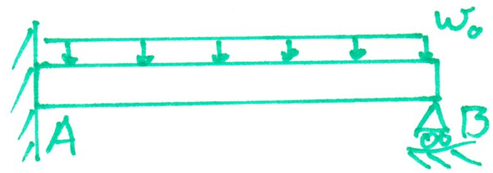
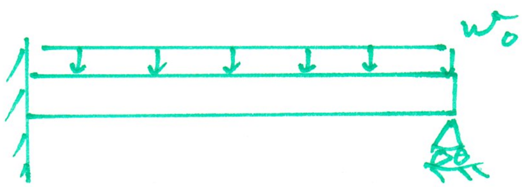
MECH 3502 MARCH 18, 2020

Last class you were asked to solve the following

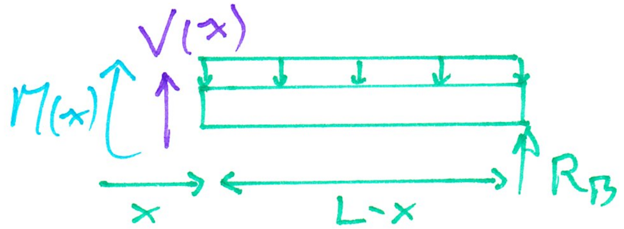


Q. How did you determine the reaction forces? If you assumed that one half of the total load would be fealt at locations A and B, you would be incorrect. This is a statically indeterminate structure and you need to solve for the displacement in order to solve for the reactions

Let's work through this problem first, then we'll go through how to recognize a statically indeterminate structure.

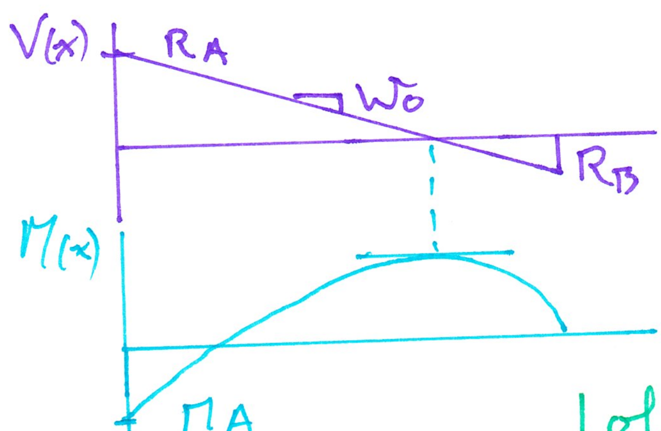


Making a cut and summing the forces and the moments



$$V(x) = w_o(L-x) - R_B$$

$$M(x) = -w_o\frac{(L-x)^2}{2} + R_B(L-x)$$



We don't know R_A R_B and M_A

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Now let's integrate the moment wrt x to get an expression for the rotation

$$\theta(x) = \frac{1}{EI}\int M(x)\,dx = \frac{1}{EI}\int -\frac{w_0}{2}L^2 + w_0 L x - \frac{w_0 x^2}{2} + R_B(L-x)\,dx$$

$$= \frac{1}{EI}\left\{ -\frac{w_0 L^2}{2}x + \frac{w_0 L x^2}{2} - \frac{w_0 x^3}{6} + R_B L x - \frac{R_B L x^2}{2} + C_1 \right\}$$

same

Solving for C_1 by applying the boundary condition for the slope at $x=0$

$$\theta(x=0) = 0 \Rightarrow C_1 = 0$$

Integrating again wrt x gives us an expression for the deflection

$$u_y(x) = \int \theta(x)\,dx$$

$$= \frac{1}{EI}\left\{ -\frac{w_0 x^4}{24} + \frac{R_B L x^2}{2} - \frac{R_B L x^3}{6} + C_2 \right\}$$

Applying the b.c. at $x=0$ gives

$$u_y(x=0) = 0 \Rightarrow C_2 = 0$$

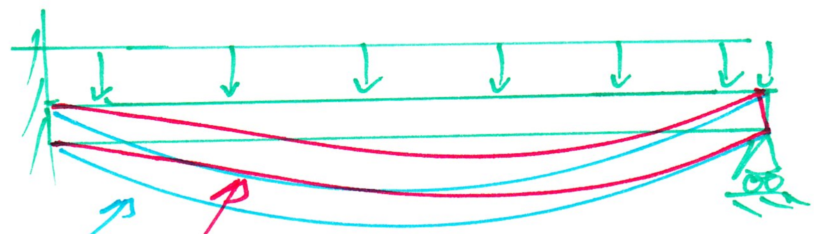
Now the deflection is also zero at $x=L$

$$u_y(x=L) = 0 \Rightarrow -\frac{w_0 L^4}{24} + \frac{R_B L^3}{2} - \frac{R_B L^3}{6}$$

gives us $R_B = \frac{3}{8} w_0 L$

and from $\Sigma F \Rightarrow R_A = \frac{5}{8} w_0 L$

So clearly the load is not equally split between the two supports. The reason why is the addition of the rotational constraint pulls away som of the force that would occur at location B



Deformed shape if the rotational constraint at location A, equal reaction forces at locations A and B , $R_A = R_B = \frac{w_o L}{2}$

Actual Deformed shape with rotational constraint at location A. Beam has extra support from this added B.C. which clearly 'lifts-up' the beam more thereby reducing the load at B

$$R_B = \frac{3}{8} w_o L < R_A = \frac{5}{8} w_o L$$

Recognizing a statically indeterminate structure often can be done by looking (and counting) the number of individual constraits that are applied to a structure

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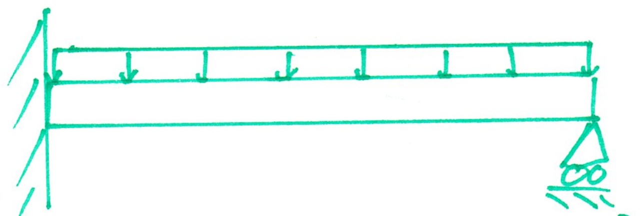
The boundary conditions for this problem are



fixed rotation
fixed deflection in x-direction
fixed deflection in y direction

fixed deflection in y direction

There are 4 constraints in total,

Q. How many rigid body motions are there for an object in 3-D?

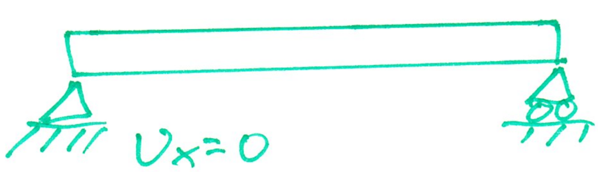
A. There are 6; 3 displacement U_x U_y U_z and 3 rotations θ_x θ_y θ_z

and in 2 dimensions there would be only 3

U_x U_y and θ

So at minimum, we need to have at least 3 individual constraints to suppress all rigid body motions. More than that may render the structure to be over constrained.

Let's look at a few examples



$U_x = 0$
$U_y = 0$

$U_y = 0$

3 b.c.'s
stat. det.

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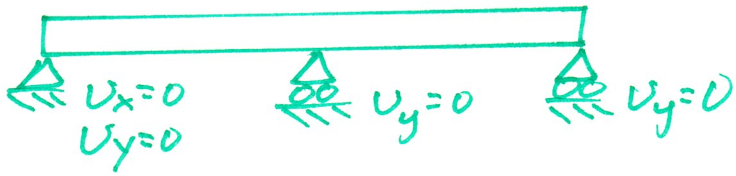
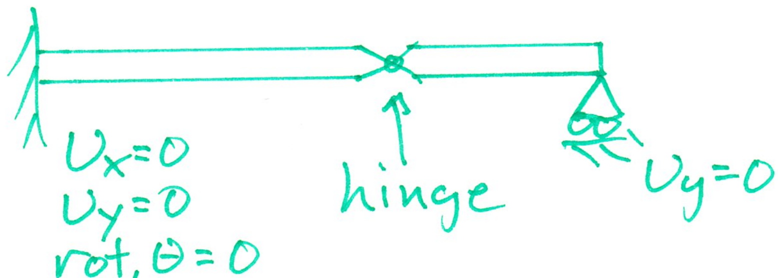


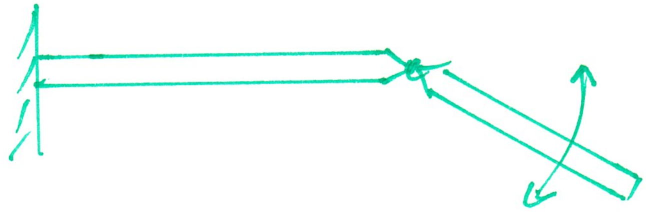
4 b.c.'s
Stat.
Indet.



4 bc's
Stat
det.

Q Whoa wait, this has 4 b.c.'s yet is statically determinate?

A Well yes, the presence of the hinge in this structure actually adds a degree of freedom to the structure. In this case we actually need 4 b.c.'s to sufficiently constrain the structure. Imagine what would happen if the 4th constraint at the right side were missing



end would be free to rotate.

Being able to recognize whether a problem is or isn't statically determinate is going to be critical in being able to recognize what has to be done to analize a structure. If is is statically indeterminate, deflections are going to have to be determined.

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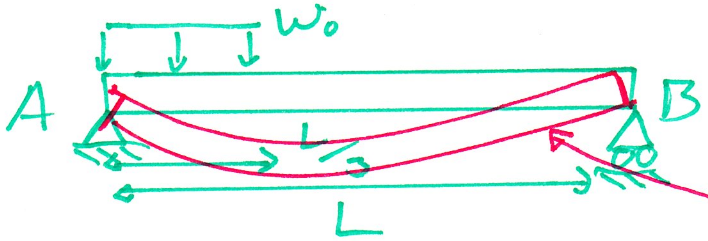
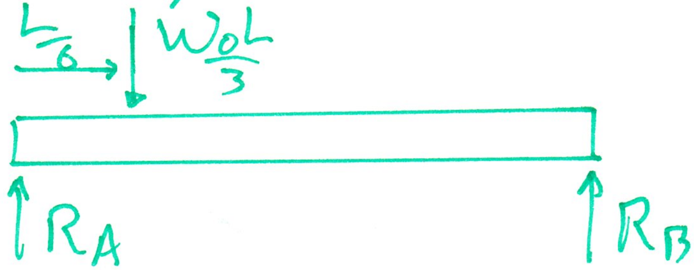
Let's do another problem



Count b.c.'s at A $v_x=0$ $v_y=0$ and at B $v_y=0$ so there are 3 = statically determinate

Let's sketch the anticipated deformed shape since this helps us get a feeling for what our answers might look like

Solve for the reactions at A and B using a free body diagram. Note that we can use an equivalent point load for the distributed load



$$\Sigma M_A \Rightarrow R_B = \frac{w_0 L}{18}$$

$$\Sigma F \Rightarrow R_A + R_B = \frac{w_0 L}{3} \text{ so } R_A = \frac{5}{18} w_0 L$$

Now if we are also after the equation for the beam's deflection, we are going to need to start with equations for the shear and the bending moment.

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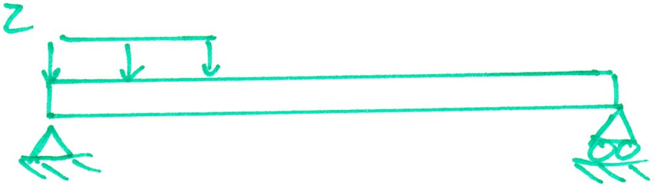
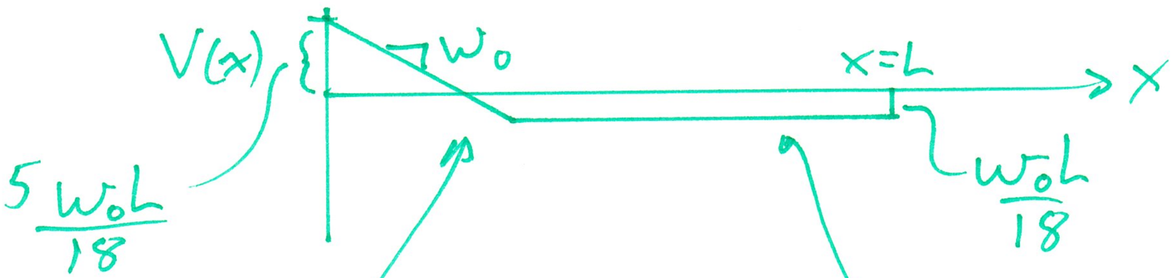
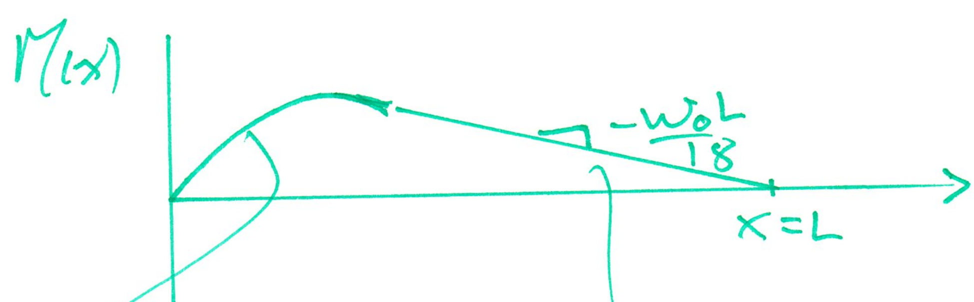




$V(x) = \frac{5 w_0 L}{18} - w_0 x$ $\qquad$ $V(x) = \frac{-w_0 L}{18}$

We have 2 separate equations for the shear; one between 0 and $L/3$ and another between $L/3$ and L. We are also going to have 2 equations for the moment.



$M(x) = \frac{5 w_0 L x}{18} - \frac{w_0 x^2}{2}$ $\qquad$ $M(x) = -\frac{w_0 L x}{18} + \frac{w_0 L^2}{18}$

I got these expressions from my plot of the moment and knowing that the moment equations should be zero at both x=0 and at x=L

I could have easily gotten these also from making 2 diferent cuts, one in each section.

Now we are going to integrate both of these equations twice, once to get the slopes and a second time to get the deflections

in section 1 $\quad 0 < x < \frac{L}{3}$

$$\theta(x) = \frac{1}{EI}\int M(x)\,dx = \frac{1}{EI}\left[\frac{5 w_0 L x^2}{36} - \frac{w_0 x^3}{6} + C_1\right]$$

$$U_y(x) = \int \theta(x)\,dx = \frac{1}{EI}\left[\frac{5}{108} w_0 L x^3 - \frac{w_0 x^4}{24} + C_1 x + C_2\right]$$

and in section 2 $\quad \frac{L}{3} < x < L$

$$\theta(x) = \frac{1}{EI}\left[-\frac{w_0 L x^2}{36} + \frac{w_0 L^2}{18} x + C_3\right]$$

$$U_y(x) = \frac{1}{EI}\left[-\frac{w_0 L x^3}{108} + \frac{w_0 L^2 x^2}{36} + C_3 x + C_4\right]$$

Note that we have 4 constants from all that integrating. We have to solve these from the boundary conditions

$$U_x(x=0) = 0 \quad \text{and} \quad U_y(x=L) = 0$$

this applies to the equation from section 1

this applies to the equation from section 2

We need 2 more...

Those are going to have to come from how the equations from the two sections are connected to one another.

$$\theta(x=L/3) \text{ for section 1} = \theta(x=\tfrac{L}{3}) \text{ for section 2}$$

and $$v_y(x=\tfrac{L}{3}) \text{ for section 1} = v_y(x=\tfrac{L}{3}) \text{ for section 2}$$

this is certainly going to use up a lot of trees

$$v_y(x=0)=0 \text{ for section 1} \Rightarrow C_2=0$$

$$v_y(x=L)=0 \text{ for section 2} \Rightarrow C_4=-C_3L$$

$$\theta(x=\tfrac{L}{3})=\theta(x=\tfrac{L}{3}) \Rightarrow C_3=C_1+\frac{7}{972}wL^4$$

$$v_y(x=\tfrac{L}{3})=v_y(x=\tfrac{L}{3}) \Rightarrow C_1=-\frac{25}{5832}wL^4$$

Ouch that took a lot of work and the likelyhood that we've made a math error in there is pretty high. Let's look at another way of solving these problems that is a bit easier. This is based on applying the principle of superposition with some typical 'known' solutions to beams.