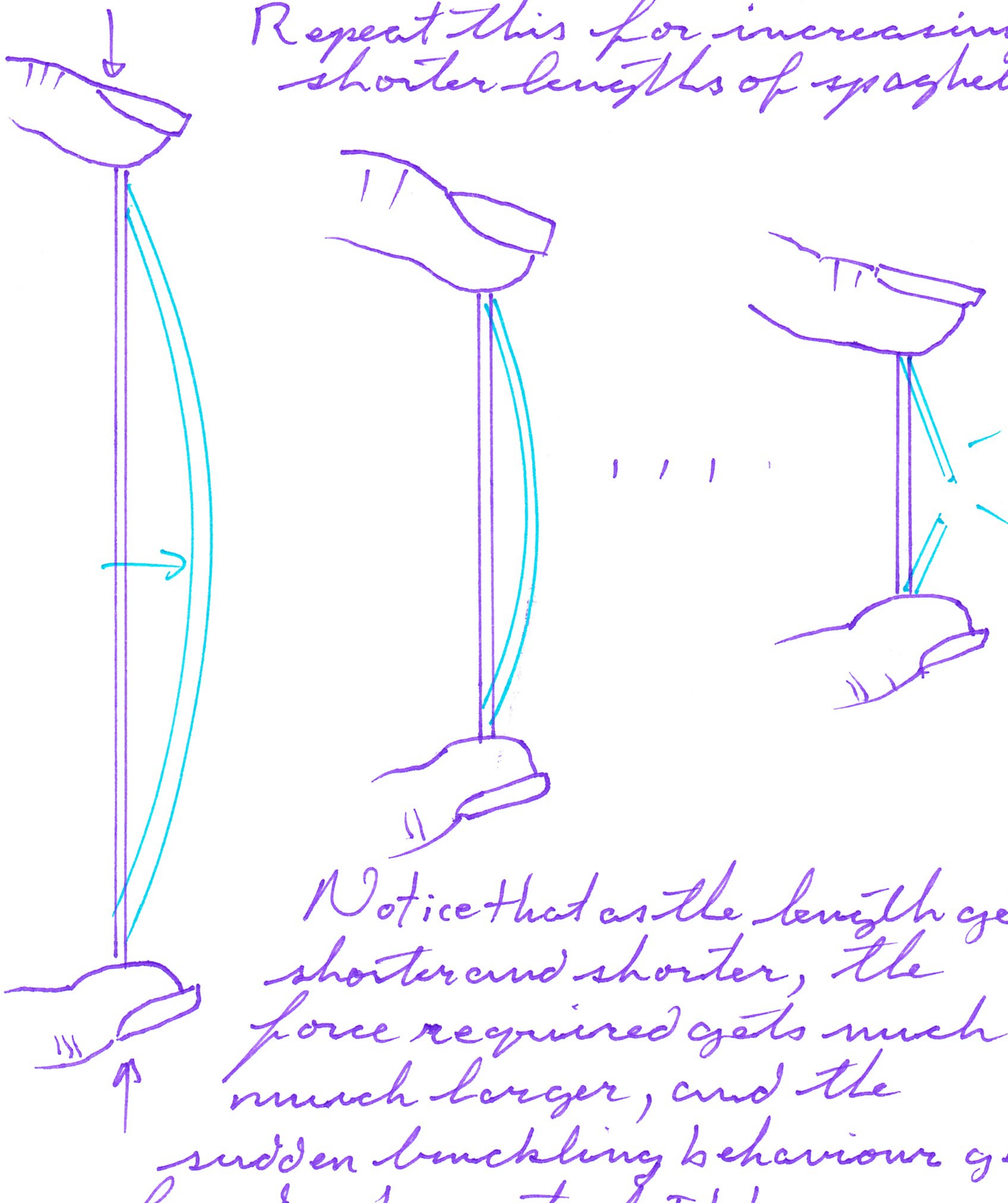
MECH 3502 MARCH 23, 2020

BUCKLING

Get yourself a few pieces of uncooked spaghetti and try this simple experiment. Push on the two ends of one piece that is the full length ~25cm

Repeat this for increasingly shorter lengths of spaghetti.



Notice that as the length gets shorter and shorter, the force required gets much much larger, and the sudden buckling behaviour gets harder to control. It becomes more unstable.

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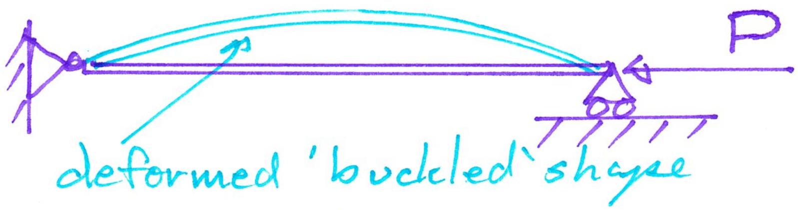
Buckling represents a structural instabity that occurs in a long slender structure subjected to a compressive force. The text refers to these as columns. Now lets see if we can determine an expression for the load required to cause this to happen.

Consider a 'beam' that is supported by pin connections at either end and is subjected to a compressive load



If we make a 'cut' midspan in the beam and draw the free body diagram

There are two equal and opposite forces, P separated by a distance

$$M_1 = P \cdot u_y$$

To counteract this, there must also be some internal bending moment, M that wants to restore the beam to being straight.

This moment, M is related to the beams deflection

$$M_2 = EI \frac{\partial^2 v_y}{\partial x^2}$$

Note, we used this equation before, namely the moment is proportional to the second derivative of the displacement.

These two moments have to balance

$$M_1 + M_2 = 0$$

$$P u_y + EI \frac{\partial^2 v_y}{\partial x^2} = 0$$

or

$$v_y'' + \frac{P}{EI} v_y = 0$$

This is a second order homogeneous diferential equation.

The general solution is

$$v_y = C_1 \sin\left(\sqrt{\frac{P}{EI}}\, x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}\, x\right)$$

applying the boundary conditions @ $x = 0, L$ where $v_y = 0$ gives

$$v_y(x=0) = 0 \implies C_2 = 0$$

$$v_y(x=L) = 0 \implies C_1 \sin\left(\sqrt{\frac{P}{EI}}\, L\right) = 0$$

Since $C_1 \neq 0$ we need $\sin\left(\sqrt{\frac{P}{EI}}\, L\right) = 0$

Which is zero when $\sqrt{\frac{P}{EI}}\,L = n\pi$

For any given beam, the dimensions are given, I and L and obviously, so is the stiffness, E. The only thing left is the load, P. Solving for P gives

$$P = \frac{n^2 \pi^2 EI}{L^2}$$

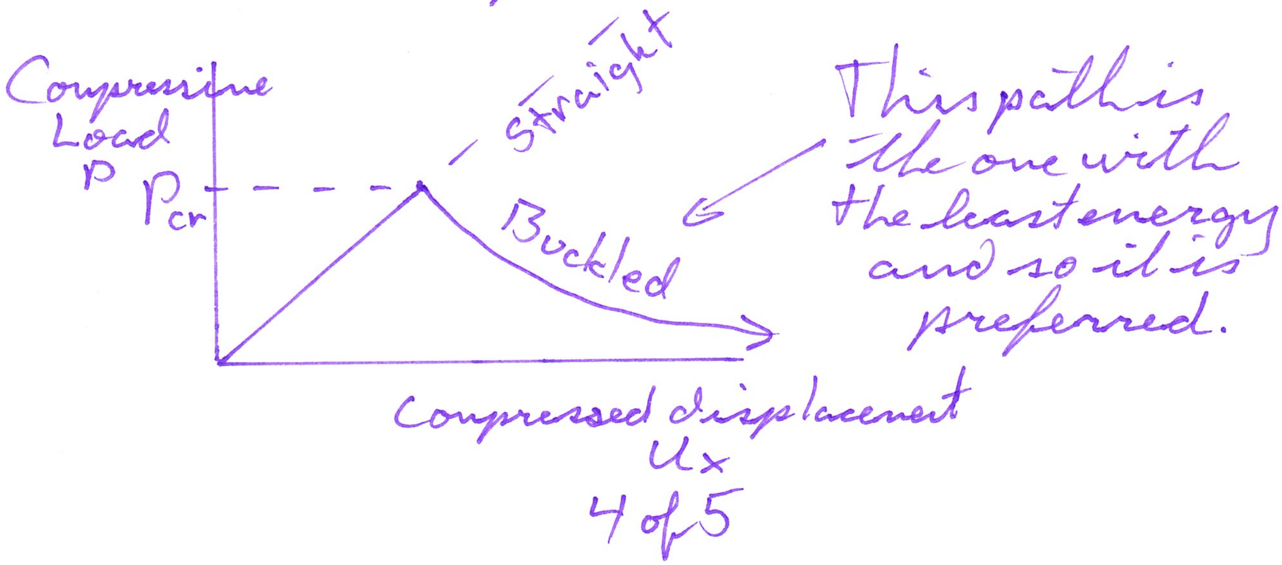
This is called Euler's Buckling Equation

The smallest non zero value is when $n=1$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

this is called the Critical Buckling Load

It represents the compressive load at which a beam will no longer remain straight, but will want to take on a buckled shape



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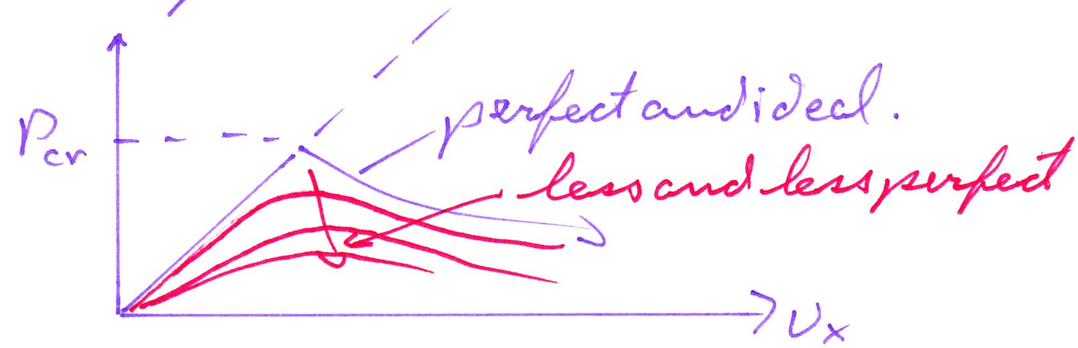
Now in reality, buckling will always occur at a load that is less than P_{cr} because no beam is perfectly straight, nor can it have a perfectly applied compressive load, so we get



Now back to our spaghetti experiment, the critical buckling load says that $P_{cr} \propto \frac{1}{L^2}$ so think about how the load went up

Length of Spaghetti	Ideal Buckling Load	Measured
25cm	0.35kg	.08kg
12.5cm	1.4kg	.5kg
6cm	6.0kg	~5kg
4cm	13.6kg	~11kg
2cm	54.3kg	No data
1cm	217kg	

These #'s are way off. The longer the spaghetti, the less perfect it is

More Buckling to come next lecture