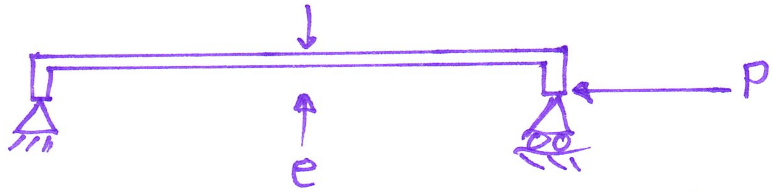
Eccentric Loading of Columns

Consider the following loading scenario:



The Moment produced by the applied load is

$$M = P(e + v_y)$$

The Internal Restoring Moment due to the deflection of the beam is:

$$M = EI \frac{\partial^2 v_y}{\partial x^2}$$

These must sum to zero, so we get

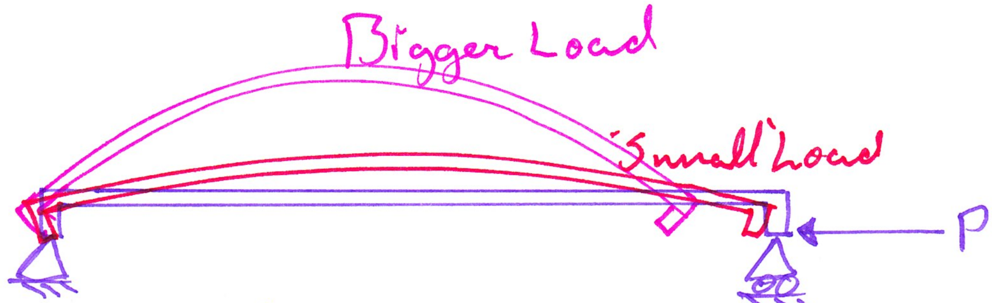
$$EI \frac{\partial^2 v_y}{\partial x^2} + P v_y + Pe = 0$$

This is a second order Non Homogeneous D.E. and the solution is:

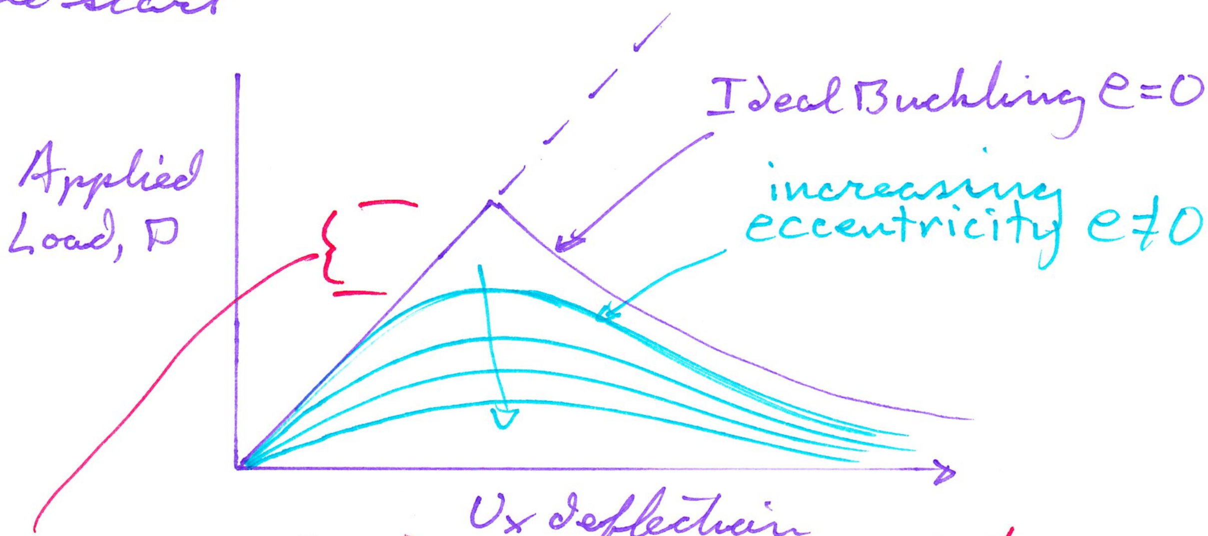
$$v_{y_{max}} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\,\frac{L}{2}\right) - 1\right]$$

In this case, the beam <u>does not</u> experience sudden buckling, but rather simply bends as soon as the load is applied.

This equation is referred to as the secant formula in the text.

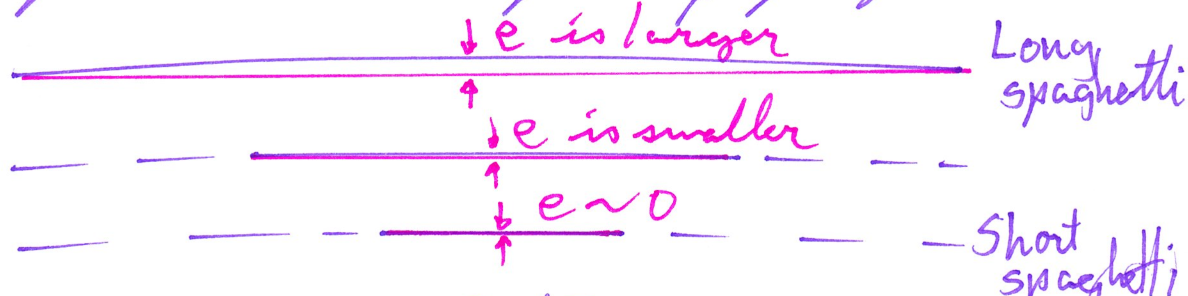


In this case, the beam bends right from the start



any eccentricity results in a substantial decrease is the load from the Ideal critical buckling load.

This is why the force required to cause buckling of a longer piece of spaghetti is much lower than the ideal value when compared to a shorter piece of spaghetti.



Now in terms of stress, clearly the maximum stress would occur where the bending moment is maximum, which is at the midspan of the beam. There are two sources of stress

$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

due to compressive loading

due to the bending moment

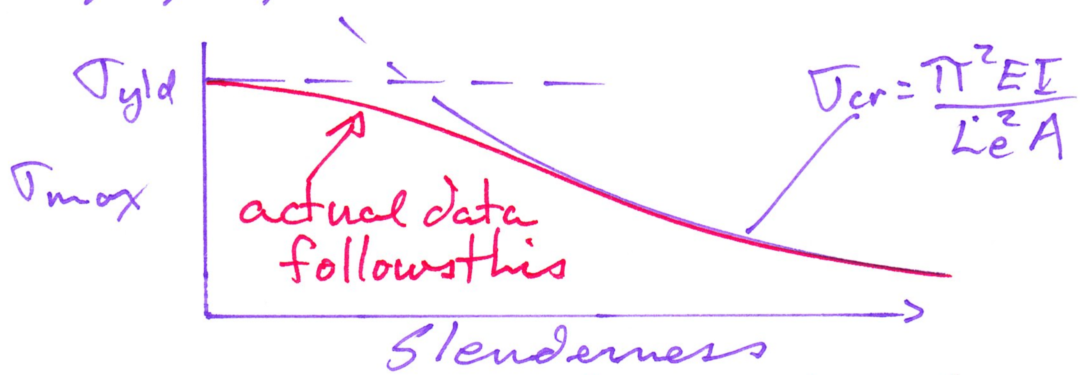
The maximum Moment $= P(v_{y\,max} + e)$

so

$$\sigma_{max} = \frac{P}{A} + \frac{P(v_{y\,max} + e)\cdot c}{I}$$

$$= \frac{P}{A} + \frac{e \cdot c \cdot \sec\left(\sqrt{\frac{P}{EI}}\,\frac{L}{2}\right)}{I}$$

$= \sigma_{yld}$ Set this expression equal to the yield strength of the material in order to optimize the dimensions from a design perspective



Different fits for different materials are presented in the textbook pp. 726 for 6061-T6.