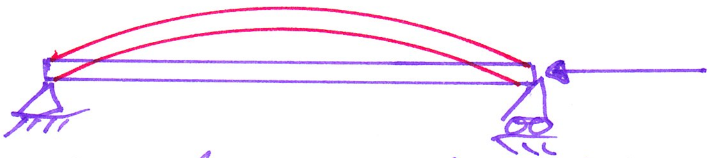
MECH 3502 MARCH 25, 2020

Effect of Supports on Buckling

The type of supports on either end of the beam influence the buckling behaviour of a 'column' in compression.

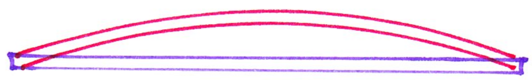
For a pin-pin support we had:



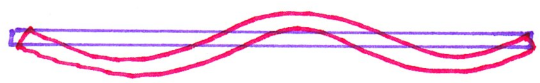
$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

and for the general Euler's Buckling eqn we had

$$P = \frac{n^2 \pi^2 EI}{L^2}$$ where the n's represent the various buckling modes

1  $n=1$ $P_{cr} = P_{cr1}$

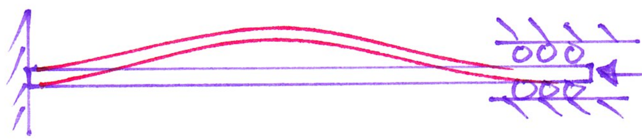
2  $n=2$ $P_{cr} = 4P_{cr1}$

3  $n=3$ $P_{cr} = 9P_{cr1}$

4  $n=4$ $P_{cr} = 16P_{cr1}$

Remember that the first case $n=1$ is the one that will always happen first and will ultimately result in failure of the structure. These are analogous to the vibration modes of a beam, specifically the various natural frequencies.

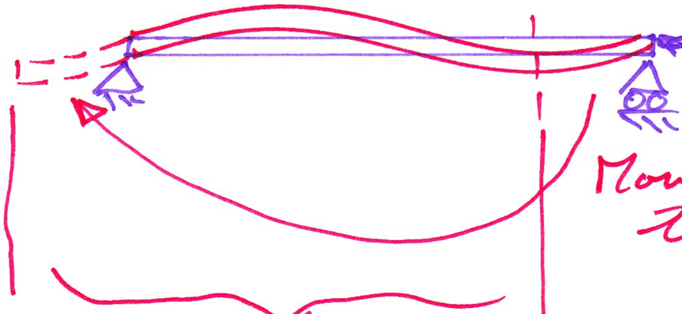
Now consider the scenario where the end conditions are such that the rotations are held fixed as shown below:



P

Called fixed-fixed

The curvature look more severe than the pin-pin scenario. Looking at the pictures of the deformed shapes for the various buckling modes, specificlly at the mode 2, n=2, case



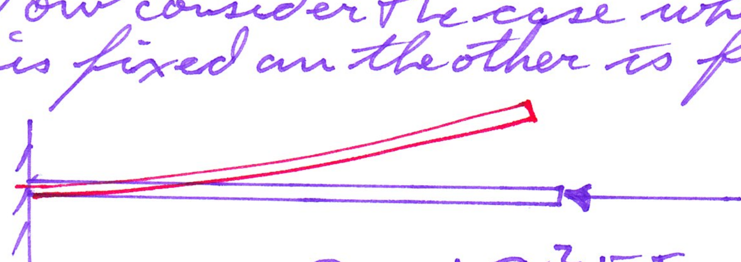
$n=2$

Move this piece over to the other side.

This then looks exactly like the above deformed shape

Thus for the fixed-fixed case we get $P_{cr} = \dfrac{4\pi^2 EI}{L^2}$

Now consider the case where one end is fixed an the other is free as:

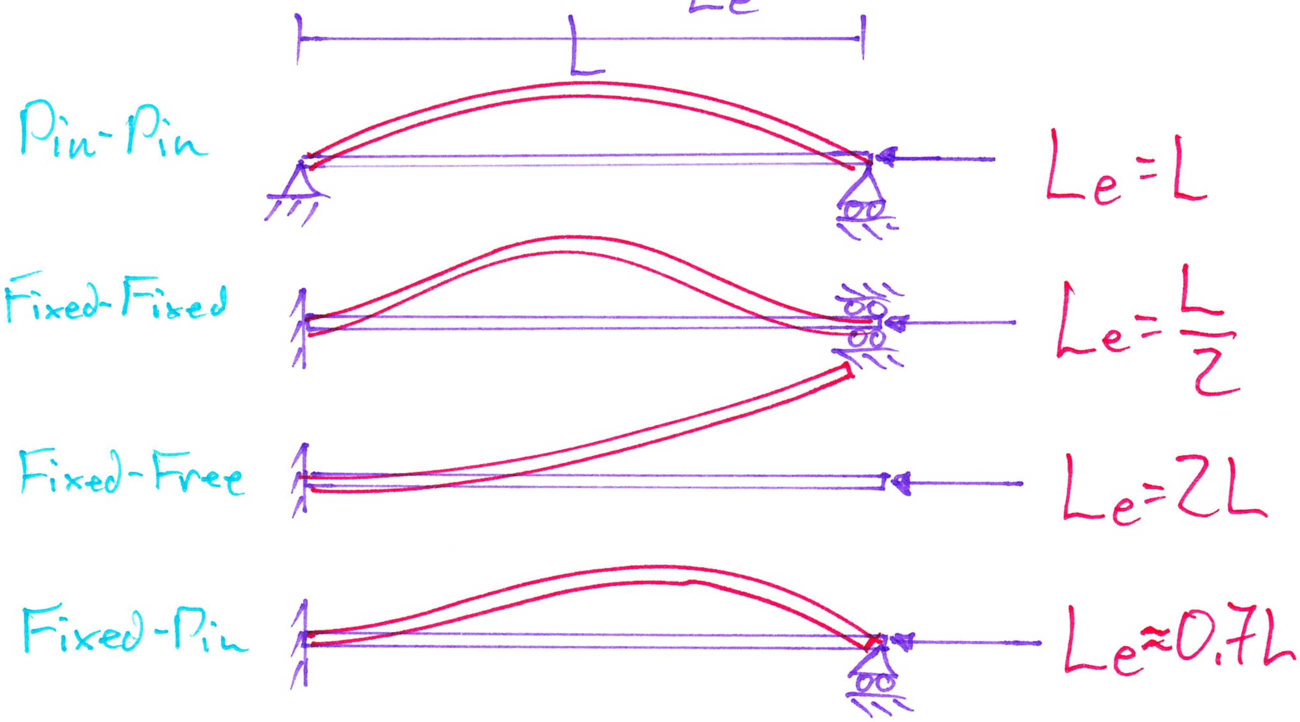


Called fixed-free

$$P_{cr} = \frac{1}{4}\frac{\pi^2 EI}{L^2}$$

One approach to relate these scenarios is through the use of an effective length L_e

$$P_{cr} = \frac{\pi^2 E I}{L_e^2}$$



Go get yourself a piece of uncooked spaghetti and try these out. You will see that these are exactly what you observe in terms of deformations.

(Remember to keep that spaghetti afterwards, not sure how long this situation is going to last)

Now from a design perspective, just how thin can we make a member in compression and not have to worry about buckling:

Consider stress $\sigma = P/A$

or $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{L^2} \frac{I}{A}$ Properties of the cross section

Define the radius of gyration $k = \sqrt{\frac{I}{A}}$

so $\sigma_{cr} = \frac{\pi^2 E}{L^2} k^2$ Function of the beams geometry

Define the slenderness ratio L/k

so we can determine a minimum slenderness ratio $\left(\frac{L}{k}\right)_{min.} = \sqrt{\frac{\pi^2 E}{\sigma_{cr}}}$

use σ_{yld} for the chosen material, this way it will be equally (from an ideal perspective) likely to yield as buckle.