| = r \quad \leftarrow \text{HW} \right\}.$$

$$\boxed{\underset{\substack{D \\ \text{(in terms of } x \text{ and } y\text{)}}}{\iint} f(x,y)\, dA = \underset{D \text{ (in terms of } r \text{ and } \theta\text{)}}{\iint} f(r\cos\theta, r\sin\theta)\cdot r\, dr\, d\theta}$$

<u>#3, p. 939</u> $\iint_R \sqrt{x^2+y^2}\, dA$, where

R is bounded by $y = \sqrt{9-x^2}$, $y = x$, $x = 0$.



In polar coordinates,

16:

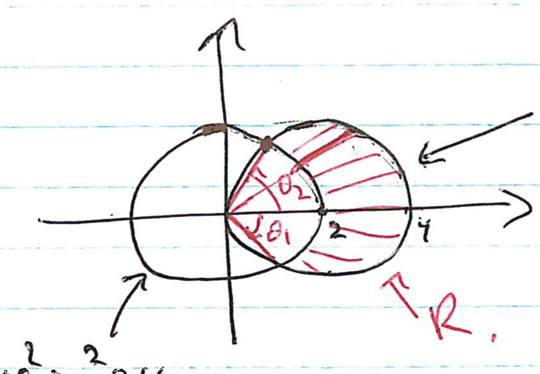
$$R=\left\{(r,\theta)\,\middle|\,\frac{\pi}{4}\le\theta\le\frac{\pi}{2},\ 0\le r\le 3\right\}.$$

$$\iint_R \sqrt{x^2+y^2}\,dA=\int_0^3\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} r\cdot r\,d\theta\,dr=$$

$$=\int_0^3 r^2\left(\frac{\pi}{2}-\frac{\pi}{4}\right)dr=\frac{\pi}{4}\cdot\left(\frac{1}{3}r^3\Big|_0^3\right)=$$

$$=\frac{\pi}{12}\left(3^3-0^3\right)=\frac{27}{12}\pi=\frac{9}{4}\pi.$$

<u>#4, p. 939</u>. $I=\iint_R \frac{1}{\sqrt{x^2+y^2}}\,dA$, where R is the region outside $x^2+y^2=4$ and inside $x^2+y^2=4x$.



$x^2+y^2=4x \iff$

$\iff x^2-4x+y^2=0 \iff$

$\iff (x-2)^2+y^2=4.$

In polar coordinates,

Curve 1 $(x^2+y^2=4)$: $r=2$

Curve 2 $(x^2+y^2=4x)$: $r^2=4r\cos\theta$

$\iff r=4\cos\theta$

$\therefore$ Pts of intersection: $2=4\cos\theta \iff \cos\theta=\frac{1}{2}$

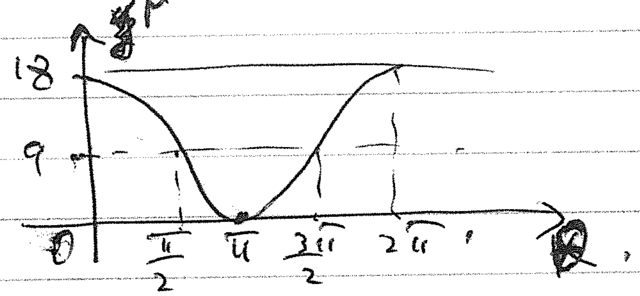
$\iff \theta=\pm\frac{\pi}{3}.$

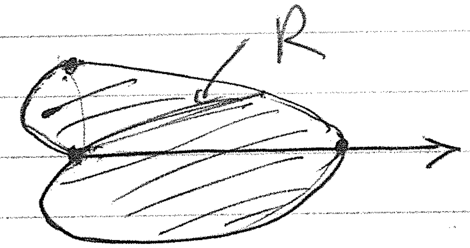
$$R = \left\{ (r,\theta) \,\middle|\, -\frac{\pi}{3} \le \theta \le \frac{\pi}{3} \text{ and } 2 \le r \le 4\cos\theta \right\}.$$

$$\therefore\ I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \int_{2}^{4\cos\theta} \frac{1}{r} \cdot r\, d\overset{r}{\cancel{\theta}}\, d\theta =$$

$$= \int_{-\frac{\pi}{3}}^{\pi/3} (4\cos\theta - 2)\, d\theta = \ldots \text{HW}.$$

#9, p. 939. Area bounded by $r = 9(1+\cos\theta)$.





$$\text{Area} = \iint_R dA = \left\{ R = \{(r,\theta) \mid 0 \le \theta \le 2\pi,\ 0 \le r \le 9(1+\cos\theta)\} \right\}$$

$$= \int_0^{2\pi} \int_0^{9(1+\cos\theta)} r\, dr\, d\theta =$$

$$= \int_0^{2\pi} \left[\frac{1}{2} r^2 \Big|_{r=0}^{r=9(1+\cos\theta)} \right] d\theta =$$

$$= \frac{1}{2} \int_0^{2\pi} 81 (1+\cos\theta)^2\, d\theta = \ldots \text{HW}.$$

HW: #13, 18, 22 p. 939.