$$v_2 + \frac{\phi_2 L}{2} - \frac{1}{2}(-\phi_1)L - v_1 = a_1 \ \frac{-1}{2}L^3$$

$$a_1 = \frac{2}{L^3} \ -v_2 - \frac{\phi_2 L}{2} - \frac{\phi_1 L}{2} + v_1$$

$$v_2 = \frac{2}{\cancel{L^3}}\left(-v_2 - \frac{\phi_2 L}{2} - \frac{\phi_1 L}{2} + v_1\right)\cancel{L^3} + a_2 L^2 - \phi_1 L + v_1$$

$$a_2 L^2 = \ v_2 + 2v_2 + \phi_2 L + \phi_1 L - 2v_1 + \phi_1 L - v_1$$

$$\therefore a_2 = \frac{3}{L^2}(v_2 - v_1) + \frac{(2\phi_1 + \phi_2)}{L^{\cancel{2}}}\cancel{L}$$

$$v = \left[\frac{2}{L^3} \ v_1 - v_2 \ - \frac{1}{L^2}(\phi_1 + \phi_2)\right]x^3 + \left[\frac{-3}{L^2} \ v_1 - v_2 \ + \frac{1}{L}(2\phi_1 + \phi_2)\right]x^2$$

$$- \phi_1 x + v_1$$

Note: Terms with $\phi_i s$ are opposite signs from v in Equation (4.1.4)

$$f_{1y} = V = \frac{EI}{L^3}\frac{d^3 v(0)}{dx^3} = \frac{EI}{L^3}(12v_1 - 12v_2 - 6L\phi_1 - 6L\phi_2)$$

$$m_1 = m = EI\frac{d^2 v(0)}{dx^2} = \frac{EI}{L^3}(-6Lv_1 + 6Lv_2 + 4L^2\phi_1 + 2L^2\phi_2)$$

$$f_{2y} = -V = -EI\frac{d^3 v(L)}{dx^3} = \frac{EI}{L^3}(-12v_1 + 12v_2 + 6\phi_1 L + 6\phi_2 L)$$

$$m_2 = -m = -EI\frac{d^2 v(L)}{dx^2} = \frac{EI}{L^3}(-12Lv_1 + 12Lv_2 + 6\phi_1 L^2 +$$

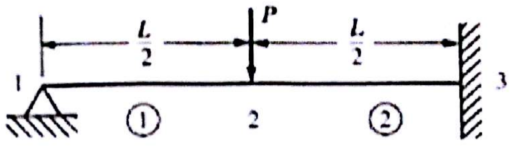
$$6\phi_2 L^2 + 6Lv_1 - 6Lv_2 - 4\phi_1 L^2 - 2\phi_2 L^2)$$

$$m_2 = \frac{EI}{L^3} \ (-6Lv_1 + 6Lv_2 + 2L^2\phi_1 + 4L^2\phi_2)$$

$$\begin{Bmatrix} f_{1y} \\ m_1 \\ f_{2y} \\ m_2 \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_1 \\ \phi_1 \\ v_2 \\ \phi_2 \end{Bmatrix}$$

Note: All 6L terms have opposite signs from Equation (4.1.13), m_1 and m_2 are now negative of previous results.

4.5



Let $\dfrac{L}{2} = l$

Element 1–2

$$[k_{1\text{-}2}] = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix};$$

Element 2–3

$$[k_{2\text{-}3}] = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

$$\begin{Bmatrix} F_{1y} = ? \\ M_1 = 0 \\ F_{2y} = -P \\ M_2 = 0 \\ F_{3y} = ? \\ M_3 = ? \end{Bmatrix} = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l & 0 & 0 \\ 6l & 4l^2 & -6l & 2l^2 & 0 & 0 \\ -12 & -6l & 24 & 0 & -12 & 6l \\ 6l & 2l^2 & 0 & 8l^2 & -6l & 2l^2 \\ 0 & 0 & -12 & -6l & 12 & -6l \\ 0 & 0 & 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}\begin{Bmatrix} v_1 = 0 \\ \phi_1 = ? \\ v_2 = ? \\ \phi_2 = ? \\ v_3 = 0 \\ \phi_3 = 0 \end{Bmatrix}$$

$$\begin{Bmatrix} 0 \\ -P \\ 0 \end{Bmatrix} = \frac{EI}{l^3}\begin{bmatrix} 4l^2 & -6l & 2l^2 \\ -6l & 24 & 0 \\ 2l^2 & 0 & 8l^2 \end{bmatrix}\begin{Bmatrix} \phi_1 \\ v_2 \\ \phi_2 \end{Bmatrix}$$

Rearrange

$$\begin{Bmatrix} 0 \\ 0 \\ \hline -P \end{Bmatrix} = \frac{EI}{l^3}\begin{array}{c} \begin{array}{cc} \alpha\alpha & \alpha\beta \end{array} \\ \left[\begin{array}{cc:c} 4l^2 & 2l^2 & -6l \\ 2l^2 & 8l^2 & 0 \\ \hdashline -6l & 0 & 24 \end{array}\right] \\ \begin{array}{cc} \beta\alpha & \beta\beta \end{array} \end{array}\begin{Bmatrix} \phi_1 \\ \phi_2 \\ \hline v_2 \end{Bmatrix}$$

Apply partition method

$$N = [k_{\beta\beta}] - [k_{\beta\alpha}][k_{\alpha\alpha}]^{-1}[k_{\alpha\beta}]$$

$$= \frac{EI}{l^3}\, 24 - [-6l \;\; 0\begin{bmatrix} 4l^2 & 2l^2 \\ 2l^2 & 8l^2 \end{bmatrix}^{-1}\begin{Bmatrix} -6l \\ 0 \end{Bmatrix}] = 13.7148\frac{EI}{l^3}$$

$$d_\beta = N^{-1}\, F \Rightarrow v_2 = \frac{l^3}{13.7148\, EI}(-P)$$

$$= \frac{-7Pl^3}{96EI} \Rightarrow \boxed{v_2 = \frac{-7PL^3}{768EI}}$$

$$\{d_\alpha\} = -[k_{\alpha\alpha}]^{-1}[k_{\alpha\beta}]\{d_\beta\}$$

$$\Rightarrow \{d_\alpha\} = \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix} = -\begin{bmatrix} 4l^2 & 2l^2 \\ 2l^2 & 8l^2 \end{bmatrix} \begin{Bmatrix} -6l \\ 0 \end{Bmatrix} \left[\frac{-7Pl^3}{96EI} \right]$$

$$= \frac{1}{l^2} \begin{bmatrix} 0.2857 & -0.0714 \\ -0.0714 & 0.1429 \end{bmatrix} \begin{Bmatrix} -6l \\ 0 \end{Bmatrix} \left[\frac{-7Pl^3}{96EI} \right]$$

$$\Rightarrow \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix} = \begin{bmatrix} -\frac{Pl^2}{8EI} \\ \frac{Pl^2}{32\,EI} \end{bmatrix} = \begin{bmatrix} -\frac{PL^2}{32\,EI} \\ \frac{PL^2}{128\,EI} \end{bmatrix}$$

Substituting back in the global matrix equation we have

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{Bmatrix} = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l & 0 & 0 \\ 6l & 4l^2 & -6l & 2l^2 & 0 & 0 \\ -12 & -6l & 24 & 0 & -12 & 6l \\ 6l & 2l^2 & 0 & 8l^2 & -6l & 2l^2 \\ 0 & 0 & -12 & -6l & 12 & -6l \\ 0 & 0 & 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{Bmatrix} 0 \\ -\frac{Pl^2}{8EI} \\ \frac{-7\,Pl^3}{96\,EI} \\ \frac{Pl^2}{32\,EI} \\ 0 \\ 0 \end{Bmatrix}$$

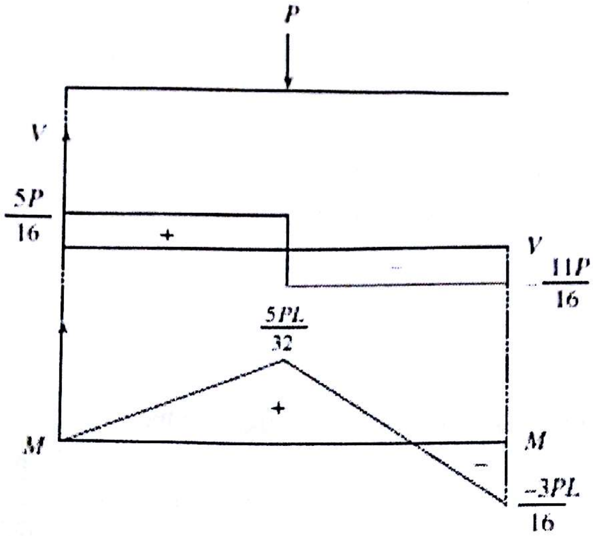
$$F_{1y} = \frac{EI}{l^3}\left[\frac{-6Pl^3}{8EI} + \frac{7Pl^3}{8EI} + \frac{6Pl^3}{32EI}\right] = \frac{EI}{l^3}\frac{10Pl^3}{32EI} \Rightarrow \boxed{F_{1y} = \frac{5P}{16}}$$

Similarly

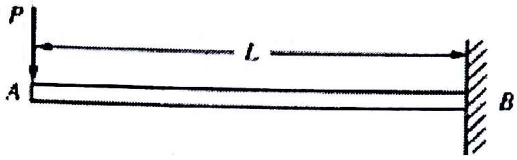
$$\boxed{M_1 = 0} \qquad \boxed{F_{3y} = \frac{11P}{16}}$$

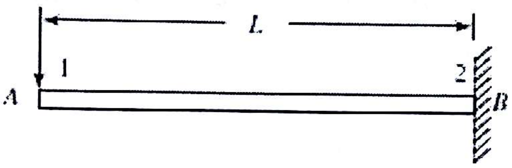
$$\boxed{F_{2y} = -P} \qquad \boxed{M_3 = \frac{-3PL}{16}}$$

$$\boxed{M_2 = 0}$$



4.6





$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & 6L & 2L^2 \\ & & 12 & -6L \\ \text{Symmetry} & & & 4L^2 \end{bmatrix}$$

Boundary conditions

$$v_2 = \phi_2 = 0$$

$$\begin{Bmatrix} -P \\ 0 \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L \\ 6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_1 \\ \phi_1 \end{Bmatrix} \Rightarrow \boxed{\phi_1 = \frac{PL^2}{2EI}}$$

$$\boxed{v_1 = \frac{-PL^3}{3EI}}$$

Matrix forces

$$\begin{Bmatrix} F_{1x} \\ M_1 \\ F_{2y} \\ M_2 \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & 12 & -6L \\ \text{Symmetry} & & & 4L^2 \end{bmatrix}\begin{Bmatrix} \frac{-PL^3}{3EI} \\ \frac{PL^2}{2EI} \\ 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \; F_{1y} = \frac{EI}{L^3}\left[-12\left(\frac{-PL^3}{3EI}\right) + 6L\left(\frac{-PL^2}{2EI}\right)\right]$$

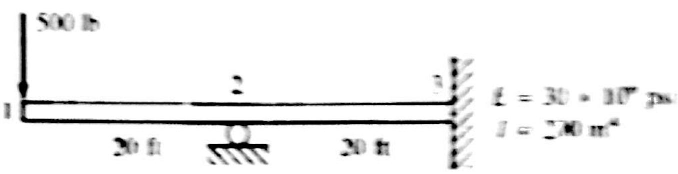
$$\Rightarrow \; F_{1y} = -P$$

Similarly $\quad M_1 = 0$

$F_{2y} = P$

$M_2 = -PL$

4.7



$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ & 4L^2 & -6L & 2L^2 & 0 & 0 \\ & & 24 & 0 & -12 & 6L \\ & & & 8L^2 & -6L & 2L^2 \\ & & & & 12 & -6L \\ \text{Symmetry} & & & & & 4L^2 \end{bmatrix}$$

$E = 30 \times 10^6, \quad I = 200 \text{ in.}^4, \quad L = 20 \text{ ft} = 240 \text{ in.}$

$$\{F\} = [K]\{d\} \;\Rightarrow\; \begin{Bmatrix} F_{1y} = -10 \\ M_1 = 0 \\ F_{2y} = ? \\ M_2 = 0 \\ F_{3y} = ? \\ M_3 = ? \end{Bmatrix} = [K]\begin{Bmatrix} v_1 \\ \phi_1 \\ v_2 \\ \phi_2 \\ v_3 \\ \phi_3 \end{Bmatrix} \text{ where } v_2 = v_3 = \phi_3 = 0$$

$$\begin{Bmatrix} -500 \\ 0 \\ 0 \end{Bmatrix} = \frac{30 \times 10^6 (200)}{(240)^3}\begin{bmatrix} 12 & 6L & 6L \\ 6L & 4L^2 & 2L^2 \\ 6L & 2L^2 & 8L^2 \end{bmatrix}\begin{Bmatrix} v_1 \\ \phi_1 \\ \phi_2 \end{Bmatrix}$$

Solving for the displacements we have

$\phi_1 = 0.0072$ rad, $\phi_2 = 0.0024$ rad, $v_1 = -1.344$ in.

Substituting in the equation $\{F\} = [K]\{d\}$ we have

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{Bmatrix} = \frac{30\times 10^6(200)}{(240)} \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ & 4L^2 & -6L & 2L^2 & 0 & 0 \\ & & 24 & 0 & -12 & 6L \\ & & & 8L^2 & -6L & 2L^2 \\ & & & & 12 & -6L \\ & \text{Symmetry} & & & & 4L^2 \end{bmatrix} \begin{Bmatrix} -1.344 \text{ in.} \\ 0.0072 \\ 0 \\ 0.0024 \\ 0 \\ 0 \end{Bmatrix}$$

$F_{1y} = -500$, $M_1 = 0$,

$F_{2y} = 1250$ lb, $M_2 = 0$, $F_{3y} = -750$ lb, $M_3 = 60$ kip-in.

Element 1–2

$$\begin{Bmatrix} f_{1y} \\ m_1 \\ f_{2y} \\ m_2 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & 12 & -6L \\ & & & 4L^2 \end{bmatrix} \begin{Bmatrix} -1.344 \\ 0.0072 \\ 0 \\ 0.0024 \end{Bmatrix}$$

$$\begin{aligned} f_{1y} &= -500 \text{ lb} \\ m_1 &= 0 \\ f_{2y} &= 500 \text{ lb} \\ m_2 &= -12000 \text{ lb-in.} \end{aligned}$$

Element 2–3

$$\begin{Bmatrix} f_{2y} \\ m_2 \\ f_{3y} \\ m_3 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & 12 & -6L \\ & & & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.0024 \\ 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{aligned} f_{2y} &= 750 \text{ lb} \\ m_2 &= 120{,}000 \text{ lb-in.} \\ f_{3y} &= -750 \text{ lb} \\ m_3 &= 60{,}000 \text{ lb-in.} \end{aligned}$$

$$\phi_3 = -0.0135 \text{ rad}$$

(B) in (A)

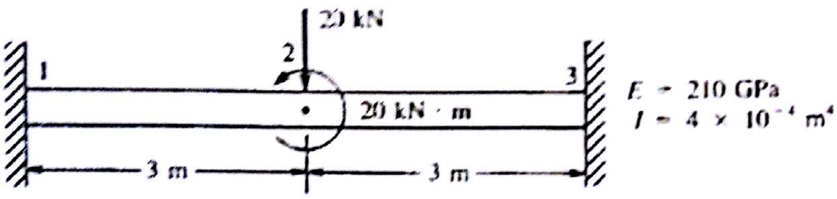
$$\Rightarrow F_{1y} = \frac{(10\times10^6)(200)}{60^3(1000)}[-24(-0.315)+720(-0.009)]$$

$$= 10 \text{ kip}$$

$$M_1 = \frac{(10\times10^6)(200)}{60^3(1000\times12)}[-720(-0.315)+14400(-0.009)]$$

$$= 75 \text{ kip}\cdot\text{ft}$$

4.10



$$[k_{1\text{-}2}] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & 12 & -6L \\ \text{Symmetry} & & & 4L^2 \end{bmatrix}$$

$$[k_{2\text{-}3}] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & 12 & -6L \\ \text{Symmetry} & & & 4L^2 \end{bmatrix}$$

Boundary conditions

$$v_1 = \phi_1 = v_3 = \phi_3 = 0$$

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 24 & 0 \\ 0 & 8L^2 \end{bmatrix}$$

$$\begin{Bmatrix} F_{2y} = -20000 \text{ N} \\ M_2 = 20000 \text{ N}\cdot\text{m} \end{Bmatrix} = \frac{(210\times10^9)(4\times10^{-4})}{(3)^3}\begin{bmatrix} 24 & 0 \\ 0 & 8L^2 \end{bmatrix}\begin{Bmatrix} v_2 \\ \phi_2 \end{Bmatrix}$$

$$-0.006428 = 24\, v_2 \Rightarrow v_2 = -2.68\times10^{-4} \text{ m}$$

$$0.0064285 = 72\, \phi_2 \Rightarrow \phi_2 = 8.93\times10^{-5} \text{ rad}$$

$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{Bmatrix} = \frac{(210\times10^9)(4\times10^{-4})}{3} \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -2.68\times10^{-4} \\ 8.93\times10^{-5} \\ 0 \\ 0 \end{Bmatrix}$$

$\Rightarrow F_{1y} = 3.1\times10^6(-12(-2.68\times10^{-4}) + 6(3)(8.93\times10^{-5})) \Rightarrow F_{1y} = 15000$ N

$M_1 = 3.1\times10^6(-6(3)(-2.68\times10^{-4}) + 2(3)^2(8.93\times10^{-5})) \Rightarrow M_1 = 20000$ N·m

Similarly

$F_{2y} = -20000$ N

$M_2 = 20000$ N·m

$F_{3y} = 5000$ N

$M_3 = -10000$ N·m

Element 1–2

$$\begin{Bmatrix} f_{1y} \\ m_1 \\ f_{2y} \\ m_2 \end{Bmatrix} = \frac{(210\times10^9)(4\times10^{-4})}{(3)^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -2.68\times10^{-4} \\ 8.93\times10^{-5} \end{Bmatrix}$$

$\Rightarrow f_{1y} = 15000$ N

$m_1 = 20000$ N·m

$f_{2y} = -15000$ N

$m_2 = 25000$ N·m

Element 2–3

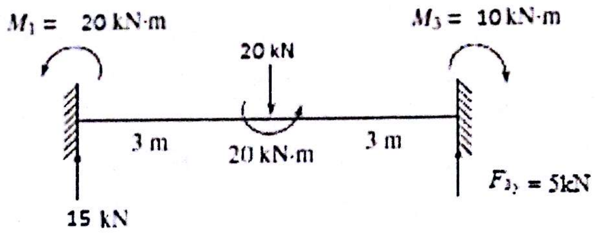
$$\begin{Bmatrix} f_{2y} \\ m_2 \\ f_{3y} \\ m_3 \end{Bmatrix} = \frac{(210\times10^9)(4\times10^{-4})}{(3)^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} -1.34\times10^{-4} \\ 8.93\times10^{-5} \\ 0 \\ 0 \end{Bmatrix}$$

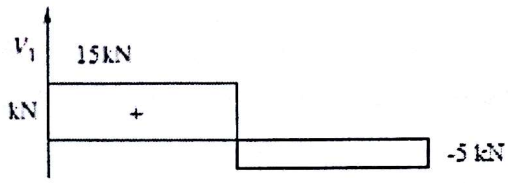
$\Rightarrow f_{2y} = -5000$ N

$m_2 = -5000$ N·m

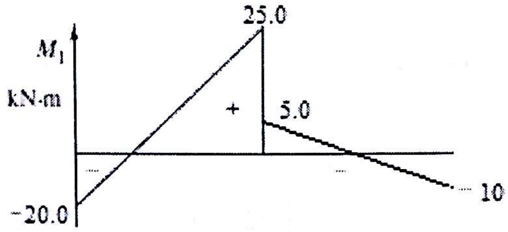
$f_{3y} = 5000$ N

$m_3 = -10000$ N·m



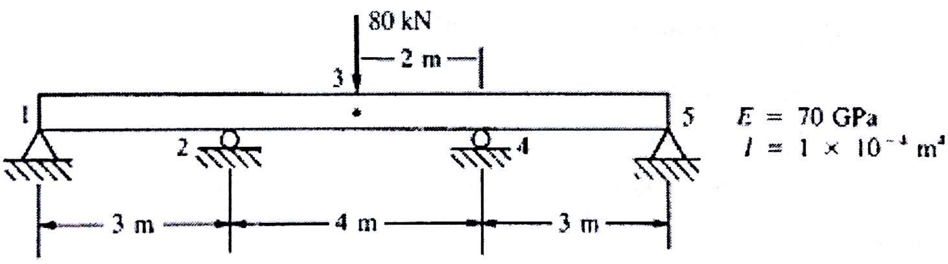


Shear diagram

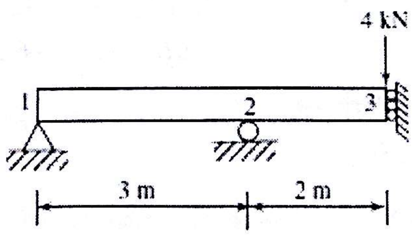


Moment diagram

4.11



Using symmetry



$$[k_{1-2}] = EI \begin{bmatrix} \frac{12}{27} & \frac{6}{9} & \frac{-12}{27} & \frac{6}{9} \\ & \frac{4}{3} & \frac{-6}{9} & \frac{2}{3} \\ & & \frac{12}{27} & \frac{-6}{9} \\ \text{Symmetry} & & & \frac{4}{3} \end{bmatrix}$$

$$\begin{Bmatrix} f_{1y} \\ m_1 \\ f_{2y} \\ m_2 \end{Bmatrix} = (70 \times 10^9)\,(1 \times 10^{-4}) \begin{bmatrix} \frac{4}{9} & \frac{2}{3} & \frac{-4}{9} & \frac{2}{3} \\ \frac{2}{3} & \frac{4}{3} & \frac{-2}{3} & \frac{2}{3} \\ \frac{-4}{9} & \frac{-2}{3} & \frac{4}{9} & \frac{-2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{-2}{3} & \frac{4}{3} \end{bmatrix} \begin{Bmatrix} 0 \\ 1.904 \times 10^{-3} \\ 0 \\ -3.809 \times 10^{-3} \end{Bmatrix}$$

$$\Rightarrow f_{1y} = -8890 \text{ N}$$
$$m_1 = 0$$
$$f_{2y} = 8890 \text{ N}$$
$$m_2 = -26670 \text{ N}\cdot\text{m}$$

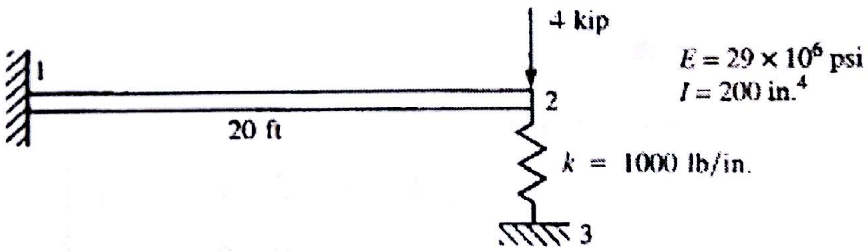
Element 2–3

$$\begin{Bmatrix} f_{2y} \\ m_2 \\ f_{3y} \\ m_3 \end{Bmatrix} = (70 \times 10^9)\,(1 \times 10^{-4}) \begin{bmatrix} \frac{3}{2} & \frac{3}{2} & \frac{-3}{2} & \frac{3}{2} \\ \frac{3}{2} & 2 & \frac{-3}{2} & 1 \\ \frac{-3}{2} & \frac{-3}{2} & \frac{3}{2} & \frac{-3}{2} \\ \frac{3}{2} & 1 & \frac{-3}{2} & 2 \end{bmatrix} \begin{Bmatrix} 0 \\ -3.809 \times 10^{-3} \\ -7.619 \times 10^{-3} \\ 0 \end{Bmatrix}$$

$$\Rightarrow f_{2y} = 40000 \text{ N} = -f_{3y}$$
$$m_2 = 26670 \text{ N}\cdot\text{m}, \quad m_3 = 53330 \text{ N}\cdot\text{m}$$

Elements 3–4 and 4–5 have same forces due to symmetry. Moments though will have opposite signs.

4.12



$$[K] = \frac{EI}{L^3} \begin{array}{c} \begin{array}{cccc} v_1 & \phi_1 & v_2 & \phi_2 \end{array} \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & 12 + \frac{KL^3}{EI} & -6L \\ \text{Symmetry} & & & 4L^2 \end{bmatrix} \end{array} \begin{array}{c} v_1 \\ \phi_1 \\ v_2 \\ \phi_2 \end{array}$$

Boundary conditions

$$v_1 = \phi_1 = 0$$

Applying the boundary conditions on equation $\{F\} = [K]\,\{d\}$

$$\begin{Bmatrix} F_2 = -4000 \text{ lb} \\ M_2 = 0 \end{Bmatrix} = \frac{(29 \times 10^6)(200)}{(20 \times 12)^3} \begin{bmatrix} 12 + \frac{KL^3}{EI} & -6L \\ -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} v_2 \\ \phi_2 \end{Bmatrix}$$

$$0 = -6(240)v_2 + 4(240)^2\,\phi_2$$

$$\Rightarrow 0 = -6v_2 + 4(240)\phi_2 \Rightarrow v_2 = 160\phi_2$$

$$-4000 = 419.56\,[(12 + (2.38)\,160\phi_2 - 6(240)\phi_2]$$

$$\Rightarrow \quad \phi_2 = -0.01106 \text{ rad}$$

$$\Rightarrow \quad v_2 = 160(-0.001106) \Rightarrow v_2 = -1.772 \text{ in.}$$

Beam element

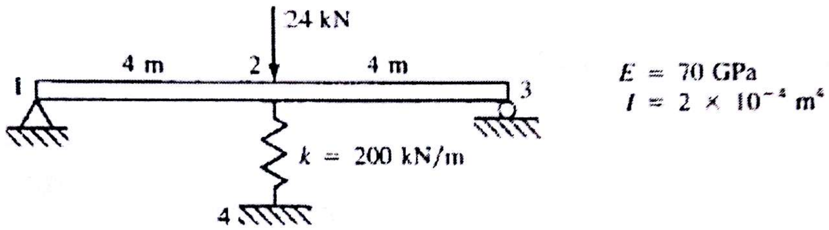
$$\begin{Bmatrix} f_{1y} \\ m_1 \\ f_{2y} \\ m_2 \end{Bmatrix} = \frac{(29\times10^6)(200)}{(20\times12)^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ -1.772 \\ -0.01106 \end{Bmatrix}$$

$$f_{1y} = 2230 \text{ lbs} \uparrow,\ m_1 = -534 \text{ kip}\cdot\text{in.}$$

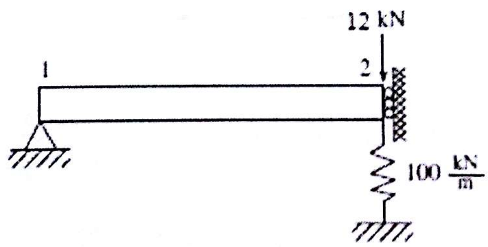
$$f_{2y} = -2230 \text{ lbs} \downarrow,\ m_2 = 0$$

The extra force at node 2 is resisted by the spring, ie., $F_s = -kv_2 = -1000\,(-1.772) =$ 1770 lb

4.13



Applying symmetry



$$[K] = \frac{EI}{L_3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ & 4L^2 & -6L & 2L^2 \\ & & \frac{12+KL^3}{EI} & -6L \\ \text{Symmetry} & & & 4L^2 \end{bmatrix}$$

Applying the boundary conditions $v_1 = 0$, $\phi_2 = 0$ we have

$$\begin{Bmatrix} M_1 = 0 \\ F_{2y} = -12000 \text{ N} \end{Bmatrix} = \frac{(70\times10^9)(2\times10^{-4})}{4^3}\begin{bmatrix} 4L^2 & -6L \\ -6L & \frac{12+KL^3}{EI} \end{bmatrix}\begin{Bmatrix} \phi_1 \\ v_2 \end{Bmatrix}$$