Rubrics of home work #1

10 points in total

Q1 (A3)

10/10

Verify by multiplying $[A]\,[A]^{-1} = [I]$

2 **A.3** $[D]^{-1} = \dfrac{[C]^T}{|[D]|}$

$$[D] = \begin{bmatrix} 5 & 2 & 1 \\ 2 & 10 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$

$$\text{Cofactor matrix, } [C] = \begin{bmatrix} 50 & -10 & -10 \\ -10 & 24 & 2 \\ -10 & 2 & 46 \end{bmatrix} \quad 1$$

$$|[D]| = 5(50) + (2)(-10) + (1)(-10) = 220$$

$$[D]^{-1} = \frac{1}{220}\begin{bmatrix} 50 & -10 & -10 \\ -10 & 24 & 2 \\ -10 & 2 & 46 \end{bmatrix} = \begin{bmatrix} \frac{5}{22} & -\frac{1}{22} & -\frac{1}{22} \\ -\frac{1}{22} & \frac{6}{55} & \frac{1}{110} \\ -\frac{1}{22} & \frac{1}{110} & \frac{23}{110} \end{bmatrix} \quad 1$$

A.4 Nonsense

A.5 $[B] = \begin{bmatrix} 2 & 0 \\ 2 & 4 \end{bmatrix}$

(1) $\left[\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{array}\right]$ divide 1st row by 2

(2) $\left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & 0 \\ 2 & 4 & 0 & 1 \end{array}\right]$ □multiply 1st row by -2 and add to row 2

(3) $\left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 4 & -1 & 1 \end{array}\right]$ divide 2nd row by 4

(4) $\left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{4} & \frac{1}{4} \end{array}\right] \quad \therefore [B]^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix}$

A.6 $[D]^{-1}$ by row reduction

$$[D] = \begin{bmatrix} 5 & 2 & 1 \\ 2 & 10 & 0 \\ 1 & 0 & 5 \end{bmatrix} \rightarrow \left[\begin{array}{ccc|ccc} 5 & 2 & 1 & 1 & 0 & 0 \\ 2 & 10 & 0 & 0 & 1 & 0 \\ 1 & 0 & 5 & 0 & 0 & 1 \end{array}\right]$$

a. Divide R_1 by 5

$$\left[\begin{array}{ccc|ccc} 1 & \frac{2}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 \\ 2 & 10 & 0 & 0 & 1 & 0 \\ 1 & 0 & 5 & 0 & 0 & 1 \end{array}\right]$$

b. Multiply R_1 by -2 + Add to R_2

Q2 (A8) and Q3 (A9)

$$[B]^T[A]^T = \begin{bmatrix} b_{11}(a_{11})+b_{21}(a_{12}) & b_{11}(a_{21})+b_{21}(a_{22}) \\ b_{12}(a_{11})+b_{22}(a_{12}) & b_{12}(a_{21})+b_{22}(a_{22}) \\ b_{13}(a_{11})+b_{23}(a_{12}) & b_{13}(a_{21})+b_{23}(a_{22}) \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}(b_{11})+a_{12}(b_{21}) & a_{21}(b_{11})+a_{22}(b_{21}) \\ a_{11}(b_{12})+a_{12}(b_{22}) & a_{21}(b_{12})+a_{22}(b_{22}) \\ a_{11}(b_{13})+a_{12}(b_{23}) & a_{21}(b_{13})+a_{22}(b_{23}) \end{bmatrix}$$

Answer: $([A]\,[B])^T = [B]^T\,[A]^T$

2 **A.8** $[T] = \begin{bmatrix} C & S \\ -S & C \end{bmatrix}$

$[C] = \begin{bmatrix} C & S \\ -S & C \end{bmatrix}$ $\qquad [C]^T = \begin{bmatrix} C & -S \\ S & C \end{bmatrix}$ 0.5

$|[T]| = C^2 + S^2 = 1$ 0.5

$[T]^{-1} = \dfrac{[C]^T}{|[T]|} = \begin{bmatrix} C & -S \\ S & C \end{bmatrix}$ 0.5

and

$[T]^T = \begin{bmatrix} C & -S \\ S & C \end{bmatrix}$ 0.5

$\therefore \quad [T]^T = [T]^{-1}$ and T is an orthogonal matrix

2 **A.9** Show $\{X\}^T[A]\{X\}$ is symmetric. Given

$\{X\} = \begin{bmatrix} x & y \\ 1 & x \end{bmatrix}, [A] = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$\{X\}^T = \begin{bmatrix} x & 1 \\ y & x \end{bmatrix}$ 0.5

$\{X\}^T[A]\{X\} = \begin{bmatrix} x & 1 \\ y & x \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} x & y \\ 1 & x \end{bmatrix}$ 0.5

$= \begin{bmatrix} ax+b & bx+c \\ ay+bx & by+cx \end{bmatrix}\begin{bmatrix} x & y \\ 1 & x \end{bmatrix}$

Q3 (A9) and Q4 (A11)

$$= \begin{bmatrix} ax^2 + bx + bx + c & axy + by + bx^2 + cx \\ axy + bx^2 + by + cx & ay^2 + bxy + bxy + cx^2 \end{bmatrix}$$ 0.5

as the 1–2 term = 2–1 term $\{X\}^T[A]\{X\}$ is symmetric. 0.5

A.10 Evaluate $[K] = \int_0^L [B]^T \; E\,[B]\,dx$, $[B] = \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix}$

$$[K] = \int_0^L \begin{Bmatrix} -\frac{1}{L} \\ \frac{1}{L} \end{Bmatrix} E \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} dx$$

$$[K] = \int_0^L \begin{bmatrix} \frac{1}{L^2} & \frac{-1}{L^2} \\ \frac{-1}{L^2} & \frac{1}{L^2} \end{bmatrix} E\,dx$$

$$[K] = E \begin{bmatrix} \frac{1}{L} & \frac{1}{L} \\ \frac{-1}{L} & \frac{1}{L} \end{bmatrix} = \frac{E}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

(Should multiply by A to get actual $[K]$ for a bar)

2 **A.11** The following integral represents the strain energy in a bar

$$U = \frac{A}{2} \int_0^L \{d\}^T \, [B]^T [D]\,[B]\,\{d\}\,dx$$

where $\{d\} = \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}$ $\quad [B] = \begin{bmatrix} \frac{-1}{L} & \frac{1}{L} \end{bmatrix}$ $\quad [D] = E$

Show that $\frac{dU}{d\{d\}}$ yields $[k]\{d\}$, where $[k]$ is the bar stiffness matrix given by

$$[k] = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$\{d\}^T = [d_1 \; d_2]$ $\qquad [B]^T = \begin{Bmatrix} \frac{-1}{L} \\ \frac{1}{L} \end{Bmatrix}$

$$U = \frac{A}{2} \int_0^L \{d\}^T \, [B]^T [D]\,[B]\,\{d\}\,dx$$ 0.5

$$U = \frac{AL}{2} \; \{d\}^T [B]^T [D]^T [B]\,\{d\} = \frac{AL}{2} \; [d_1 \quad d_2] \begin{Bmatrix} \frac{-1}{L} \\ \frac{1}{L} \end{Bmatrix} E \begin{bmatrix} \frac{-1}{L} & \frac{1}{L} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}$$

$$U = \frac{AL}{2} \begin{bmatrix} \frac{d_2 - d_1}{L} \end{bmatrix} [E] \begin{bmatrix} \frac{-1}{L} & \frac{1}{L} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}$$

$$U = \frac{AEL}{2} \begin{bmatrix} \frac{d_2 - d_1}{L} \end{bmatrix} \begin{bmatrix} \frac{-1}{L} & \frac{1}{L} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} = \frac{AEL}{2} \begin{bmatrix} \frac{d_1 - d_2}{L^2} & \frac{-d_1 + d_2}{L^2} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}$$

Q4 (A11) and Q5 (A12)

$$U = \frac{AEL}{2}\left[\frac{d_1^2 - d_1d_2 - d_1d_2 + d_2^2}{L^2}\right] = \frac{AE}{2L}\,[d_1^2 - 2d_1d_2 + d_2^2]$$ 0.5

$$\frac{dU}{d\ d} = \begin{Bmatrix}\frac{\partial U}{\partial d_1}\\ \frac{\partial U}{\partial d_2}\end{Bmatrix} = \begin{Bmatrix}\frac{AE}{2L}\,2d_1 - 2d_2\\ \frac{AE}{2L}\,2d_2 - 2d_1\end{Bmatrix} = \frac{AE}{L}\begin{Bmatrix}d_1 - d_2\\ -d_2 - d_1\end{Bmatrix} = \frac{AE}{L}\begin{Bmatrix}d_1 - d_2\\ -d_1 + d_2\end{Bmatrix}$$

$$= \frac{AE}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}d_1\\ d_2\end{Bmatrix}$$ 0.5

$$\frac{dU}{d\ d} = \frac{AE}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{Bmatrix}d_1\\ d_2\end{Bmatrix} = [k]\begin{Bmatrix}d_1\\ d_2\end{Bmatrix} = [k]\,\{d\} \text{ knowing that } [k] = \frac{AE}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$ 0.5

Thus $\frac{dU}{d\ d} = [k]\,\{d\}$

② A12.

det|K| = 0 0.5

positive semidefinite 0.5

sigular 0.5

det | reduced K | ≠ 0 0.5