COMONOTONE POLYNOMIAL APPROXIMATION IN $L_p[-1, 1], 0 < p \leq \infty$

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1. Introduction and main result

Let $P_n$ denote the set of all algebraic polynomials of degree $\leq n$, $L_p[a, b]$, $0 < p \leq \infty$, be the set of all measurable functions on $[a, b]$ such that the (quasi)norm $\|f\|_{L_p[a, b]}$ is finite, where as always,

$$\|f\|_{L_p[a, b]} := \left( \int_a^b |f(x)|^p \, dx \right)^{1/p}, \quad 0 < p < \infty,$$

and it is the sup-norm for $p = \infty$. Thus throughout the paper, $L_\infty[a, b]$ is understood to be $C[a, b]$ with the usual uniform norm. Also for brevity, we denote $\| \cdot \|_p := \| \cdot \|_{L_p[-1, 1]}$.

Let $Y_r := \{y_1, \ldots, y_r, y_0 := -1 < y_1 < y_2 < \ldots < y_r < 1 =: y_{r+1}\}, \, r \geq 0$. We denote by $\Delta^1(Y_r)$ the set of all functions $f$ such that $f$ is nondecreasing on $[y_r-2k, y_r-2k+1]$, and is nonincreasing on $[y_r-2k-1, y_r-2k]$, i.e., those that have $0 \leq r < \infty$ monotonicity changes at the points in $Y_r$ and are nondecreasing near $1$. Also, let $\Delta^1 := \Delta^1(Y_0)$ denote the set of all nondecreasing functions on $[-1, 1]$. Functions from the class $\Delta^1(Y_r)$ are said to be comonotone with one another.

Comonotone polynomial approximation is the approximation of a function $f \in \Delta^1(Y_r)$, by polynomials which are comonotone with it. For $f \in L_p[-1, 1] \cap \Delta^1(Y_r), \, r \geq 0$, let

$$E_n^{(1)}(f, Y_r)_n := \inf_{P_n \in \mathcal{R}_n \cap \Delta^1(Y_r)} \|f - P_n\|_n,$$

be the degree of comonotone polynomial approximation of $f$. (In particular,

$$E_n^{(1)}(f)_p := E_n^{(1)}(f, Y_0)_p := \inf_{P_n \in \mathcal{R}_n \cap \Delta^1} \|f - P_n\|_p.$$  

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is the degree of monotone approximation.\)

Recall that the \( m \)-th order Ditzian–Totik modulus of smoothness \( \omega_m^\varphi(f, \delta)_p \) (see [6]) is given by

\[
\omega_m^\varphi(f, \delta)_p = \sup_{n < k \leq \delta} \| \Delta_n^m \varphi(f, \cdot) \|_p,
\]

where \( \varphi(x) := \sqrt{1 - x^2} \), and

\[
\Delta_n^m(f, x) := \begin{cases} \sum_{i=0}^{m} \binom{m}{i} (-1)^{m-i} f(x - \frac{m}{2} \eta + i \eta), & \text{if } x \pm \frac{m}{2} \eta \in [-1, 1], \\ 0, & \text{otherwise}, \end{cases}
\]

is the symmetric \( m \)-th difference. (Note that if we set \( \varphi(x) \equiv 1 \), then (1) becomes the definition of the usual \( m \)-th modulus of smoothness \( \omega_m(f, \delta)_p := \omega_1^m(f, \delta)_p \).)

The following result on continuous approximation of continuous functions in the sup-norm (i.e., in the case when \( p = \infty \)) is known.

**Theorem A.** Let \( f \in C[-1, 1] \cap \Delta^1(Y_r), r \geq 1 \). Then

\[
E_n^{(1)}(f, Y_r, \infty) \leq C^*(r, d(r)) \omega_2(f, n^{-1}), \quad n \geq 1,
\]

and

\[
E_n^{(1)}(f, Y_r) \leq C^{**}(r) \omega_2(f, n^{-1}), \quad n \geq 1,
\]

where

\[
d(r) := \min\{y_1 + 1, y_2 - y_1, \ldots, y_r - y_{r-1}, 1 - y_r\}.
\]

Estimate (3) was first proved by Leviatan [10], with a constant \( C^{**}(r, d_0) \), where \( d_0 := \min\{y_1, 1, y_r\} \). In its present form it appears in a recent paper by Leviatan and Shevchuk [11]. Estimate (2) is due to Shvedov [18] (see also Yu [20]). It was also shown by Shvedov [18] that the constant \( C^* \) in (2) cannot be replaced by one independent of \( d(r) \) (if no extra conditions are put on \( n \)). Moreover, estimate (2) is exact in the sense that \( \omega_2 \) cannot be replaced by \( \omega_3 \) as follows immediately from a result of Zhou [21].

For other relevant results see the list of references.

The purpose of this paper is to prove the following generalization of Theorem A in \( L_p, 0 < p \leq \infty \).

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Theorem 1. Let \( f \in L_p([-1, 1] \cap \Delta^1(Y_r)), 0 < p \leq \infty \). Then, for each \( n > C(r)/d(r) \), where \( d(r) \) is defined in (4)

\[
E^{(1)}(f, Y_r)_p \leq C(r)\omega_2^p(f, n^{-1})_p.
\]

The constant \( C(r) \) depends on \( p \) when \( p \to 0 \).

We emphasize that the constant in (5) does not depend on \( Y_r \). (This does not contradict the above mentioned negative result by Shvedov since (5) is valid only for \( n > n(Y_r) = C(r)/d(r) \).)

Proof. We are going to prove Theorem 1 in stages. We first approximate \( f \in L_p([-1, 1] \cap \Delta^1(Y_r)) \) by a continuous piecewise-linear spline \( s \in \Delta^1(Y_r) \) such that \( \|f - s\|_p \leq \omega_2^p(f, n^{-1})_p \). Then we will show how to approximate \( s \) by polynomials in \( \Delta^1(Y_r) \).

We begin with the partition of the interval \([-1, 1]\) by the Chebyshev nodes \( x_k := x_{kn} := \arccos \pi k/n \) which we augment with \( Y_r \). Then we delete \( x_i \) and \( x_{i-1} \) for which there is a \( y_j, j = 1, \ldots, r \) such that \( x_i \leq y_j < x_{i-1} \), and we end up with a new partition which we denote \( Z_{r,n} \). Explicitly,

\[
Z_{r,n} := Y_r \cup \{ x_{k}^{n} \}_{k=0}^{p} \backslash \{ x_{i}, x_{i-1} : x_i \leq y_j < x_{i-1} \text{ for some } j = 1, \ldots, r \}.
\]

Now we have,

Lemma 2. Let a function \( f \in L_p \cap \Delta^1(Y_r), 0 < p \leq \infty \), be given. Then for every \( n \geq C(r)/d(r) \), there exists a continuous piecewise-linear spline \( s \in \Delta^1(Y_r) \) on the knot sequence \( Z_{r,n} \) satisfying

\[
\|f - s\|_p \leq C(r)\omega_2^p(f, n^{-1})_p,
\]

and

\[
\omega_2^p(s, n^{-1})_p \leq C(r)\omega_2^p(f, n^{-1})_p,
\]

where \( C(r) \) is as in Theorem 1.

Note first that (7) is stated here only for convenient reference as it follows immediately by (6) since

\[
\omega_2^p(s, n^{-1})_p \leq C\omega_2^p(f, n^{-1})_p + C\|s - f\|_p.
\]

Also, when \( p = \infty \), then the construction of the spline \( s \) is trivial. Indeed, we can simply take \( s \) as the piecewise-linear interpolant to \( f \) on \( Z_{r,n} \). For then \( s \in \Delta^1(Y_r) \), and using Whitney's theorem (see [4, p. 183, Theorem 4.2]) we conclude that \( s \) satisfies (5). Therefore we will concentrate on proving (6) for the case \( 0 < p < \infty \).
Proof. Assume $0 < p < \infty$. Then it is not difficult to construct a spline $\tilde{s}$ which satisfies all of the conditions of the theorem except that it may be discontinuous at $\{y_i\}_{i=1}^n$. Moreover, $\tilde{s}$ can be so chosen that, for every interval $I$ of the partition $Z_p$, the restriction of $\tilde{s}$ to $I$ is a near best linear approximant to $f$ in $L_p(I)$. There are different ways to construct such a spline. In particular, it can be constructed by following the line of proof of [2, Theorem 3]. (The only difference is that in [2] a continuous piecewise-quadratic spline was constructed and this demanded a much more elaborate work.)

Our next step is to alter the spline $\tilde{s}$ in the neighborhood of the $y_i$'s in order to obtain a continuous spline $s$ satisfying all the requirements of the lemma. To simplify the notation we describe the construction of $s$ in the neighborhood of a generic knot $\tilde{y}$, which will denote any of the $y_i$'s, and under the assumption that $s$ is nondecreasing in some small neighborhood on the left of $\tilde{y}$ and nonincreasing in some small neighborhood on the right. Let $\tilde{y} \in [x_i, x_{i+1}]$, say and let $p_1(\tilde{y})$ be the linear piece of $s$ in $[x_{i-1}, x_i]$, while $p_2(\tilde{y})$ be the linear piece of $s$ in $[x_i, x_{i+1}]$. Note that $n$ should be sufficiently large so that $x_{i+2}$ and $x_{i-3}$ are well defined. In fact we assume it is so large that $p_1$ is nondecreasing and $p_2$ is nonincreasing. This is the first occurrence where the dependence of $n$ on $d(r)$, comes in. Now, we define $s$ in $[x_{i+2}, x_{i-3}]$ as the piecewise-linear continuous spline $s(x) = \tilde{s}(x)$ for $x \notin (x_{i-1}, x_{i+1})$, and $s(\tilde{y}) = p_1(\tilde{y})$, if $p_2(x_{i-3}) \leq p_1(x_{i+1})$, or $s(\tilde{y}) = p_2(\tilde{y})$, if $p_2(x_{i-3}) > p_1(x_{i+1})$. Putting $s(x) := \tilde{s}(x)$ outside the neighborhoods of the $y_i$'s, we obtain a continuous piecewise-linear spline $s \in \Delta^1(Y_r)$.

It remains to verify that $s$ satisfies (6). To this end it suffices to show that the degree of local approximation of $f$ (near $\tilde{y}$) by $s$ is not worse than that by $\tilde{s}$. We consider the case where $p_2(x_{i-3}) \leq p_1(x_{i+1})$, the other case is analogous. Since in this case $s|_{[x_{i+1}, \tilde{y}]} = \tilde{s}|_{[x_{i+1}, \tilde{y}]}$, all we have to show is that

$$
\| f - s \|_{L_p[\tilde{y}, x_{i+1}]} \leq C \left( \| f - p_1 \|_{L_p[\tilde{y}, x_{i+1}]} + \| f - p_2 \|_{L_p[\tilde{y}, x_{i+1}]} \right).
$$

Indeed,

$$
\| s - p_2 \|_{C[\tilde{y}, x_{i-2}]} = \| s(\tilde{y}) - p_2(\tilde{y}) \| \leq \| p_1 - p_2 \|_{C[\tilde{y}, x_{i-2}]} \leq C h_i^{-1/p} \| p_1 - p_2 \|_{L_p[\tilde{y}, x_{i-2}]}.
$$

Thus,

$$
\| s - p_2 \|_{L_p[\tilde{y}, x_{i-2}]} \leq C h_i^{-1/p} \| s - p_2 \|_{C[\tilde{y}, x_{i-2}]} \leq C \| p_1 - p_2 \|_{L_p[\tilde{y}, x_{i-2}]}.
$$

and

$$
\| f - s \|_{L_p[\tilde{y}, x_{i-2}]} \leq C \left( \| f - p_2 \|_{L_p[\tilde{y}, x_{i-2}]} + \| s - p_2 \|_{L_p[\tilde{y}, x_{i-2}]} \right).
$$

\[ \leq C \left( \|f - p_1\|_{L^p([\bar{y}_1, x_{i-2}])} + \|f - p_2\|_{L^p([\bar{y}_1, x_{i-2}])} \right). \]

Lemma 2 implies that from now on, we may assume that the function \( f \) in the statement of Theorem 1 is a continuous piecewise-linear spline on the knot sequence \( Z_{\tau, n} \). Evidently, this assumption considerably simplifies all subsequent considerations. Furthermore, replacing \( f \) by \( f - f(y_1) \), we may assume without loss of generality, that \( f(y_1) = 0 \).

Hence, in the rest of the paper \( f \) is going to be a continuous piecewise-linear function on the knot sequence \( Z_{\tau, n} \) which belongs to \( \Delta^1(Y_r) \) and satisfies \( f(y_1) = 0 \).

Let \( y_1 \in \mathcal{J}_j := [x_j, x_{j-1}] \) and set \( h_j := |\mathcal{J}_j| = x_{j-1} - x_j \). We will show that

\[ \|f\|_{L^p([y_1, x_{i-2}])} \leq C \omega_2(f, h_j, J_j)_p, \tag{9} \]

where \( J_j = [x_{j+2}, x_{j-2}] \).

Clearly, while \( \mathcal{J}_j \subset J_j \), we have \( |J_j| \leq C|\mathcal{J}_j| - Ch_j \), and for \( n \geq C(r)/d(r) \) with a sufficiently large \( C(r) \) we obtain

\[ \omega_2(f, h_j, J_j)_p \leq C \omega_2(f, n^{-1})_p. \tag{10} \]

In order to prove (9), we take \( L \) to be the straight line such that \( L|_{[y_1, x_{i-2}]} = f|_{[y_1, x_{i-2}]} \), and we get

\[ \|f\|_{L^p([y_1, h_j/y_1])} \leq \|f - L\|_{L^p([y_1, h_j/y_1])} \]

\[ = \|f(\cdot) - L(\cdot) + \left( L(\cdot) - 2L(\cdot + h_j/6) + L(\cdot + h_j/3) \right)\|_{L^p([y_1, h_j/y_1])} \]

\[ = \|f(\cdot) - 2f(\cdot + h_j/6) + f(\cdot + h_j/3)\|_{L^p([y_1, h_j/y_1])} \]

\[ = \|\Delta^2_{h_j/6}f\|_{L^p([y_1, h_j/y_1])} \leq C \omega_2(f, h_j, J_j)_p, \]

where in the first inequality we used the fact that \( f \leq 0 \) in \([y_1 - h_j/6, y_1]\), since it is nondecreasing there and \( f(y_1) = 0 \), while \( L \geq 0 \) in that interval because it is a nonincreasing straight line and \( L(y_1) = f(y_1) = 0 \).

Similarly, one can show that

\[ \|f\|_{L^p([y_1, y_1 + h_j/6])} \leq C \omega_2(f, h_j, J_j)_p, \]

and, hence, (9) is satisfied.

We now define the flipped function

\[ \tilde{f}(x) := \begin{cases} -f(x), & \text{if } x < y_1, \\ f(x), & \text{if } x \geq y_1. \end{cases} \]

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Then evidently, the function $\hat{f}$ is a continuous piecewise-linear spline from the class $\Delta^1(Y_r \setminus \{y_1\})$, and (10) implies that for $n \geq C(r)/d(r)$,

$$\omega^p_n(f, n^{-1}) \leq C \omega^p_n(f, n^{-1})_p.$$  

We can now apply the method from [1] and [10] (see also [9]) and prove Theorem 1 by induction on $r$. For $r = 0$ Theorem 1 becomes a theorem on monotone polynomial approximation in $L_p$, $0 < p \leq \infty$, which was proved in [19] (see also [12]) for $1 \leq p \leq \infty$, and in [3] for $0 < p < 1$. To complete the proof it remains to show that if Theorem 1 is valid for $\hat{f} \in \Delta^1(Y_r \setminus \{y_1\})$, then it remains valid for $f \in \Delta^1(Y_r)$. We will use the construction from [9]. Namely, let $q_n \in P_n$ be a polynomial which is comonotone with $\hat{f}$ and such that

$$||\hat{f} - q_n||_p \leq C \omega^p_n(\hat{f}, n^{-1})_p. \tag{11}$$

Similarly to [9] one can show that for sufficiently large $\mu = \mu(r)$ ($\mu = 15r$ will do), there exist polynomials $V_n(x)$ and $W_n(x)$ of degree $\leq C(r)n$ such that the polynomial

$$p_n(x) := (a_n(x) - a_n(y_1))V_n(x) + a_n(y_1)W_n(x)$$

is comonotone with $f$, and the following inequalities are satisfied:

$$|\text{sgn}(x - y_1) - V_n(x)| \leq C(r)\psi_j^p(x),$$

and

$$|\text{sgn}(x - y_1) - W_n(x)| \leq C(r)\psi_j^p(x),$$

where $\psi_j(x) := \frac{h_j}{|x - x_j|^{1 + h_j}}$ (recall that $y_1 \in [x_j, x_{j-1}]$).

To complete the proof it remains to show that

$$||f - p_n||_p \leq C(r)\omega^p_n(f, n^{-1})_p. \tag{19}$$

Now,

$$||f - p_n||_p^2 = ||(\hat{f} - q_n)\text{sgn}(\cdot - y_1) + q_n(\text{sgn}(\cdot - y_1) - V_n(\cdot)) + q_n(y_1)(V_n - W_n)||_p^2$$

$$\leq C \omega^p_n(\hat{f}, n^{-1})_p^2 + C \int_{-1}^1 |f(x)|^p \psi_j^p(x) \, dx + C \int_{-1}^1 |q_n(y_1)|^p \psi_j^p(x) \, dx$$

$$=: I_1 + I_2 + I_3, \text{ say.}$$

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To estimate $I_3$ we observe that $\int_{y_1}^1 \varphi_j^{\nu, \sigma}(x) \, dx \leq C h_j$ and that $q_n$ is monotone near $y_1$. Hence,

$$I_3 = \int_{y_1}^1 |q_n(y_1)|^p \varphi_j^{\nu, \sigma}(x) \, dx \leq C h_j |q_n(y_1)|^p \leq C \|q_n\|_{L^p[y_1 - h_j / 6, y_1 + h_j / 6]}^p \leq C \|q_n - \bar{f}\|_{L^p[y_1 - h_j / 6, y_1 + h_j / 6]}^p + C \|\bar{f}\|_{L^p[y_1 - h_j / 6, y_1 + h_j / 6]}^p \leq C \omega^p_{2}(\bar{f}, n^{-1})_p,$$

where we applied (9) and (10).

It remains to estimate $I_2$, for which we need one more lemma.

**Lemma 3.** Let $f \in \Delta^1(Y_r)$ be a continuous piecewise-linear spline on the knot sequence $Z_{y_1, n}$, $y_1 \in [x_i, x_{i-1})$, and $f(y_1) = 0$. Then for all $x \in [-1, 1]$,

$$|f(x)| \leq C \left(1 + \frac{|x - x_j|}{\delta_n(x, x_j)}\right)^2 \delta_n(x, x_j)^{-1/p} \omega^p_{2}(f, n^{-1})_p,$$

where $\delta_n(x, x_j) = \min \{\Delta_n(x), \Delta_n(x_j)\}$.

**Proof.** For the sake of convenience, notation set $B_{y_1, n} = \{-1 - \varepsilon_m < z_m - 1 < z_1 < z_0 = 1\}$ and $\tilde{h}_i := z_{i-1} - z_i$. Then $\tilde{h}_i \leq 12 \tilde{h}_i$. Fix $x > y_1$ (the case $x \leq y_1$ is similar) and denote

$$Z_{y_1, n}(y_1, x) := \{i \mid z_i \in Z_{y_1, n} : y_1 \leq z_i \leq x\}.$$

Since $f$ is piecewise linear, we have

$$|f(x)| = |f(x) - f(y_1)| \leq |f'(\xi)||x - y_1|$$

for some $\xi \in (y_1, x)$. Now,

$$|f'(\xi)| \leq |f'(\xi) - f'(y_1 +)| + |f'(y_1 +)|$$

$$\leq |f'(\xi) - f'(y_1 +)| + |f'(y_1 +) - f'(y_1 -)|$$

$$\leq \sum_{i \in Z_{y_1, n}(y_1, \xi)} |f'(\xi_i) - f'(\xi_{i-1})| \leq \sum_{i \in Z_{y_1, n}(y_1, x)} |f'(\xi_i) - f'(\xi_{i-1})|$$

$$\leq C \left(1 + \frac{|x - x_j|}{\delta_n(x, x_j)}\right) \max_{i \in Z_{y_1, n}(y_1, x)} |f'(\xi_i) - f'(\xi_{i-1})|,$$

where in the second inequality we used the fact that $f'(y_1 +)$ have opposite signs, and in the last inequality, that $\tilde{h}_i \geq \delta_n(x, x_j)$, for all $i \in Z_{y_1, n}(y_1, x)$. Therefore,

$$|f(x)| \leq C \left(1 + \frac{|x - x_j|}{\delta_n(x, x_j)}\right)|x - x_j| \max_{i \in Z_{y_1, n}(y_1, x)} |f'(\xi_i) - f'(\xi_{i-1})|.$$

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Thus, to complete the proof we need an estimate of \( \max_{i \in \mathbb{Z}_{\nu,n}(y_1,x)} |f'(z_i^+) - f'(z_i^-)| \). To this end, let \( i \in Z_{\nu,n}(y_1,x) \) be fixed, and let \( f_i \) be the linear function defined by \( f_i|_{z_{i+1},x_i+1} := f|_{z_{i+1},x_i+1} \). We observe that with \( \tilde{h} := 1/100n \) and \( \alpha = 1/1000 \), say, the set

\[
A := \{ x : z_{i+1} \leq x - \tilde{h} \varphi(x) < x < z_i \leq (1 - \alpha)z_i + \alpha z_{i-1} \leq x + \tilde{h} \varphi(x) < z_{i-1} \},
\]

is of measure \( \text{meas} A \sim \tilde{h} \), and for every \( x \in A \),

\[
|\Delta_{\tilde{h} \varphi(x)}^2(f_i,x) - f_i(x + \tilde{h} \varphi(x)) - f_i(x + \tilde{h} \varphi(x))| \geq \left| f_i((1 - \alpha)z_i + \alpha z_{i-1}) - f((1 - \alpha)z_i + \alpha z_{i-1}) \right|
\]

\[
= C_{\alpha} \tilde{h} |f'(z_{i+1}) - f'(z_{i-1})|.
\]

This in turn implies,

\[
\omega_2^p(f,n^{-1}) = \sup_{0 < h \leq n^{-1}} \int_{-1}^1 |\Delta_{\tilde{h} \varphi(x)}^2(f_i,x)|^p dx \geq \int_{-1}^1 |\Delta_{\tilde{h} \varphi(x)}^2(f_i,x)|^p dx \geq C \text{meas } A \tilde{h} \omega_2^p |f'(z_{i+1}) - f'(z_{i-1})|^p.
\]

Thus,

\[
|f'(z_{i+1}) - f'(z_{i-1})| \leq C \tilde{h}_i \omega_2^p |f'(z_{i+1}) - f'(z_{i-1})|^p.
\]

and since \( \tilde{h}_i \geq \delta_n(x,x_j), i \in Z_{\nu,n}(y_1,x) \), then

\[
|f(x)| \leq C \left( 1 + \frac{|x - x_j|}{\delta_n(x,x_j)} \right)^{\frac{1}{\nu}} |x - x_j| \delta_n(x,x_j)^{-\nu+1} \omega_2^p \omega_2^p(f,n^{-1})_p
\]

\[
\leq C \left( 1 + \frac{|x - x_j|}{\delta_n(x,x_j)} \right)^2 \delta_n(x,x_j)^{-1/\nu} \omega_2^p(f,n^{-1})_p.
\]

The proof is complete. \( \square \)

We are ready to complete the proof of Theorem 1 by showing the proper estimate of \( I_2 \). By the same arguments as in the proof of Lemma 3.4 of [7], we see that

\[
\psi_j(x)^2 \leq C \frac{\delta_n(x,x_j)}{|x - x_j| + \delta_n(x,x_j)}.
\]

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Hence,

$$I_0 \leq C \|f\|_{L_p(x, x_j)} \int_{-1}^{1} \left( 1 + \frac{|x - x_j|}{\delta(x, x_j)} \right)^{-2p} \delta(x, x_j)^{-1} \left( \frac{\delta(x, x_j)}{|x - x_j| + \delta(x, x_j)} \right)^{\nu p/2} \, dx \leq C \omega^p_{2} \|f\|_{L_p(x, x_j)}^p,$$

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