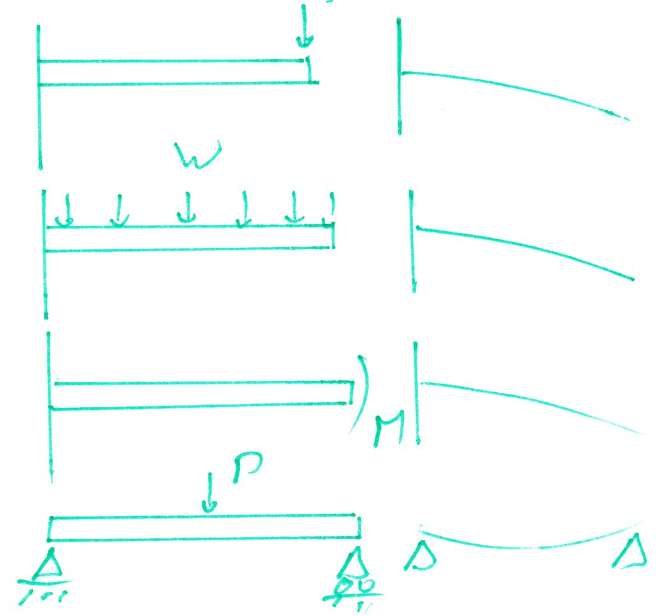
MECH 3502 MARCH 20, 2020

Using the Principle of Superposition to solve Beam Deflection Problems.

Approach involves using a number of 'known' solutions to assemble a solution for a more complex problem. Our text book provides a table of these 'known' solutions in Appendix F in 8th Ed.

This looks like



Beam & Loading	Elastic Curve	Max δ	Slope	Eqn
		$-\frac{PL^3}{3EI}$	$-\frac{PL^2}{2EI}$	$y=$
		$-\frac{wL^4}{8EI}$		
		$-\frac{ML^2}{2EI}$		
		$-\frac{PL^3}{48EI}$		

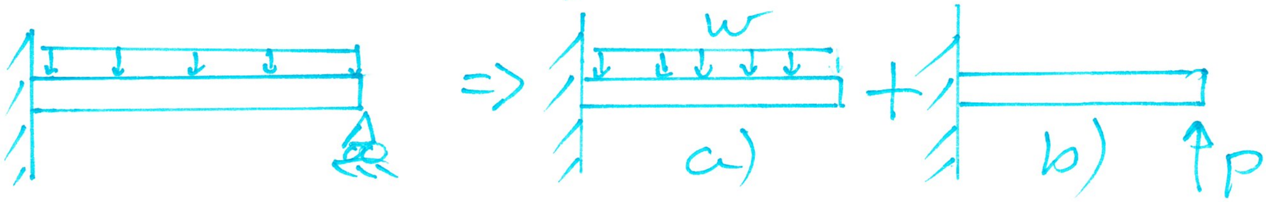
etc.

NB the "equation of the Elastic Curve" is the equation for the beam's deflection as a function of position, x along the length of the beam.

I would provide this complete table on the 'Final Exam'

The general idea is to 'break up' the applied loading and/or applied constraints that can be identified among the 'known' solutions, pull the info from the table, and then assemble the final solution by applying the Principle of Superposition.

Let's look at an example:



This was the same problem we solved last class, remember that it was statically indeterminate.

I split this into the two problems on the right, case a) and case b) which are both solutions provided in the appendix. Nt case b) is flipped upside down and the load P in this case would represent the reaction force at the second support and by treating this as a second force, we have turned the original problem into 2 statically determinate cases.

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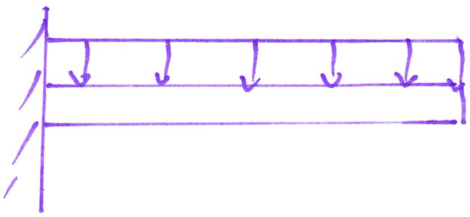
for case a) we have

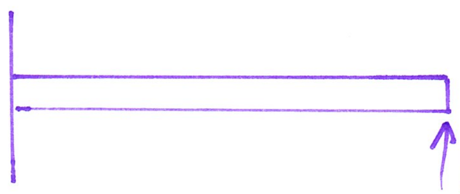


$$u_{y\,max} = -\frac{wL^4}{8EI}$$

and

$$u_y = -\frac{w}{24EI}\left(x^4 - 4Lx^3 + 6L^2x^2\right)$$

and for case b) we have



$\uparrow P = R_B$

$$u_{y\,max} = \frac{PL^3}{3EI}$$

$$u_y = \frac{P}{6EI}\left(x^3 - 3Lx^2\right)$$

equating the sum of the deflections at the end of the beam to zero (where it was originally supported by a pin support:

$$u_{y\,max\ case\ a)} + u_{y\,max\ case\ b)} = 0$$

$$-\frac{wL^4}{8EI} + \frac{PL^3}{3EI} = 0$$

we get $P = \frac{3}{8} wL = R_B$

Et Voila! same answer we had from last time, and much easier too!

The only hard part is being able to determine how to 'break up' the beam

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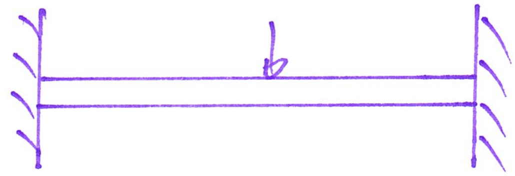
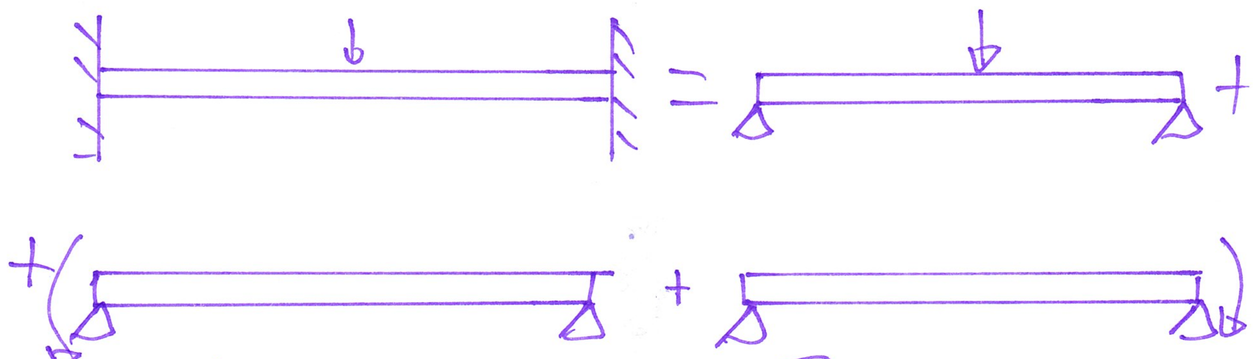
Let's look at a few more examples



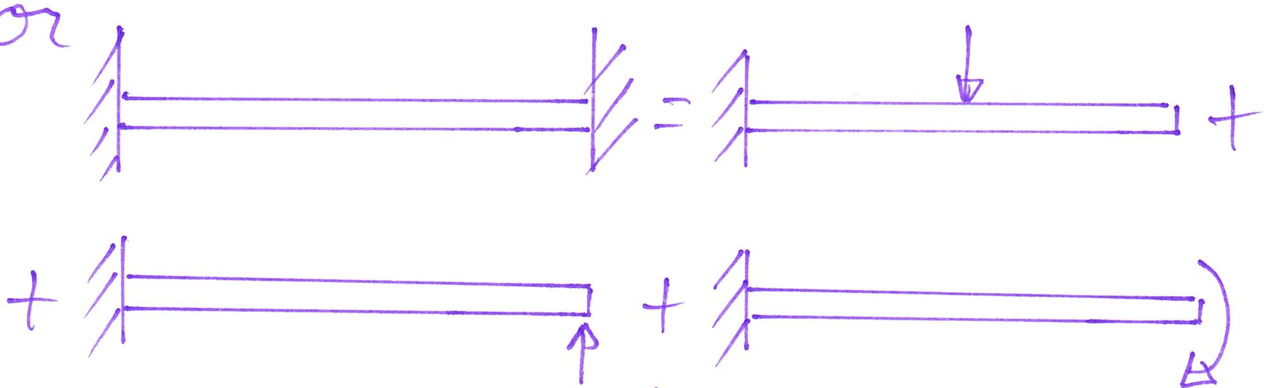
This one is super over constrained, if we add ups the constraints, we get 6

So we are likely going to have to break this ups into more than 2 separate cases. There are multiple ways in which this can be done. Here are 2 ways that I come ups with:



Where these two moments helps make the slopes 0 at the ends

or



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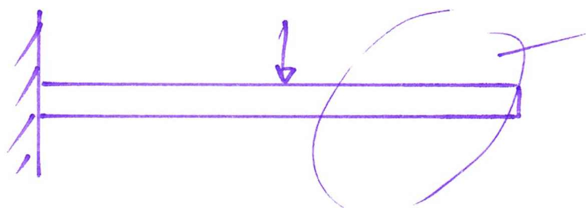
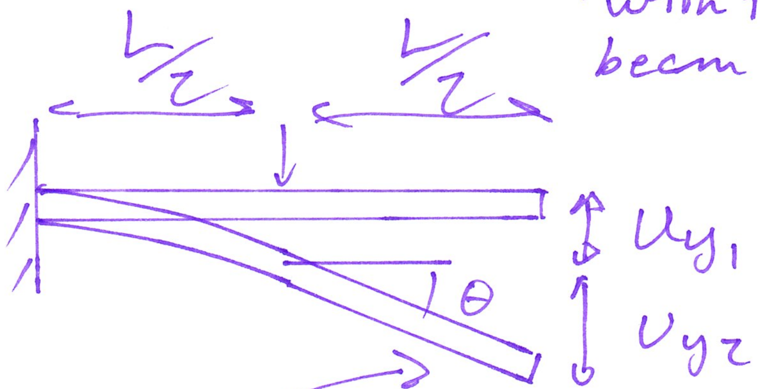
Notice that in the second case, we have a situation that is not strickttly among the known solutions, namely this one:



What does the displacement look like for this section?
Just a straight part with the slope of the beam being constant



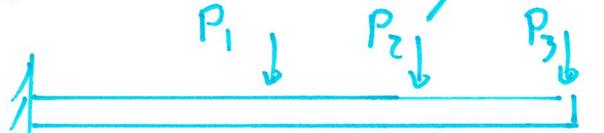
So if we wanted to find the deflection at this end we could use the deflection at the applied load v_{y1} and the slope times length to get v_{y2} and add them together

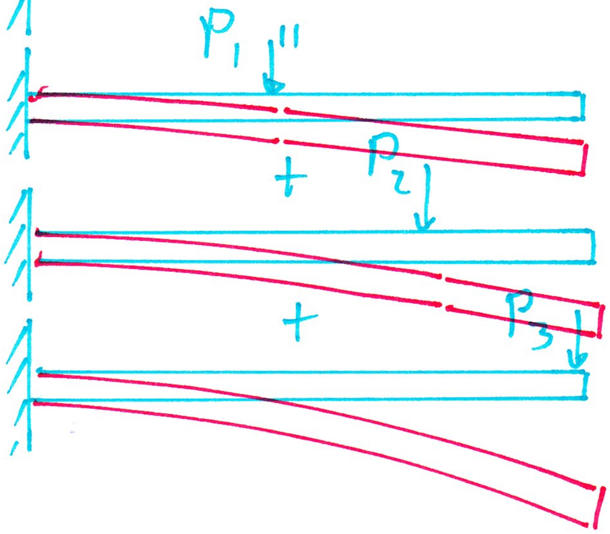
$$v_{ymax} = v_{y1} + v_{y2} =$$

$$\frac{-P\left(\frac{L}{2}\right)^3}{3EI} + \frac{-P\left(\frac{L}{2}\right)^2}{2EI} \cdot \frac{L}{2}$$

But you have to use $\frac{L}{2}$ instead of L in the known solution.

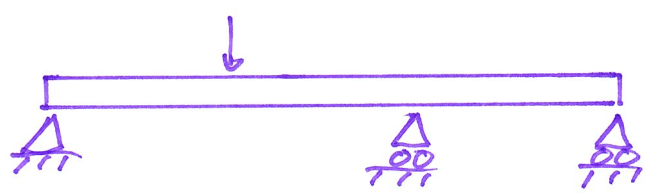
How about the following



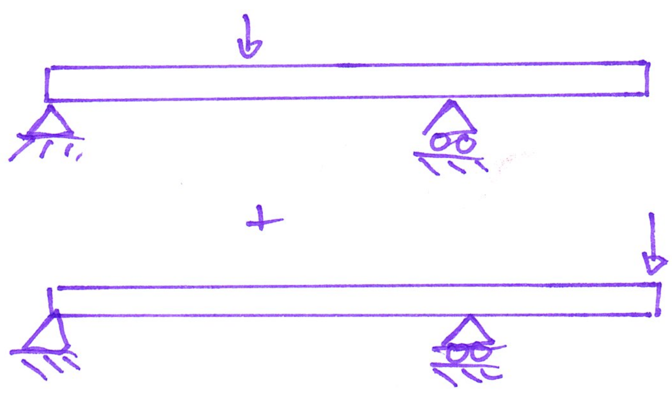


This won't be too bad to get the max deflection, but will certainly be some work if we wanted the final equation for the deflection. Fortunately, that's not something I'll be asking for.

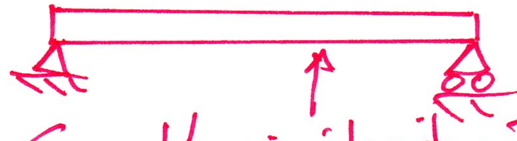
And this one



Here's one way, can you think of another?



Notice this one is also not in the known solutions but this one is



See the similarity?