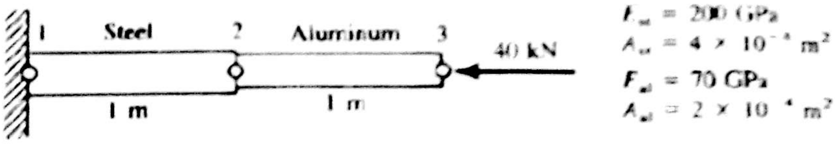
3.8



$$[k^{(1)}] = \frac{(4\times 10^{-4}\,\text{m}^2)(200\times 10^{6}\,\frac{\text{kN}}{\text{m}^2})}{1\,\text{m}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k^{(1)}] = 800\times 10^{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\frac{\text{kN}}{\text{m}}$$

$$[k^{(2)}] = \frac{(2\times 10^{-4}\,\text{m}^2)(70\times 10^{6}\,\frac{\text{kN}}{\text{m}^2})}{1\,\text{m}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k^{(2)}] = 140\times 10^{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\frac{\text{kN}}{\text{m}}$$

$$\begin{Bmatrix} F_{1x} = 0 \\ F_{2x} \\ F_{3x} = -40\text{ kN} \end{Bmatrix} = 10^{2}\begin{bmatrix} 800 & -800 & 0 \\ -800 & 940 & -140 \\ 0 & -140 & 140 \end{bmatrix}\begin{Bmatrix} u_1 = 0 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\Rightarrow \quad 0 = 10^{2}\,(940\,u_2 - 140\,u_3) \;\Rightarrow\; u_3 = 6.741\,u_2$$

$$\Rightarrow -40000 = 10^{2}\,(-140\,u_2 + 140\,u_3)$$

Substituting (1) into (2)

$$\Rightarrow \quad -40000 = 10^{2}\,(-140\,u_2 + 140\,(6.714)\,u_2)$$

$$\Rightarrow \quad u_2 = -0.50\times 10^{-3}\text{ m}$$

$$\Rightarrow \quad u_3 = -3.356\times 10^{-3}\text{ m}$$

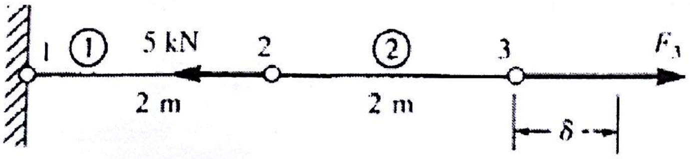
$$F_{1x} = 10^{2}\,(-800\times(-0.50\times 10^{-3}))$$

$$\Rightarrow \quad F_{1x} = 40\text{ kN}$$

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = 800 \times 10^2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ -0.50 \times 10^{-3} \end{Bmatrix} \Rightarrow \begin{matrix} f_{1x}^{(1)} = 40 \text{ kN} \\ f_{2x}^{(1)} = -40 \text{ kN} \end{matrix}$$

$$\begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = 140 \times 10^2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} -0.50 \times 10^{-3} \\ -3.356 \times 10^{-3} \end{Bmatrix} \Rightarrow \begin{matrix} f_{2x}^{(2)} = 40 \text{ kN} \\ f_{3x}^{(2)} = -40 \text{ kN} \end{matrix}$$

3.9



E = 210 GPa
$A = 4 \times 10^{-4}$ m²
δ = 25 mm

$$[k_{1-2}] = [k_{2-3}] = 4.2 \times 10^4 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \frac{\text{kN}}{\text{m}}$$

$$\{F\} = [K]\{d\}$$

$$\begin{Bmatrix} F_{1x} = 0 \\ F_{2x} = -5 \text{ kN} \\ F_{3x} = ? \end{Bmatrix} = 4.2 \times 10^4 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 = ? \\ u_3 = 0.025 \text{ m} \end{Bmatrix}$$

$$\Rightarrow \frac{-5 \text{ kN}}{4.2 \times 10^4} = 2\,u_2 - 1(0.025)$$

$$\Rightarrow \quad u_2 = 0.01244 \text{ m}$$

$$F_{3x} = 4.2 \times 10^4 \,[-1 \;\; 1] \begin{Bmatrix} 0.01244 \\ 0.025 \end{Bmatrix} \Rightarrow F_{3x} = 527.5 \text{ kN}$$

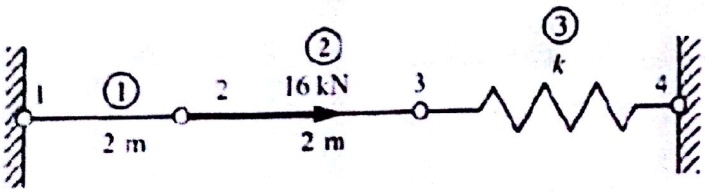
$$F_{1x} = 4.2 \times 10^4 \,[1 \; -1] \begin{Bmatrix} 0 \\ 0.01244 \end{Bmatrix} \Rightarrow F_{1x} = -522.5 \text{ kN}$$

Element forces

$$\begin{Bmatrix} f_{1x}^{(1)} \\ f_{2x}^{(1)} \end{Bmatrix} = 4.2 \times 10^4 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.01244 \end{Bmatrix} \Rightarrow \begin{matrix} f_{1x}^{(1)} = -522.5 \text{ kN} \\ f_{2x}^{(1)} = 522.5 \text{ kN} \end{matrix}$$

$$\begin{Bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{Bmatrix} = 4.2 \times 10^4 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} 0.01244 \\ 0.025 \end{Bmatrix} \Rightarrow \begin{matrix} f_{2x}^{(2)} = -527.5 \text{ kN} \\ f_{3x}^{(2)} = 527.5 \text{ kN} \end{matrix}$$

3.10



E = 70 GPa
$A = 2 \times 10^{-4}$ m²
k = 2000 kN/m

$$[k^{(1)}] = [k^{(2)}] = 7000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k^{(2)}] = 2000\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\{F\} = [K]\{d\}$$

$$\begin{Bmatrix} F_{1x} = ? \\ F_{2x} = 16\text{kN} \\ F_{3x} = 0 \\ F_{4x} = ? \end{Bmatrix} = 10^3\begin{bmatrix} 7 & -7 & 0 & 0 \\ -7 & 14 & -7 & 0 \\ 0 & -7 & 9 & -2 \\ 0 & 0 & -2 & 2 \end{bmatrix}\begin{Bmatrix} u_1 = 0 \\ u_2 = ? \\ u_3 = ? \\ u_4 = 0 \end{Bmatrix}$$

$$\Rightarrow \quad 16 = 10^3\,[14\,u_2 - 7\,u_3] \qquad (1)$$

$$0 = 10^3\,[-7\,u_2 + 9\,u_3]$$

$$\Rightarrow \quad u_3 = \frac{7}{9}\,u_2 \qquad (2)$$

Substituting (2) into (1)

$$\Rightarrow \quad \frac{8}{10^3} = 14\,u_2 - 7 \times \frac{7}{9}\,u_2$$

$$\Rightarrow \quad u_2 = 1.870 \times 10^{-3}\text{ m}$$

$$\Rightarrow \quad u_3 = 1.454 \times 10^{-3}\text{ m}$$

Element (1)

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = 7 \times 10^3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 0 \\ 1.870\times10^{-3} \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \begin{Bmatrix} -13.10 \\ 13.10 \end{Bmatrix}\text{kN}$$

Element (2)

$$\begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = 7 \times 10^3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 1.870\times10^{-3} \\ 1.454\times10^{-3} \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{2x} \\ f_{3x} \end{Bmatrix} = \begin{Bmatrix} 2.90 \\ -2.90 \end{Bmatrix}\text{kN}$$

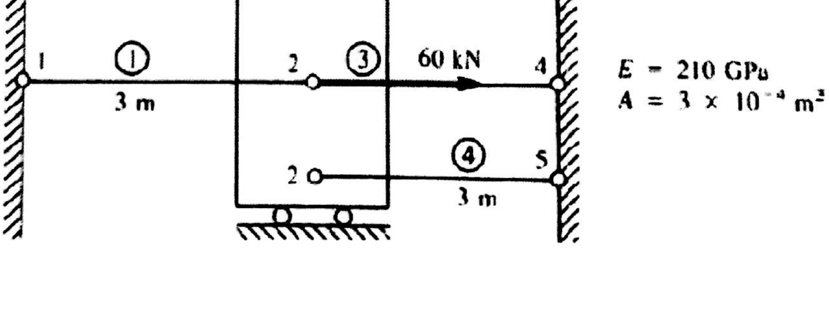
Element (3)

$$\begin{Bmatrix} f_{3x} \\ f_{4x} \end{Bmatrix} = 2 \times 10^3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} 1.454\times10^{-3} \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} f_{3x} \\ f_{4x} \end{Bmatrix} = \begin{Bmatrix} 2.90 \\ -2.90 \end{Bmatrix}\text{kN}$$

$$F_{1x} = 10^3\,[7\ \ -7]\begin{Bmatrix} 0 \\ 1.87\times10^{-3} \end{Bmatrix} = F_{1x} = -13.10\text{ kN}$$

$$F_{4x} = 10^3\,[-2\ 2]\begin{Bmatrix} 1.454\times10^{-3} \\ 0 \end{Bmatrix} \Rightarrow F_{4x} = -2.90\text{ kN}$$

3.11



$$[k_{1-2}] = [k_{2-3}] = [k_{2-4}] = [k_{2-5}] = 2.1 \times 10^7 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\{F\} = [K]\,\{d\}$$

$$\begin{Bmatrix} F_{1x} = 0 \\ F_{2x} = 60\text{ kN} \\ F_{3x} = 0 \\ F_{4x} = 0 \\ F_{5x} = 0 \end{Bmatrix} = 2.1 \times 10^7 \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 4 & -1 & -1 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ u_2 \\ u_3 = 0 \\ u_4 = 0 \\ u_5 = 0 \end{Bmatrix}$$

$$\Rightarrow \quad u_2 = 7.144 \times 10^{-4}\text{ m}$$

Reactions

$$F_{1x} = (2.1 \times 10^7)(-1)(u_2) \;\Rightarrow\; F_{1x} = -15000\text{ N}$$

$$F_{3x} = (2.1 \times 10^7)(-1)(u_2) \;\Rightarrow\; F_{3x} = -15000\text{ N}$$

$$F_{4x} = (2.1 \times 10^7)(-1)(u_2) \;\Rightarrow\; F_{4x} = -15000\text{ N}$$

$$F_{5x} = (2.1 \times 10^7)(-1)(u_2) \;\Rightarrow\; F_{5x} = -15000\text{ N}$$

Element forces

$$f_{1-2} = -f_{2-3} = -f_{2-4} = -f_{2-5} = (2.1 \times 10^7)(u_2)$$

$$\Rightarrow \quad f_{1-2} = 15000\text{ N}$$

$$f_{2-3} = -15000\text{ N}$$

$$f_{2-4} = -15000\text{ N}$$

$$f_{2-5} = -15000\text{ N}$$

3.12

$$\frac{P}{A(x)} = E\frac{du}{dx}$$

$$u = \int \frac{P}{A(x)E}\,dx$$

$$u = \int \frac{P}{A_0\left(1 + \frac{x}{L}\right)E}\,dx$$

$$= \int \frac{PL}{A_0 L\left(1 + \frac{x}{L}\right)E}\,dx$$

$$= \int \frac{PL}{A_0 (L + x) E}\,dx$$

$$= \int \frac{PL}{A_0 E\, u}\,du \quad \text{(Change variable } u = L + x \text{ and } du = dx\text{)}$$

$$= \frac{PL}{A_0 E}\int \frac{1}{u}\,du$$

$$= \frac{PL}{A_0 E}\, ln\, u$$

$$\Rightarrow \quad u = \frac{PL}{A_0E}\, ln\,(L + x)$$

$$u = \frac{(-1000)(20)}{2 \times 10 \times 10^6}\, ln\,(20 + x)$$

$$u = -10^{-3}\, ln\,(20 + x)$$

$$u(x = 0) = (-\,ln\,20) \times 10^{-3}$$

$$= -2.996 \times 10^{-3} \text{ in.}$$

$$u(x = 10) = (-ln\,(20 + 10))\,(-10^{-3})$$

$$= -3.401 \times 10^{-3} \text{ in.}$$

Two elements

$$A\ \frac{L}{4} = A_0\left(1 + \frac{\frac{L}{4}}{L}\right) = A_0\ 1 + \frac{1}{4} = \frac{5}{4}A_0$$

$$A\ \frac{3}{4}L = A_0\left(1 + \frac{\frac{3}{4}L}{L}\right) = A_0\ 1 + \frac{3}{4} = \frac{7}{4}A_0$$

$$[k^{(1)}] = \frac{5}{4}\frac{A_0E}{\frac{L}{2}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[k^{(2)}] = \frac{7}{4}\frac{A_0E}{\frac{L}{2}}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{Bmatrix} -P \\ 0 \\ F_{3x} \end{Bmatrix} = \frac{A_0E}{\frac{L}{2}}\begin{bmatrix} \frac{5}{4} & \frac{-5}{4} & 0 \\ \frac{-5}{4} & \frac{5}{4} + \frac{7}{4} & \frac{-7}{4} \\ 0 & \frac{-7}{4} & \frac{7}{4} \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_{3=0} \end{Bmatrix}$$

$$\Rightarrow \quad \frac{A_0E}{\frac{L}{2}}\ (\frac{5}{4}u_1 - \frac{5}{4}\ u_2) = -P \qquad (1)$$

$$\Rightarrow \quad \frac{A_0E}{\frac{L}{2}}\ (\frac{-5}{4}\ u_1 + 3\ u_2) = 0$$

$$\Rightarrow \quad u_2 = \frac{5}{4 \times 3}\ u_1 = \frac{5}{12}u_1 \qquad (2)$$

Substituting (2) into (1)

$$\Rightarrow \quad \frac{A_0E}{\frac{L}{2}}\ (\frac{5}{4}u_1 - \frac{5}{4}\ (\frac{5}{12}u_1)) = -P$$

$$\Rightarrow \quad \left[\frac{12}{12}\ \frac{5}{4} - \frac{5}{4}\ \frac{5}{12}\right]u_1 = \frac{-PL}{2A_0E}$$

$$\Rightarrow \quad \left[\frac{60-25}{48}\right]u_1 = \frac{-PL}{2A_0E}$$

$$\Rightarrow \quad u_1 = \frac{-PL}{\not{2}A_0E}\ \frac{\overset{24}{\not{48}}}{35}$$

$$\Rightarrow \quad u_1 = \frac{-PL}{A_0E}\ \frac{24}{35}$$

$$\Rightarrow \quad u_2 = \frac{5}{12}\ \frac{24}{35}\left(\frac{-PL}{A_0E}\right)$$

$$\Rightarrow \quad u_2 = -\frac{2}{7}\ \frac{PL}{A_0E}$$

Now $\quad A_0 = 2\text{ in.}^2,\ L = 20\text{ in.},\ E = 10 \times 10^6\text{ psi}$

$P = 1000\text{ lb}$

$$u_1 = -\frac{(1000)(20)}{2(10 \times 10^6)} \times \frac{24}{35}$$

$$\Rightarrow \quad u_1 = -0.6857 \times 10^{-3}\text{ in.}$$

$$\Rightarrow \quad u_2 = \frac{5}{12}\ (-0.6857 \times 10^{-3})$$

$$\Rightarrow \quad u_2 = -0.2857 \times 10^{-3}\text{ in.}$$

One element

$$A = A_0\left(1 + \frac{\frac{L}{2}}{L}\right) = A_0\left(1 + \frac{1}{2}\right) = \frac{3}{2}\ A_0$$

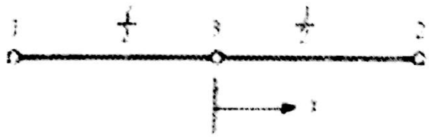
$$\begin{Bmatrix} -P \\ 0 \end{Bmatrix} = \frac{\frac{3}{2}A_0E}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 = 0 \end{Bmatrix}$$

$$\Rightarrow \quad u_1 = \frac{-PL}{\frac{3}{2}A_0E}$$

$$\Rightarrow \quad u_1 = -\frac{2}{3}\ \frac{(1000)(20)}{(2)(10 \times 10^6)}$$

$$\Rightarrow \quad u_1 = -0.667 \times 10^{-3}\text{ in.}$$

3.13



$$u = a_1 + a_2x + a_3x^2 \qquad \text{(A)}$$

$$u(0) = u_2 = a_1 \qquad (1)$$

$$u\left(-\frac{l}{2}\right) = u_1 = u_2 + a_2\left(-\frac{l}{2}\right) + a_3\left(-\frac{l}{2}\right)^2 \qquad (2)$$

$$u\left(\frac{l}{2}\right) = u_3 = u_2 + a_2\left(\frac{l}{2}\right) + a_3\left(\frac{l}{2}\right)^2 \qquad (3)$$

Solving for a_2 and a_3 from (2) and (3)

$$a_2 = \frac{u_3 - u_1}{l}, \; a_3 = \frac{2(u_1 + u_3 - 2u_2)}{l^2} \quad (4)$$

By (1) and (4) into (A)

$$u = u_2 + \left(\frac{u_3 - u_1}{l}\right) x + \frac{2(u_1 + u_3 - 2u_2)}{l^2} x^2 \quad (5)$$

$$u = [N]\{d\} \quad (6)$$

$$u = \begin{bmatrix} \frac{-x}{l} + \frac{2x^2}{l^2} & 1 - \frac{4x^2}{l^2} & \frac{x}{l} + \frac{2x^2}{l^2} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \quad (7)$$

$$\{\epsilon\} = \frac{\partial u}{\partial x} = [B]\{d\} = \frac{\partial N}{\partial x}\{d\} \quad (8)$$

Using (7) in (8)

$$\{\epsilon\} = \begin{bmatrix} -\frac{1}{l} + \frac{4x}{l^2} & \frac{-8x}{l^2} & \frac{1}{l} + \frac{4x}{l^2} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \quad (9)$$

$$\therefore \; [B] = \begin{bmatrix} -\frac{1}{l} + \frac{4x}{l^2} & \frac{-8x}{l^2} & \frac{1}{l} + \frac{4x}{l^2} \end{bmatrix} \quad (10)$$

$$[K] = A \int_{-l/2}^{l/2} [B^T]\, E\, [B]\, dx \quad (11)$$

A = cross sectional area of the bar

E = Young's Modulus of the bar

3.14 Given $u = a + bx^2$ for 2 noded bar

$$\epsilon = \frac{du}{dx} = 2bx$$

$$u(0) = u_1 = a$$

$$u(L) = u_2 = u_1 + bL^2$$

$$\therefore \quad b = \frac{u_2 - u_1}{L^2}$$

$$u = u_1 + \left[\frac{u_2 - u_1}{L^2}\right] x^2$$

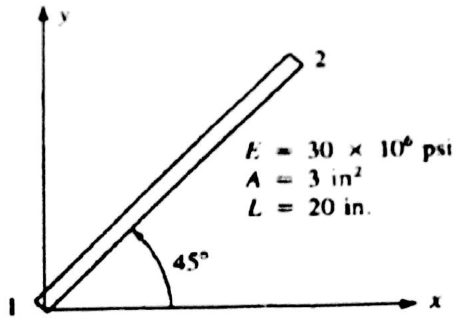
This displacement function allows for a rigid body displacement as the $a = u_1$ term does this. Also should allow for constant strain, but have $\epsilon = 2bx$ or a linear strain. Therefore, not complete. Need to complete 2nd degree polynomial and 3rd node for compatible function.

Try $\quad u = a_1 + a_2 x + a_3 x^2$

$$\frac{du}{dx} = a_2 + 2a_3 x$$

'a_2' allows for constant strain term.

3.15 (a)

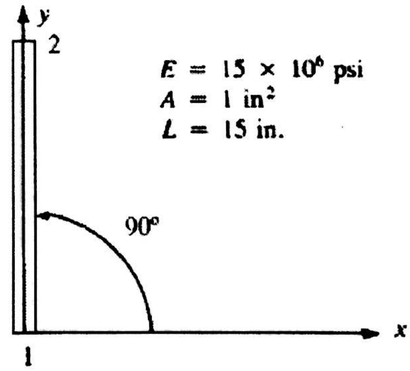


$$C = \frac{1}{\sqrt{2}},\ S = \frac{1}{\sqrt{2}}$$

$$[K] = \frac{EA}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ & S^2 & -CS & -S^2 \\ & & C^2 & CS \\ & & & S^2 \end{bmatrix}$$

$$[K] = 2.25 \times 10^6 \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \frac{\text{lb}}{\text{in.}}$$

(b)

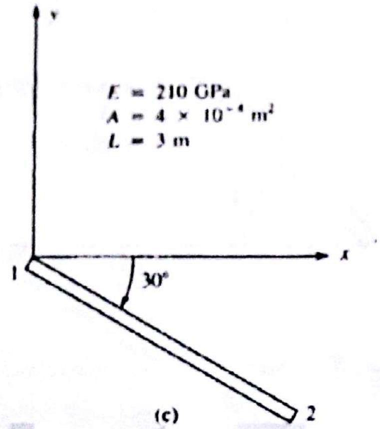


$$C = 0,\ S = 1$$

$$[K] = \frac{15 \times 10^6 \times 1}{15}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$[K] = \frac{10^6}{4}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \frac{\text{lb}}{\text{in.}}$$

(c)

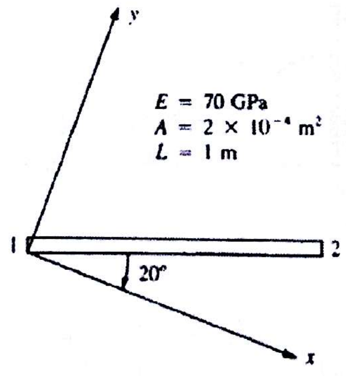


$$C = \frac{\sqrt{3}}{2}, \quad S = -\frac{1}{2}$$

$$[K] = \frac{(210 \times 10^6)(4 \times 10^{-4})}{3} \begin{bmatrix} \frac{3}{4} & -\frac{\sqrt{3}}{4} & -\frac{3}{4} & \frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{1}{4} & \frac{\sqrt{3}}{4} & -\frac{1}{4} \\ -\frac{3}{4} & \frac{\sqrt{3}}{4} & \frac{3}{4} & -\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & -\frac{1}{4} & -\frac{\sqrt{3}}{4} & \frac{1}{4} \end{bmatrix}$$

$$\underline{K} = 7000 \begin{bmatrix} 3 & -\sqrt{3} & -3 & \sqrt{3} \\ -\sqrt{3} & 1 & \sqrt{3} & -1 \\ -3 & \sqrt{3} & 3 & -\sqrt{3} \\ \sqrt{3} & -1 & -\sqrt{3} & 1 \end{bmatrix} \frac{\text{kN}}{\text{m}}$$

(d)



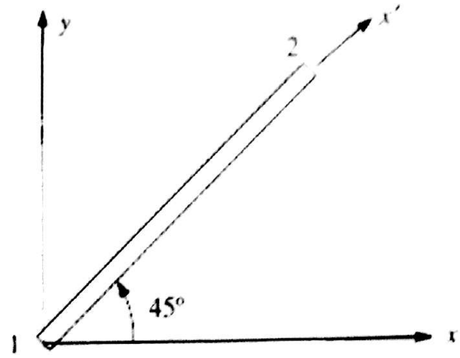
$C = 0.9397 \qquad C^2 = 0.883 \qquad CS = 0.321$

$S = 0.3420 \qquad S^2 = 0.117$

$$[K] = \frac{(70 \times 10^4)(2 \times 10^{-4})}{1} \begin{bmatrix} 0.883 & 0.321 & -0.883 & -0.321 \\ 0.321 & 0.883 & -0.321 & -0.883 \\ -0.883 & -0.321 & 0.883 & 0.321 \\ -0.321 & -0.883 & 0.321 & 0.883 \end{bmatrix}$$

$$[K] = 1.4 \times 10^7 \begin{bmatrix} 0.883 & 0.321 & -0.883 & -0.321 \\ 0.321 & 0.883 & -0.321 & -0.883 \\ -0.883 & -0.321 & 0.883 & 0.321 \\ -0.321 & -0.883 & 0.321 & 0.883 \end{bmatrix} \frac{\text{N}}{\text{m}}$$

3.16 (a)



$C = 0.707$ $u_1 = 0.5$ in. $v_1 = 0.0$ in.

$S = 0.707$ $u_2 = 0.25$ in. $v_2 = 0.75$ in.

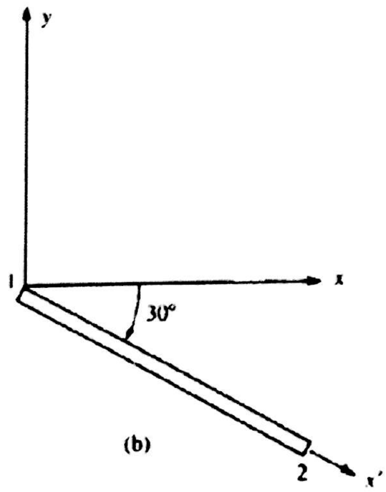
$$u'_1 = u_1\,C + v_1\,S = 0.5\,(0.707) + (0.0)\,(0.707)$$

$$\Rightarrow \quad u'_1 = 0.3536 \text{ in.}$$

$$u'_2 = u_2\,C + v_2\,S = (0.25)\,(0.707) + (0.75)\,(0.707)$$

$$u'_2 = 0.707 \text{ in.}$$

(b)



$$C = \frac{\sqrt{3}}{2}, \quad S = -\frac{1}{2}$$

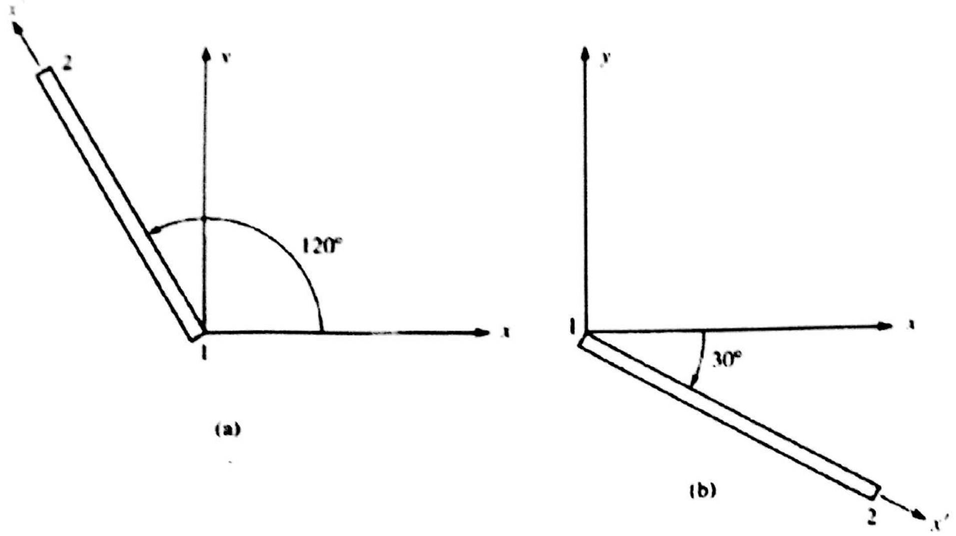
$$u'_1 = u_1\,C + v_1\,S = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + (0)\; -\frac{1}{2}$$

$$\Rightarrow \quad u'_1 = 0.433 \text{ in.}$$

$$u'_2 = u_2 C + v_2 S = \left(\frac{1}{4}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{3}{4}\right) - \frac{1}{2}$$

$$\Rightarrow \quad u'_2 = -0.1585 \text{ in.}$$

3.17



$u_1 = 0.0$ $u_2 = 5.0$ mm $E = 210$ GPa

$v_1 = 2.5$ mm $v_2 = 3.0$ mm $A = 10 \times 10^{-4}$ m^2

$L = 3$ m

(a) We know that $\{d'\} = [T]\{d\}$

$$[T] = \begin{bmatrix} C & S & 0 & 0 \\ -S & C & 0 & 0 \\ 0 & 0 & C & S \\ 0 & 0 & -S & C \end{bmatrix}$$

$$C = \cos 120° = -0.5, S = \sin 120° = 0.866$$

$$\begin{Bmatrix} u'_1 \\ v'_1 \\ u'_2 \\ v'_2 \end{Bmatrix} = \begin{bmatrix} -0.5 & 0.866 & 0 & 0 \\ -0.866 & -0.5 & 0 & 0 \\ 0 & 0 & -0.5 & 0.866 \\ 0 & 0 & -0.866 & -0.5 \end{bmatrix} \begin{Bmatrix} 0.0 \\ 0.0025 \\ 0.005 \\ 0.003 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} u'_1 \\ v'_1 \\ u'_2 \\ v'_2 \end{Bmatrix} = \begin{Bmatrix} 0.002165 \\ -0.00125 \\ 0.000098 \\ -0.00583 \end{Bmatrix} \text{m} = \begin{Bmatrix} 2.165 \\ -1.25 \\ 0.098 \\ -5.830 \end{Bmatrix} \text{mm}$$

(b) $C = \cos(-30°) = 0.866, S = -0.5$

$$\begin{Bmatrix} u'_1 \\ v'_1 \\ u'_2 \\ v'_2 \end{Bmatrix} = \begin{bmatrix} 0.866 & -0.5 & 0 & 0 \\ 0.5 & 0.866 & 0 & 0 \\ 0 & 0 & 0.866 & -0.5 \\ 0 & 0 & 0.5 & 0.866 \end{bmatrix} \begin{Bmatrix} 0.0 \\ 0.0025 \\ 0.005 \\ 0.003 \end{Bmatrix}$$

$$\begin{Bmatrix} u'_1 \\ v'_1 \\ u'_2 \\ v'_2 \end{Bmatrix} = \begin{Bmatrix} -1.25 \\ 2.165 \\ 3.03 \\ 5.098 \end{Bmatrix} \text{ mm}$$

3.18

(a) $\sigma = \dfrac{E}{L} [-C \quad -S \quad C \quad S] \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}, \theta = 45°$

$C = \dfrac{\sqrt{2}}{2}, \quad S = \dfrac{\sqrt{2}}{2}, \quad E = 30 \times 10^6 \text{ psi}, \quad L = 60 \text{ in.}$

$$\sigma = \frac{30 \times 10^6}{60} \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0.02 \\ 0.04 \end{Bmatrix}$$

$\Rightarrow \sigma = 21200 \text{ psi}$

(b) $C = \dfrac{\sqrt{3}}{2}, S = \dfrac{1}{2}, E = 210 \text{ GPa}, L = 3 \text{ m}, \theta = 30°$

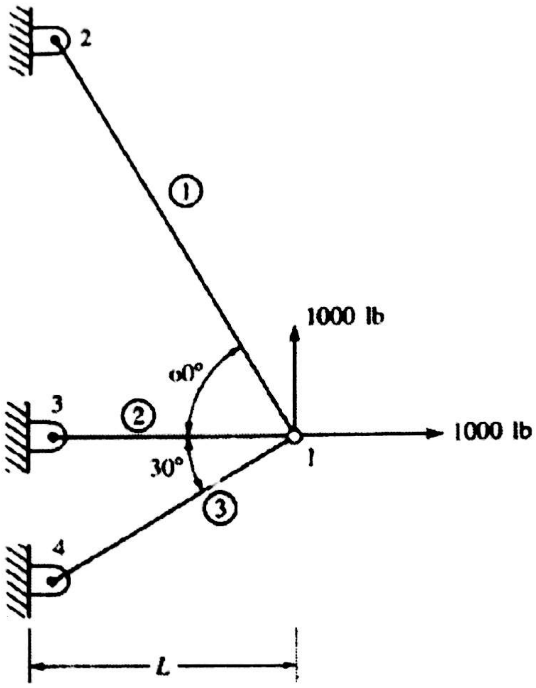
$$\sigma = \begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{Bmatrix} 0.25 \\ 0 \\ 1.00 \\ 0 \end{Bmatrix} \times 10^{-3} \times \frac{210 \times 10^6}{3}$$

$\Rightarrow \sigma = 45470 \dfrac{\text{kN}}{\text{m}^2}$

$\Rightarrow \sigma = 45.47 \text{ MPa}$

3.19

3.22



$$[K] = [T^T]\,[k']\,[T] = \frac{AE}{L}\begin{bmatrix} C^2 & CS & -C^2 & -CS \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -CS & C^2 & CS \\ -CS & -S^2 & CS & S^2 \end{bmatrix}$$

For element 1; $\theta = 120°$

$$[k^{(1)}] = \frac{AE}{2L}\begin{bmatrix} \frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} & \frac{1}{4} & -\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & -\frac{3}{4} & -\frac{\sqrt{3}}{4} & \frac{3}{4} \end{bmatrix}$$

For element 2; $\theta = 180°$

$$[k^{(2)}] = \frac{AE}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For element 3; $\theta = 210°$

$$[k^{(3)}] = \frac{\sqrt{3}AE}{2L}\begin{bmatrix} \frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} & -\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} \\ -\frac{3}{4} & -\frac{\sqrt{3}}{4} & \frac{3}{4} & \frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} & \frac{1}{4} \end{bmatrix}$$

Applying the boundary conditions

$$u_2 = v_2 = u_3 = v_3 = u_4 = v_4 = 0$$

$$\begin{Bmatrix} 1000 \\ 1000 \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} \frac{1}{8}+1+\frac{3\sqrt{3}}{8} & -\frac{\sqrt{3}}{8}+0+\frac{3}{8} \\ -\frac{\sqrt{3}}{8}+0+\frac{3}{8} & \frac{3}{8}+0+\frac{\sqrt{3}}{8} \end{bmatrix}\begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix}$$

$$\begin{Bmatrix} 1000 \\ 1000 \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} 1.77 & 0.16 \\ 0.16 & 0.59 \end{bmatrix}\begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix}$$

$$\Rightarrow u_1 = \frac{422\,(100)}{1 \times 10 \times 10^6} \Rightarrow u_1 = 0.00422 \text{ in.}$$

$$\Rightarrow v_1 = \frac{1570\,(100)}{1 \times 10 \times 10^6} \Rightarrow v_1 = 0.0157 \text{ in.}$$

Element (1)

$$\begin{Bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} \frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} & \frac{1}{4} & -\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & -\frac{3}{4} & -\frac{\sqrt{3}}{4} & \frac{3}{4} \end{bmatrix}\begin{Bmatrix} 422\frac{L}{AE} \\ 1570\frac{L}{AE} \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow f_{2x} = 287 \text{ lb}$$

$$f_{2y} = -497 \text{ lb}$$

$$f^{(1)} = \sqrt{{f_{2x}}^2 + {f_{2y}}^2} \Rightarrow f^{(1)} = 5741 \text{ lb (C)}$$

$$\sigma^{(1)} = \frac{f^{(1)}}{A} = \frac{-5741}{\text{A}} \text{ psi}$$

$$\Rightarrow \sigma^{(1)} = -5741 \text{ psi (C)}$$

Element (2)

$$\begin{Bmatrix} f_{1x} \\ f_{1y} \\ f_{3x} \\ f_{3y} \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} 422\frac{L}{AE} \\ 1570\frac{L}{AE} \\ 0 \\ 0 \end{Bmatrix}$$

$$f_{3x} = -422 \text{ lb}$$

$$f_{3y} = 0 \text{ lb}$$

$$f^{(2)} = \sqrt{{f_{3x}}^2 + {f_{3y}}^2} \Rightarrow f^{(2)} = 422 \text{ lb (T)}$$

$$\sigma^{(2)} = \frac{f^{(2)}}{A} = \frac{422}{A} \text{ psi}$$

$$\Rightarrow \sigma^{(2)} = 422 \text{ psi (T)}$$

Element (3)

$$\begin{Bmatrix} f_{1x} \\ f_{1y} \\ f_{4x} \\ f_{4y} \end{Bmatrix} = \frac{\sqrt{3}AE}{2L}\begin{bmatrix} \frac{3}{4} & \frac{\sqrt{3}}{4} & -\frac{3}{4} & -\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & \frac{1}{4} & -\frac{\sqrt{3}}{4} & -\frac{1}{4} \\ -\frac{3}{4} & -\frac{\sqrt{3}}{4} & \frac{3}{4} & \frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & -\frac{1}{4} & \frac{\sqrt{3}}{4} & \frac{1}{4} \end{bmatrix}\begin{Bmatrix} 422\frac{L}{AE} \\ 1570\frac{L}{AE} \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow f_{4x} = -862.8 \text{ lb}$$

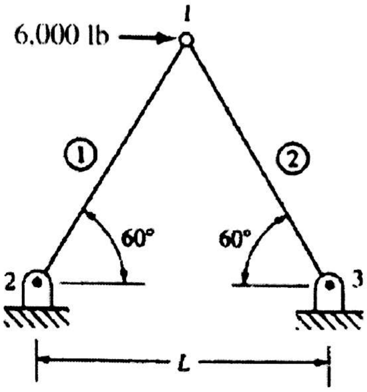
$$f_{4y} = -496 \text{ lb}$$

$$f^{(3)} = \sqrt{f_{4x}{}^2 + f_{4y}{}^2} \Rightarrow f^{(3)} = 996 \text{ lb (T)}$$

$$\sigma^{(3)} = \frac{f^{(3)}}{A} = \frac{996}{A} \text{ psi (T)}$$

$$\sigma^{(3)} = 996 \text{ psi (T)}$$

3.23



Element (1)

$$C = \frac{1}{2}; \quad S = \frac{\sqrt{3}}{2}$$

$$[k^{(1)}] = \frac{AE}{L}\left[\begin{array}{cc|cc} \frac{1}{4} & \frac{\sqrt{3}}{4} & & \\ \frac{\sqrt{3}}{4} & \frac{3}{4} & \multicolumn{2}{c}{-\lambda} \\ \hline & & \frac{1}{4} & \frac{\sqrt{3}}{4} \\ \multicolumn{2}{c|}{-\lambda} & \frac{\sqrt{3}}{4} & \frac{3}{4} \end{array}\right]$$

Element (2)

$$[k^{(2)}] = \frac{AE}{L}\left[\begin{array}{cc|cc} \frac{1}{4} & -\frac{\sqrt{3}}{4} & & \\ -\frac{\sqrt{3}}{4} & \frac{3}{4} & \multicolumn{2}{c}{-\lambda} \\ \hline & & \frac{1}{4} & -\frac{\sqrt{3}}{4} \\ \multicolumn{2}{c|}{-\lambda} & -\frac{\sqrt{3}}{4} & \frac{3}{4} \end{array}\right]$$

$$\{F\} = [K]\{d\}$$

$$\begin{Bmatrix} 6000 \\ 0 \end{Bmatrix} = \frac{AE}{L}\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{3}{2} \end{bmatrix}\begin{Bmatrix} u_1 \\ v_1 \end{Bmatrix}$$

$$\Rightarrow 6000 = \frac{AE}{L}\frac{u_1}{2}$$

$$\Rightarrow u_1 = \frac{6000 \times 100 \times 2}{1 \times 10 \times 10^6}$$

$$\Rightarrow u_1 = 0.12 \text{ in.}$$

$$v_1 = 0$$