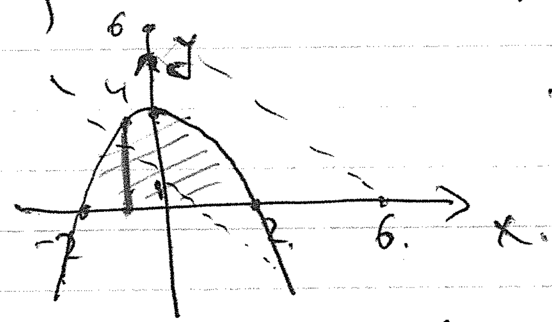
Nov. 28, 2019

Volume of V $= \iiint_V 1\,dV$, V - solid.

#4, p. 949. V is bounded by $x+y+z=6$, $y=4-x^2$, $z=0$, $y=0$.

Recall: ~~Surface~~ $V = \{(x,y,z) \mid \text{"inequalities"}\}$
$= \{(x,y,z) \mid g(x,y) \le z \le h(x,y),\ (x,y) \in D\}$,
where $D = \{(x,y) \mid x, y \text{ satisfy "inequalities"}\}$



$z=0$: $x+y=6$.

HW: show that $x+y=6$ does not intersect $y=4-x^2$

$V = \{(x,y,z) \mid 0 \le z \le 6-x-y,\ 0 \le y \le 4-x^2,\ -2 \le x \le 2\}$.

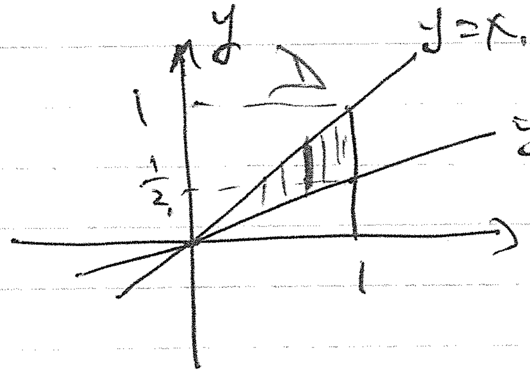
$$\text{Volume} = \int_{-2}^{2}\int_{0}^{4-x^2}\int_{0}^{6-x-y} 1 \; dz\,dy\,dx =$$

$$= \int_{-2}^{2}\int_{0}^{4-x^2}(6-x-y)\,dy\,dx = \int_{-2}^{2}\left[6y - xy - \frac{1}{2}y^2\right]\Big|_0^{4-x^2} dx$$

$$= \int_{-2}^{2}\left[6(4-x^2) - x(4-x^2) - \frac{1}{2}(4-x^2)^2\right]dx =$$

$$= 2\int_{0}^{2}\left[6(4-x^2) - \frac{1}{2}(4-x^2)^2\right]dx = \dots \text{HW}.$$

\#10, p. 949. V is bounded by $z = x^2+y^2$, $x=1$, $z=0$, $x=y$, $x=2y$.



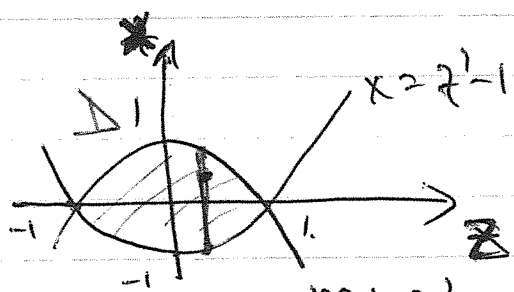
$V = \{(x,y,z) \mid 0 \le z \le x^2+y^2, (x,y) \in D\}$, where

$D = \{(x,y) \mid 0 \le x \le 1, \frac{1}{2}x \le y \le x\}$.

$$\text{Volume} = \int_0^1 \int_{\frac{1}{2}x}^{x} \int_0^{x^2+y^2} 1 \, dz \, dy \, dx = \text{-- HW.}$$

\#18, p. 950 HW

\#16, p. 950 V is bounded by $y = 1-z^2$, $y = z^2-1$, $x = 1-z^2$, $x = z^2-1$.



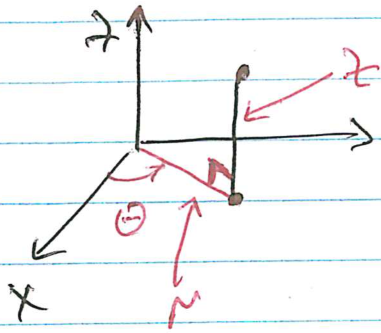
$V = \{(x,y,z) \mid z^2-1 \le y \le 1-z^2, (x,z) \in D\}$, where

$D = \{(x,z) \mid -1 \le z \le 1, z^2-1 \le x \le 1-z^2\}$

$\therefore V = \{(x,y,z) \mid z^2-1 \le y \le 1-z^2, z^2-1 \le x \le 1-z^2, -1 \le z \le 1\}$

$$\text{Volume} = \int_{-1}^{1} \int_{z^2-1}^{1-z^2} \int_{z^2-1}^{1-z^2} 1 \, dy \, dx \, dz = \text{.. HW}$$

Sect. 13.11 Triple Iterated Integrals in cylindrical Coordinates.



(r, θ, z) are cylindrical coord. If (x, y, z) are Cartesian coord. of a pt., then its cylindrical coordinates are

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$

$\left\{ \begin{array}{c} r \geq 0 \quad (!) \\ \text{Remember that} \\ 0 \leq \theta \leq 2\pi \\ \text{or } -\pi \leq \theta \leq \pi, \text{ etc.} \end{array} \right\}$

Let V be a solid in $\mathbb{R}^3$.

$$\boxed{\underset{V}{\iiint} f(x,y,z)\, dV = \underset{V}{\iiint} f(r\cos\theta, r\sin\theta, z)\, r\, dr\, d\theta\, dz}$$

(in terms of x, y, z) (in terms of r, θ, z)

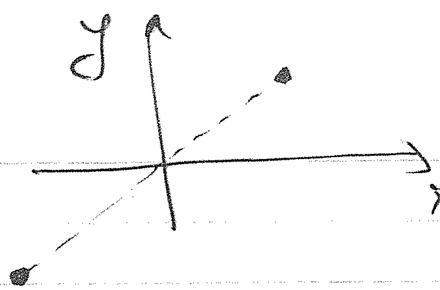
$$\boxed{dV = r\, dr\, d\theta\, dz}$$

#1, p. 958 Find eq-s for the surface in cylindrical coord. Is the surface symmetric about the z-axis.

$\underbrace{x^2 + y^2}_{r^2} + z^2 = 4$ $\quad\therefore$ In cylindrical coord.,

$$r^2 + z^2 = 4$$

Q-n: what does it mean that the surface is symmetric about the z-axis?

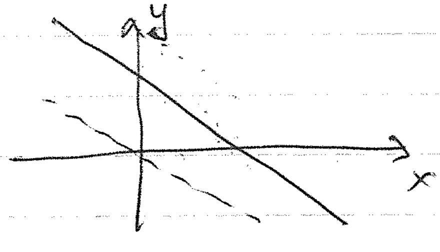


If (r, θ, z) is on the surface, then $(r, \theta + \pi, z)$ is also on the surface.

#4, p. 958 $x + y = 5$ $\therefore$ $r\cos\theta + r\sin\theta = 5$

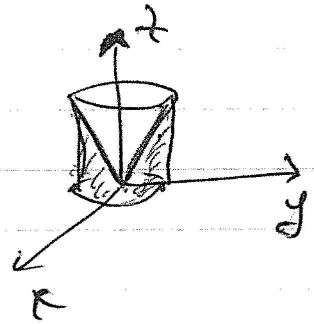
$$\Downarrow$$

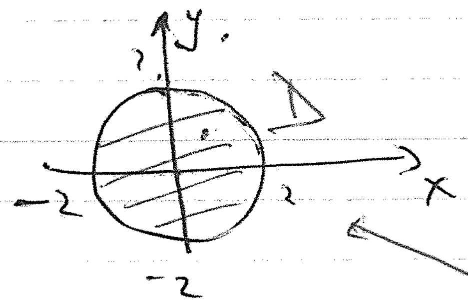
$$r = \frac{5}{\cos\theta + \sin\theta}.$$



#11, p. 958 Find the volume bounded by

$$z = \sqrt{x^2 + y^2}, \quad x^2 + y^2 = 4, \quad z = 0.$$





$$V = \{(x, y, z) \mid 0 \le z \le \sqrt{x^2 + y^2},\ (x, y) \in D\}.$$

In cylindrical coordinates,

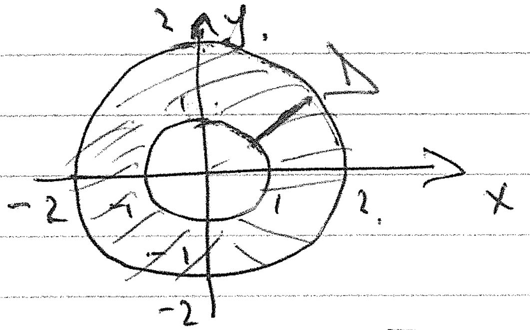
$$V = \{(r, \theta, z) \mid 0 \le z \le r,\ 0 \le \theta \le 2\pi,\ 0 \le r \le 2\}$$

$$\text{Volume} = \int_0^{2\pi} \int_0^2 \int_0^r r \, dz \, dr \, d\theta =$$

$$= 2\pi \int_0^2 r^2 \, dr = 2\pi \cdot \frac{1}{3} r^3 \Big|_0^2 = \frac{16\pi}{3}.$$

#16, p. 958 V: inside $x^2+y^2+z^2=4$
outside $x^2+y^2=1$.

$$V = \{ \overset{(x,y,z)}{\cancel{(r,\theta,z)}} \mid -\sqrt{4-x^2-y^2} \le z \le \sqrt{4-x^2-y^2},\ (x,y)\in D \}.$$



In cylinderical coord.,

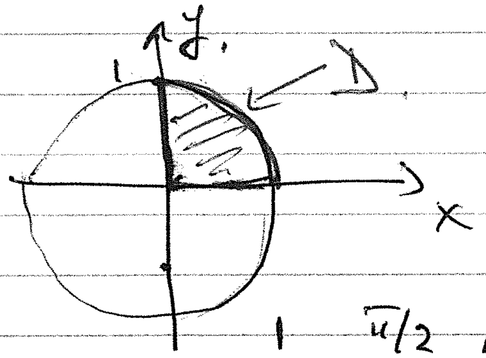
$$V = \{ (r,\theta,z) \mid -\sqrt{4-r^2} \le z \le \sqrt{4-r^2},\ 0 \le \theta \le 2\pi,\ 1 \le r \le 2 \}.$$

$$\text{Volume} = \int_0^{2\pi}\int_1^2\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta =$$

$$= 2\pi \int_1^2 2r\sqrt{4-r^2}\,dr = --$$

#25, p. 959. $\int_0^1\int_0^{\sqrt{1-y^2}}\int_0^{x^2+y^2} y^2\,dz\,dx\,dy =$

$$V = \{ (x,y,z) \mid 0 \le z \le x^2+y^2,\ 0 \le x \le \sqrt{1-y^2},\ 0 \le y \le 1 \}.$$



In cylindr. coord.,

$$V = \{ (r,\theta,z) \mid 0 \le z \le r^2,\ 0 \le \theta \le \frac{\pi}{2},\ 0 \le r \le 1 \}.$$

$$= \int_0^1\int_0^{\pi/2}\int_0^{r^2} r^2\sin^2\theta \cdot r\,dz\,d\theta\,dr =$$

$$= \int_0^1 \int_0^{\pi/2} r^5 \sin^2\theta \, d\theta \, dr =$$

$$= \underbrace{\int_0^1 r^2 dr}_{\frac{1}{3} r^3 \big|_0^1 = \frac{1}{3}} \cdot \int_0^{\pi/2} \sin^2\theta \, d\theta =$$

$$= \left\{ \begin{array}{l} \cos 2\theta = 1 - 2\sin^2\theta \quad \left\{ \begin{array}{l} = \cos^2\theta - \sin^2\theta \\ = 2\cos^2\theta - 1 \end{array} \right\} \\ \therefore \sin^2\theta = \frac{1 - \cos 2\theta}{2} \end{array} \right\}$$

$$= \frac{1}{3} \int_0^{\pi/2} \frac{1 - \cos 2\theta}{2} d\theta = \frac{1}{6}\left(\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{\pi/2} =$$

$$= \frac{1}{6}\left(\frac{\pi}{2}\right) = \frac{\pi}{12}.$$

31, 32, 38, 42 p. 959

HW.