$$\begin{Bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ -1.2569 \\ -0.003491 \\ 0 \\ 0.01396 \end{Bmatrix}$$

$$- \begin{Bmatrix} \frac{-wL}{2} = -7500 \text{ lb} \\ \frac{-wL^2}{12} = -225{,}000 \text{ lb}\cdot\text{in.} \\ -wL = -15{,}000 \text{ lb} \\ 0 \\ \frac{-wL}{2} = -7500 \text{ lb} \\ \frac{wL^2}{12} = 225{,}000 \text{ lb}\cdot\text{in.} \end{Bmatrix}$$

$F_{1y} =$ 18750 lbs, $M_1 = 1350$ lb·in.

$F_{2y} = 0$, $M_2 = 0$

$F_{3y} =$ 11250 lb, $M_3 = 0$

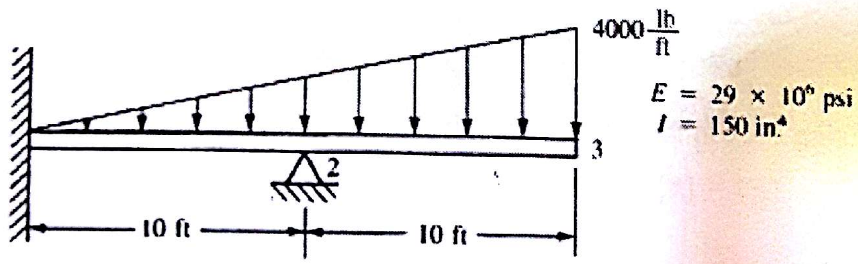
Element 1–2

$$\begin{Bmatrix} f_{1y} \\ m_1 \\ f_{2y} \\ m_2 \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ -1.2569 \\ -0.003491 \end{Bmatrix} - \begin{Bmatrix} -7500 \\ -225000 \\ -7500 \\ 225000 \end{Bmatrix} \Rightarrow \begin{matrix} f_{1y} = 18750 \text{ lb} \\ m_1 = 1350 \text{ k}\cdot\text{in.} \\ f_{2y} = -18750 \text{ lb} \\ m_2 = 675 \text{ k}\cdot\text{in.} \end{matrix}$$

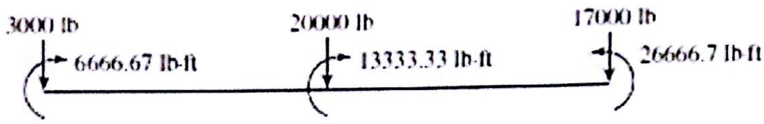
Element 2–3

$$\begin{Bmatrix} f_{2y} \\ m_2 \\ f_{3y} \\ m_3 \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} -1.2569 \\ -0.003491 \\ 0 \\ 0.01396 \end{Bmatrix} - \begin{Bmatrix} -7500 \\ -225000 \\ -7500 \\ 225000 \end{Bmatrix} \Rightarrow \begin{matrix} f_{2y} = 3750 \text{ lb} \\ m_2 = -675 \text{ k}\cdot\text{in.} \\ f_{3y} = 11250 \text{ lb} \\ m_3 = 0 \end{matrix}$$

4.24



$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ & 4L^2 & -6 & 2L^2 & 0 & 0 \\ & & 24 & 0 & -12 & 6L \\ & & & 8L^2 & -6L & 2L^2 \\ & & & & 12 & -6L \\ \text{Symmetry} & & & & & 4L^2 \end{bmatrix}$$



After applying the boundary conditions

$$v_1 = \phi_1 = v_2 = 0 \text{ in } \{F_0\} = [K]\,\{d\}$$

$$\begin{Bmatrix} -13333.33 \text{ ft} \cdot \text{lb} \\ -17000 \text{ lb} \\ 26666.67 \text{ ft} \cdot \text{lb} \end{Bmatrix} = 30208.33 \begin{bmatrix} 8L^2 & -6L & 2L^2 \\ -6L & 12 & -6L \\ 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} \phi_2 \\ v_3 \\ \phi_3 \end{Bmatrix} \begin{matrix} (1) \\ (2) \\ (3) \end{matrix}$$

Rewriting equations (1) (2) and (3) we get

$$-0.441379 = 8L^2\phi_2 - 6Lv_3 + 2L^2\phi_3 \qquad (1)$$

$$-0.562759 = -6L\phi_2 + 12v_3 - 6L\phi_3 \qquad (2)$$

$$0.882759 = 2L^2\phi_2 - 6Lv_3 + 4L^2\phi_3 \qquad (3)$$

Adding (1) to – 4 × (3) we get

$$-0.441379 = 8L^2\phi_2 - 6Lv_3 + 2L^2\phi_3$$

$$-3.53103 = -8L^2\phi_2 + 24Lv_3 - 16L^2\phi_3$$

$$-3.9724 = 18Lv_3 - 14L^2\phi_3 \qquad (4)$$

Adding L × (2) to 3 × (3) we get (where L = 10 ft)

$$-5.62759 = -6L^2\phi_2 + 12Lv_3 - 6L^2\phi_3$$

$$2.64827 = 6L^2\phi_2 - 18Lv_3 + 12L^2\phi_3$$

$$-297931 = -6Lv_3 + 6L^2\phi_3 \qquad (5)$$

Adding Equation (4) to 3 × (5) we have

$$-12.91034 = 4L^2\phi_3 \;\Rightarrow\; \boxed{\phi_3 = -3.22758 \times 10^{-2} \text{ rad}}$$

Substituting in (4)

$$\Rightarrow\; -3.9724 = 180v_3 - 1400\,(-3.22758 \times 10^{-2})$$

$$\Rightarrow\; \boxed{v_3 = -2.73103 \times 10^{-1} \text{ ft} = -3.27724 \text{ in.}}$$

Substituting in (1)

$$\Rightarrow -0.441379 = 8L^2\phi_2 - 6L(-2.73103 \times 10^{-1}) + 2L^2(-3.22758)$$

$$\Rightarrow \quad \boxed{\phi_2 = -1.29655 \times 10^{-2} \text{ rad}}$$

$$F_{1y}^{(e)} = \frac{6EI}{L^2}\phi_2 = \frac{6(29\times 10^6)(150 \text{ in.}^4)}{(120)\text{in.}^2}(-1.29655 \times 10^{-2}) = -23500 \text{ lb}$$

$$M_1^{(e)} = \frac{2EI}{L}\phi_2 = \frac{2(29\times 10^6)(150 \text{ in.}^4)}{120 \times 12''}(-1.29655 \times 10^{-2}) = 78333 \text{ lb}\cdot\text{ft}$$

$$F_{2y}^{(e)} = \frac{-12EI}{L^3}v_3 + \frac{6EI}{L^2}\phi_3$$

$$= \frac{-12(29\times 10^6)(150)}{120\times 120\times 120}(-3.27724) + \frac{6(29\times 10^6)}{(120)^2} \times 150$$

$$\times (-3.22758 \times 10^{-1}) = 40500 \text{ lb}$$

$$M_2^{(e)} = \frac{8L^2EI}{L^3}\phi_2 - \frac{6LEI}{L^3}v_2 + \frac{2L^2EI}{L^3}\phi_3 = -13333.33 \text{ ft}\cdot\text{lb}$$

$$F_{3y}^{(e)} = \frac{-6LEI}{L^3}\phi_2 + \frac{12LEI}{L^3}v_3 - \frac{6LEI}{L^3}\phi_3 = -17000 \text{ lb}$$

$$M_3^{(e)} = \frac{2L^2EI}{L^3}\phi_2 - \frac{6LEI}{L^3}v_3 + \frac{4L^2EI}{L^3}\phi_3 = 26{,}666.67 \text{ ft}\cdot\text{lb}$$

Global forces $\{F\} = \{F^{(e)}\} - \{F_0\}$

$F_{1y} = -23500 + 3000 = -20500$ lb

$M_1 = -78333.33 + 6666.67 = -71{,}666.67$ ft·lb

$F_{2y} = 40{,}500 + 20{,}000 = 60500$ lb

$M_2 = -13333.33 + 13333.33 = 0$

$F_{3y} = -17000 + 17000 = 0$

$M_3 = 26666.67 - 26666.67 = 0$

Element 1–2	Element 2–3
$f_{1y} = -20500$ lb	$f_{2y} = -30{,}000$ lb
$m_1 = -71666.67$ ft·lb	$m_2 = 2000$ kip·in.
$f_{2y} = 30{,}500$ lb	$f_{3y} = 0$
$m_2 = -2000$ kip·in.	$m_3 = 0$

4.25

$$\begin{Bmatrix} f_{1y} \\ m_1 \\ f_{2y} \\ m_2 \end{Bmatrix} = 11.574 \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -2.338 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -5000 \\ -200000 \\ -5000 \\ 200000 \end{Bmatrix}$$

$\Rightarrow f_{1y} = 5325 \text{ lb}, \; m_1 = 19914 \text{ lb}\cdot\text{ft}$

$f_{2y} = 4675 \text{ lb}, \; m_2 = -13419 \text{ lb}\cdot\text{ft}$

Element 2–3

$f_{2y} = 4675 \text{ lb}$

$m_2 = 13419 \text{ lb}\cdot\text{ft}$

from symmetry

$f_{3y} = 5325 \text{ lb}$

$m_3 = -19914 \text{ lb}\cdot\text{ft}$

Note: Spring force is

$$F_s = \left(4000 \frac{\text{lb}}{\text{in.}}\right)(2.338 \text{ in.}) = 9352 \text{ lb}$$

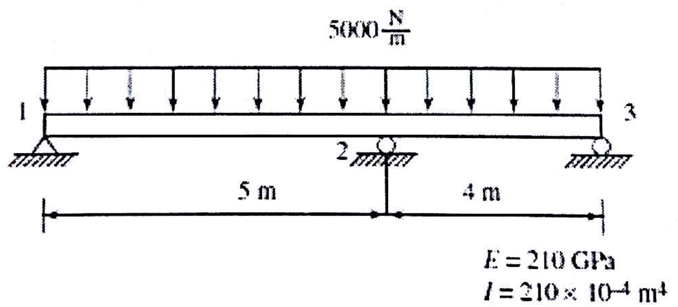
Equilibrium at node 2, $\Sigma F_y = 0$

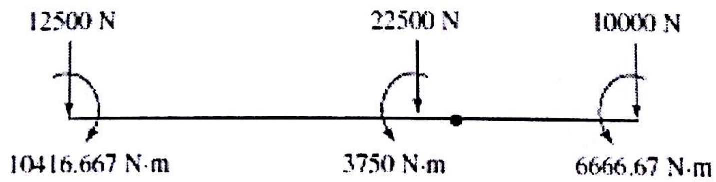
$\downarrow$ 4675 lb from element 1

$\downarrow$ 4675 lb from element 2

$\uparrow F_s = 9352$ lb

4.26





$v_1 = 0 = v_2 = v_3$

$$[k_{1\text{-}2}] = EI \begin{bmatrix} \frac{4}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix} \begin{matrix} \phi_1 \\ \phi_2 \end{matrix}$$

$$[k_{2\text{-}3}] = EI \begin{bmatrix} \frac{4}{4} & \frac{2}{4} \\ \frac{2}{4} & \frac{4}{4} \end{bmatrix} \quad (\phi_2 \;\; \phi_3)$$

$$\{F_0\} = [K]\{d\}$$

$$\begin{Bmatrix} -10416.667 \\ 3750 \\ 6666.67 \end{Bmatrix} = (210 \times 10^9)(2 \times 10^{-4}) \begin{bmatrix} 0.8 & 0.4 & 0 \\ 0.4 & 1.8 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{Bmatrix}$$

$$-10416.667 = (210 \times 10^9)(2 \times 10^{-4})[0.8\phi_1 + 0.4\phi_2) \quad (1)$$

$$3750 = (210 \times 10^9)(2 \times 10^{-4})[0.4\phi_1 + 1.8\phi_2 + 0.5\phi_3] \quad (2)$$

$$6666.67 = (210 \times 10^9)(2 \times 10^{-4})[0.5\phi_2 + \phi_3] \quad (3)$$

Multiplying – 2 x (2) and adding it to (1) we have

$$-10416.667 = 4.2 \times 10^7 [0.8\phi_1 + 0.4\phi_2]$$

$$-7500 = 4.2 \times 10^7 [-0.8\phi_1 - 3.6\phi_2 - \phi_3]$$

$$-17916.667 = 4.2 \times 10^7 [-3.2\phi_2 - \phi_3] \quad (4)$$

Adding (3) to (4) we have

$$-17916.667 = 4.2 \times 10^7 [-3.2\phi_2 - \phi_3]$$

$$6666.667 = 4.2 \times 10^7 [0.5\phi_2 + \phi_3]$$

$$-11250 = 4.2 \times 10^7 (-2.7\phi_2)$$

$$\Rightarrow \boxed{\phi_2 = 9.92 \times 10^{-5} \text{ rad}}$$

Substituting into (4) we have

$$-17916.667 = 4.2 \times 10^7 [-3.2(9.92 \times 10^{-5}) - \phi_3]$$

$$\Rightarrow \boxed{\phi_3 = 1.091 \times 10^{-4} \text{ rad}}$$

Substituting in (1)

$$\Rightarrow -10416.667 = 4.2 \times 10^7 [0.8\phi_1 + 0.4(9.92 \times 10^{-5})]$$

$$\Rightarrow \boxed{\phi_1 = -3.596 \times 10^{-4} \text{ rad}}$$

Element 1–2

$$\begin{Bmatrix} f_{1y}^{(e)} \\ m_1^{(e)} \\ f_{2y}^{(e)} \\ m_2^{(2)} \end{Bmatrix} = \frac{(210 \times 10^9)(2 \times 10^{-4})}{5^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ -3.596 \times 10^{-4} \\ 0 \\ 9.92 \times 10^{-5} \end{Bmatrix}$$

$$\Rightarrow \hat{f}_{1y}^{(e)} = -2625 \text{ N}$$

$$m_1^{(e)} = -10416.67 \text{ N}\cdot\text{m}$$

$$f_{2y}^{(e)} = 2625 \text{ N}$$

$m_2^{(e)} = -2708.33\ \text{N}\cdot\text{m}$

$$\begin{Bmatrix} f_{1y} \\ m_1 \\ f_{2y} \\ m_2 \end{Bmatrix} = \begin{Bmatrix} -2625 \\ -10416.667 \\ 2625 \\ -2708.33 \end{Bmatrix} - \begin{Bmatrix} -12500 \\ -10416.667 \\ -12500 \\ -10416.667 \end{Bmatrix}$$

$\Rightarrow f_{1y} = 9875\ \text{N},\ m_1 = 0$

$f_{2y} = 15125\ \text{N},\ m_2 = -13125\ \text{N}\cdot\text{m}$

Element 2–3

$$\begin{Bmatrix} f_{2y}^{(e)} \\ m_2^{(e)} \\ f_{3y}^{(e)} \\ m_3^{(e)} \end{Bmatrix} = \frac{(210\times10^9)(2\times10^{-4})}{4^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} 0 \\ 9.92\times10^{-5} \\ 0 \\ -1.091\times10^{-4} \end{Bmatrix}$$

$\Rightarrow f_{2y} = 13281.25\ \text{N}$

$m_2 = 13125\ \text{N}\cdot\text{m}$

$f_3 = 6718.5\ \text{N}$

$m_3 = 0$

Global

$F_{1y} = f_{1y} = 9875\ \text{N}$

$M_1 = m_1 = 0$

$F_{2y} = 1525 + 13281.25 = 28406.25\ \text{N}$

$M_2 = -13125 + 13125 = 0$

$F_{3y} = f_{3y} = 6718.75\ \text{N}$

$M_3 = m_3 = 0$

4.27

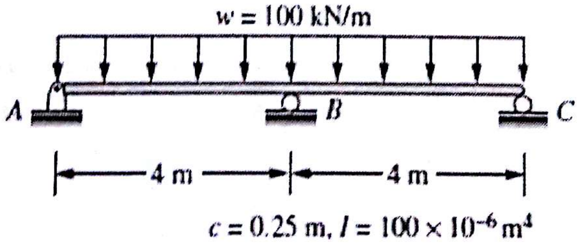


Figure P4–27

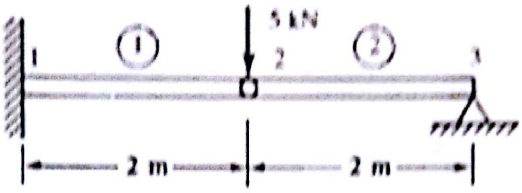
$$\Rightarrow \quad \boxed{\phi_2^{(2)} = 1.19 \times 10^{-4} \text{ rad}}$$

$$\therefore \quad v_2 = -\frac{4}{3}\phi_2$$

$$= -\frac{4}{3} \times 1.19 \times 10^{-4}$$

Hence $\boxed{v_2 = -1.57 \times 10^{-4} \text{ m}}$

4.43



$$[k^{(1)}] = \frac{3EI}{8}\begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & 4 & -2 & 0 \\ -1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[k^{(2)}] = \frac{EI}{8}\begin{bmatrix} 12 & 12 & -12 & 12 \\ 12 & 16 & -12 & 8 \\ -12 & -12 & 12 & -12 \\ 12 & 8 & -12 & 16 \end{bmatrix}$$

By superposition

$$\begin{matrix} v_1 & \phi_1 & v_2 & \phi_2 & v_3 & \phi_3 \end{matrix}$$

$$[K] = \frac{EI}{8}\begin{bmatrix} 3 & 6 & -3 & 0 & 0 & 0 \\ 6 & 12 & -6 & 0 & 0 & 0 \\ -3 & -6 & 15 & 12 & -12 & 12 \\ 0 & 0 & 12 & 16 & -12 & 8 \\ 0 & 0 & -12 & -12 & 12 & -12 \\ 0 & 0 & 12 & 8 & -12 & 16 \end{bmatrix}$$

Boundary conditions

$$v_1 = 0, \ \phi_1 = 0, \ v_3 = 0$$

$$\frac{EI}{8}\begin{bmatrix} 15 & 12 & 12 \\ 12 & 16 & 8 \\ 12 & 8 & 16 \end{bmatrix}\begin{Bmatrix} v_3 \\ \phi_2 \\ \phi_3 \end{Bmatrix} = \begin{Bmatrix} -5000 \\ 0 \\ 0 \end{Bmatrix}$$

Solving by Gaussian Elimination we have

$$\left[\begin{array}{ccc|c} 15 & 12 & 12 & -9.5\times10^{-4} \\ 12 & 16 & 8 & 0 \\ 12 & 8 & 16 & 0 \end{array}\right]$$

Select $a_{11} = 15$ as the pivot

(a) Add the multiple $\frac{-a_{21}}{a_{11}} = \frac{-12}{15} = \frac{-4}{5}$ of the first row to the second row

(b) Add the multiple $\frac{-a_{31}}{a_{11}} = \frac{-4}{5}$ of the first row to the third row.

$$\left[\begin{array}{ccc|c} 15 & 12 & 12 & -9.5\times10^{-4} \\ 0 & 6.4 & -1.6 & 7.6\times10^{-4} \\ 0 & -1.6 & 6.4 & 7.6\times10^{-4} \end{array}\right]$$

Select $a_{22} = 6.4$ as the pivot

Repeating the same procedure

$$\begin{bmatrix} 15 & 12 & 12 & -9.5\times10^{-4} \\ 0 & 6.4 & -1.6 & 7.6\times10^{-4} \\ 0 & 0 & 6 & 9.5\times10^{-4} \end{bmatrix}$$

$$\Rightarrow \quad \underline{\underline{\phi_3 = 1.583\times10^{-4}\ \text{rad}}}$$

$$\underline{\underline{\phi_2 = 1.583\times10^{-4}\ \text{rad}}}$$

$$\underline{\underline{v_2 = -3.175\times10^{-4}\ \text{m}}}$$