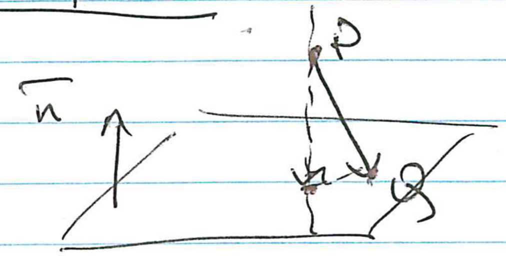
Sept. 19, 2019

#13, p. 740 Distance? From $(1,3,4)$ to plane $x+y-2z=5$



$\bar{n}=(1,1,-2)$

$Q(5,0,0)$ $\{P(1,3,4)\}$

$\overline{QP}=(-4,3,4)$.

$$\hat{n}=\frac{\bar{n}}{|\bar{n}|}=\frac{(1,1,-2)}{\sqrt{1+1+4}}=\frac{1}{\sqrt{6}}(1,1,-2).$$

$$\text{Dist}=|\overline{QP}\cdot\hat{n}|=\frac{1}{\sqrt{6}}|-4+3-8|=\frac{9}{\sqrt{6}}.$$

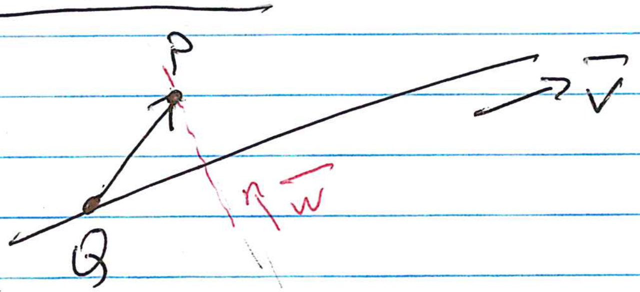
Formula
$$\text{Dist}=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$$

Eqn of plane: $x+y-2z-5=0$

$$\text{Dist}=\frac{|1+3-2\cdot4-5|}{\sqrt{1+1+4}}=\frac{9}{\sqrt{6}}.$$

#22, p. 741 Find dist from $(3,-2,0)$ to the line $x=t$, $y=3-2t$, $z=4+t$.

Method 1:



$P(3,-2,0)$

$\bar{v}=(1,-2,1)$

$Q(0,3,4)$ $\{$set $t=0\}$.

$\overline{QP}=(3,-5,-4)$

$$\bar{n}_1=\overline{QP}\times\bar{v}=\begin{vmatrix}\bar{i}&\bar{j}&\bar{k}\\3&-5&-4\\1&-2&1\end{vmatrix}=(-13,-7,-1).$$

perpend. to plane through P and containing the line

Take $\bar{n} = (13, 7, 1)$ which is $\parallel$ to $\bar{n}_1$.

$$\bar{w}_1 = \bar{n} \times \bar{v} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 13 & 7 & 1 \\ 1 & -2 & 1 \end{vmatrix} = (9, -12, -33) = 3(3, -4, -11)$$

$\bar{w} \perp$ line

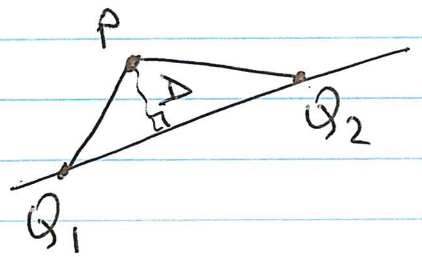
$\bar{w}$ is $\parallel$ plane through P and containing the line.

Take $\bar{w} = (3, -4, -11)$ $\{$ note: $\bar{w} \parallel \bar{w}_1$ $\}$.

$$\hat{w} = \cancel{\text{Well}} \ \frac{\bar{w}}{|\bar{w}|} = \frac{(3, -4, -11)}{\sqrt{9+16+121}} = \frac{1}{\sqrt{146}}(3, -4, -11).$$

$$\text{Dist} = |\overline{QP} \cdot \hat{w}| = \frac{9+20+44}{\sqrt{146}} = \frac{73}{\sqrt{146}} = \sqrt{\frac{73}{2}}$$

Method 2:



$$\text{Area} = \frac{1}{2}|\overline{PQ_1} \times \overline{PQ_2}|$$
$$= \frac{1}{2}|\overline{Q_1Q_2}| \cdot D$$
$$\therefore D = \frac{|\overline{PQ_1} \times \overline{PQ_2}|}{|\overline{Q_1Q_2}|}$$

$P(3, -2, 0)$

$t=0$: $Q_1(0, 3, 4)$

$t=1$: $Q_2(1, 1, 5)$

$\overline{PQ_1} = (-3, 5, 4)$

$\overline{PQ_2} = (-2, 3, 5)$

$\overline{Q_1Q_2} = (1, -2, 1)$

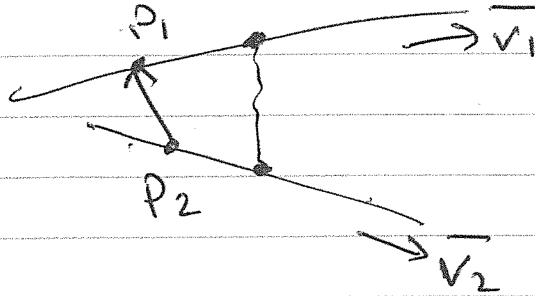
$$\overline{PQ_1} \times \overline{PQ_2} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -3 & 5 & 4 \\ -2 & 3 & 5 \end{vmatrix} = (13, 7, 1)$$

$$|\overline{PQ_1} \times \overline{PQ_2}| = \sqrt{13^2+7^2+1^2} = \sqrt{169+49+1} = \sqrt{219}$$

$$|\overline{Q_1Q_2}| = \sqrt{1+4+1} = \sqrt{6}$$

$$\therefore D = \sqrt{\frac{219}{6}} = \sqrt{\frac{73}{2}}.$$

Distance between 2 lines in 3D.



$\overline{V_1} \times \overline{V_2}$ $\perp$ both lines.

Dist = length of projection of $\overline{P_2P_1}$ onto $\overline{V_1} \times \overline{V_2}$,

i.e., $\text{dist} = \left| \overline{P_2P_1} \cdot \dfrac{\overline{V_1} \times \overline{V_2}}{|\overline{V_1} \times \overline{V_2}|} \right|$.

#26, p. 741 Find dist between the lines

$$\begin{cases} x = t \\ y = 3t - 1 \\ z = 1 + 2t \end{cases} \quad \text{and} \quad \begin{cases} x = 2t + 1 \\ y = 1 - t \\ z = 4 + 2t \end{cases}.$$

Rewrite:

$$\begin{cases} x = 0 + t \\ y = -1 + 3t \\ z = 1 + 2t \end{cases} \quad \text{and} \quad \begin{cases} x = 1 + 2t \\ y = 1 - t \\ z = 4 + 2t \end{cases}$$

$\overline{V_1} = (1, 3, 2)$, $\overline{V_2} = (2, -1, 2)$

$$\overline{V_1} \times \overline{V_2} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 3 & 2 \\ 2 & -1 & 2 \end{vmatrix} = (8, 2, -7)$$

$$|\overline{V_1} \times \overline{V_2}| = \sqrt{8^2 + 2^2 + (-7)^2} = \sqrt{64 + 4 + 49} = \sqrt{117}$$

$P_1(0, -1, 1)$, $P_2(1, 1, 4)$

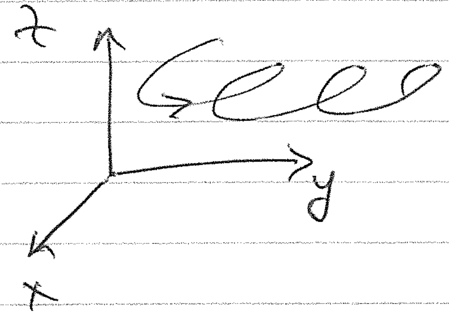
$\overline{P_1P_2} = (1, 2, 3)$

$$\text{Dist} = \left| (1, 2, 3) \cdot \frac{(8, 2, -7)}{\sqrt{117}} \right| = \frac{|8 + 4 - 21|}{\sqrt{117}} =$$

$$= \frac{9}{\sqrt{117}} = \frac{9}{\sqrt{3 \cdot 39}} = \frac{9}{3\sqrt{13}} = \frac{3}{\sqrt{13}}.$$

HW: p. 741 : # 11, 18, 19, 24, 31, 33.

Sect. 11.9: Differentiation / Integration of vectors.



position vector of a particle in 3D space at time t : $(x(t), y(t), z(t))$.

$$\bar{r}(t) = (x(t), y(t), z(t)).$$

{ vector valued f-n: $\bar{r}: R \to R^3$ }.

$$\bar{r}(t) = (x(t), y(t), z(t))$$
$$= x(t)\cdot\bar{i} + y(t)\cdot\bar{j} + z(t)\cdot\bar{k}, \quad a \le t \le b.$$

Facts: (def-s, etc.) 1) $\lim\limits_{t\to t_0} \bar{r}(t) = \left(\lim\limits_{t\to t_0} x(t), \lim\limits_{t\to t_0} y(t), \lim\limits_{t\to t_0} z(t)\right)$.

2) $\bar{r}(t)$ is continuous at t_0 if
$$\bar{r}(t_0) = \lim_{t\to t_0} \bar{r}(t) \quad \text{(definition)}$$

3) $\bar{r}(t) = (x(t), y(t), z(t))$ is continuous at t_0 <u>iff</u> x, y, z are all continuous as f-s of t.

4) Derivative:
$$\bar{r}'(t_0) = \lim_{h\to 0} \frac{\bar{r}(t_0+h) - \bar{r}(t_0)}{h}$$
provided this limit exists.

5) If $\bar{r}(t) = (x(t), y(t), z(t))$, then

$$\bar{r}'(t) = (x'(t), y'(t), z'(t)).$$

{ $\bar{r}$ is differentiable iff each component is differentiable }

6) A vector valued f-n $\bar{V}(t)$ is an antiderivative of $\bar{r}(t)$ on the interval $[a,b]$ if

$$\frac{d\bar{V}(t)}{dt} = \bar{r}(t) \text{ for all } t \in (a,b).$$

<u>Theorem</u>: If $f(t)$ is a differentiable real f-n and $\bar{v}(t)$ and $\bar{u}(t)$ are diff. vector valued f-ns, then

1) $$\frac{d}{dt}\left(f(t)\bar{v}(t)\right) = \frac{df}{dt}\bar{v} + f\frac{d\bar{v}}{dt}$$

$$\left\{ = \frac{df}{dt}(t)\bar{v}(t) + f(t)\frac{d\bar{v}}{dt}(t) \right\}$$

2) $$\frac{d}{dt}(\bar{u}\cdot\bar{v}) = \frac{d}{dt}\bar{u}\cdot\bar{v} + \bar{u}\cdot\frac{d}{dt}\bar{v}$$

3) $$\frac{d}{dt}(\bar{u}\times\bar{v}) = \frac{d\bar{u}}{dt}\times\bar{v} + \bar{u}\times\frac{d\bar{v}}{dt}.$$

<u>Ex</u>: $\bar{u} = (t, t^2, \sin t)$

$\bar{v} = (1, e^t, t)$, $t \in \mathbb{R}$.

$f(t) = t^3$.

1) $$\frac{d}{dt}\left[f(t)\bar{v}(t)\right] = \frac{d}{dt}\left[t^3(1, e^t, t)\right] =$$

1st approach

$$\stackrel{\text{1st approach}}{=} \frac{d}{dt}\left[\left(t^3, t^3e^t, t^4\right)\right] =$$

$$= \left(3t^2, 3t^2e^t + t^3e^t, 4t^3\right).$$

2nd approach

$$\stackrel{\text{2nd approach}}{=} \frac{dt^3}{dt}\left(1, e^t, t\right) + t^3\frac{d}{dt}\left[\left(1, e^t, t\right)\right]$$

$$= 3t^2\left(1, e^t, t\right) + t^3\left(0, e^t, 1\right)$$

$$= \left(3t^2, 3t^2e^t + t^3e^t, 4t^3\right).$$

2) $\frac{d}{dt}\left(\overline{u}\cdot\overline{v}\right) = \frac{d}{dt}\left[\left(t, t^2, \sin t\right)\cdot\left(1, e^t, t\right)\right]$

$$= \frac{d}{dt}\left(t + t^2e^t + t\sin t\right) =$$

$$= 1 + 2te^t + t^2e^t + \sin t + t\cos t.$$

Hw: 2nd approach using the above theorem.

<u>Ex:</u> $\overline{u}(t) = t\,\overline{i} - t^2\,\overline{j} + \cos t\,\overline{k}$

Find $\int \overline{u}(t)\,dt$.

$$\int \overline{u}(t)\,dt = \int\left(t\,\overline{i} - t^2\,\overline{j} + \cos t\,\overline{k}\right)dt =$$

$$= \left(\int t\,dt\right)\overline{i} - \left(\int t^2\,dt\right)\overline{j} + \left(\int \cos t\,dt\right)\overline{k}$$

$$= \left(\frac{t^2}{2} + C_1\right)\overline{i} - \left(\frac{1}{3}t^3 + C_2\right)\overline{j} + \left(\sin t + C_3\right)\overline{k}$$

$$= \frac{t^2}{2}\,\overline{i} - \frac{1}{3}t^3\,\overline{j} + \sin t\,\overline{k} + \overline{C},$$ where $\overline{C}$ is a constant vector.

2nd way to write:

$$\int \bar{u}(t)\,dt = \int (t, -t^2, \cos t)\,dt =$$

$$= \left(\int t\,dt, -\int t^2\,dt, \int \cos t\,dt \right) =$$

$$= \left(\tfrac{1}{2}t^2 + C_1, -\tfrac{1}{3}t^3 + C_2, \sin t + C_3 \right)$$

$$= \left(\tfrac{1}{2}t^2, -\tfrac{1}{3}t^3, \sin t \right) + \bar{C}, \text{ where}$$

$\bar{C} = (C_1, C_2, C_3)$ { constant vector in $\mathbb{R}^3$ }.

Section 11.10 Parametric and vector representation of curves.

Defn: A curve in 3D space is defined as

$C: \quad x = x(t),\ y = y(t),\ z = z(t),\quad \alpha \le t \le \beta.$

OR

$C: \quad \bar{r}(t) = (x(t), y(t), z(t)),\quad \alpha \le t \le \beta.$

$\bar{r}(t) = x(t)\,\bar{i} + y(t)\,\bar{j} + z(t)\,\bar{k},\ \alpha \le t \le \beta.$

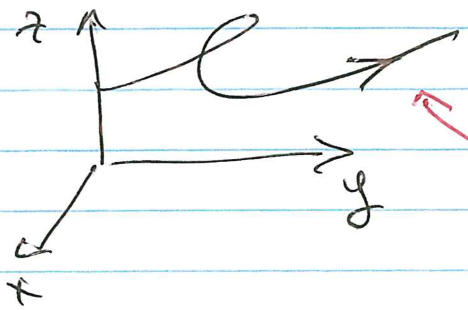
Note: A curve C given by

$\bar{r}(t) = (x(t), y(t), z(t)),\ \alpha \le t \le \beta$

is continuous iff $\bar{r} = \bar{r}(t)$ is continuous

iff $x(t), y(t)$ and $z(t)$ are all continuous for $\alpha \le t \le \beta$.

Orientation of curves:



$\vec{r}(t) = (x(t), y(t), z(t))$

direction of movement as t increases.

#1, p. 765. $\begin{cases} x + 2y + 3z = 6 \\ y - 2z = 3 \end{cases}$

Find vector representation for this curve directed so that $z \uparrow$ along the curve.

$$\begin{cases} y = 2z+3 \\ x + 2(2z+3) + 3z = 6 \end{cases} \Leftrightarrow \begin{cases} y = 2z+3 \\ x = -7z \end{cases}$$

$$\left\{ \begin{array}{l} x + 4z + \cancel{6} + 3z = \cancel{6} \\ x + 7z = 0 \end{array} \right\}$$

$$\begin{cases} x = -7t \\ y = 3 + 2t \\ z = t \end{cases}, \quad t \in \mathbb{R}.$$

(z is $\uparrow$ as $t \uparrow$).